Military Procurement, Industry Structure and the Revolution in Military Affairs*

Paul Dunne
University of the West of England

Maria Garcia-Alonso
University of Kent

Paul Levine
University of Surrey

Ron Smith
Birkbeck College

October 13, 2003

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Abstract

We examine a number of developments associated with the RMA, using a partial equilibrium model of military procurement with two-way international trade in a world where many of the recipients of this trade are non-producers engaged in regional arms races. We study the consequences for industry structure, procurement subsidies and the welfare of producers of: increased process innovation and the consequent rise in the fixed costs of R&D; increased trade between producers owing to the willingness to forgo the security benefits of domestic procurement; and a growing external market of non-producers. Our comparison of the non-cooperative and cooperative outcomes highlights two sources of inefficiencies from the position of the producers. Adopting a generalized Dixit-Stiglitz utility function to model military strength, the existence of a separate taste-for-variety parameter means governments acting independently choose to support too few firms and generate too little variety. The second source is from competition for the external market and here there are two effects. First export revenue from exports rises as the total number of firms falls. This occurs because firms then compete less intensively and can spread their fixed costs over a larger market share. As the external market increases, governments then choose to support *less* firms. Under non-cooperation, however, this reduction in firm number is too little compared with the optimum because governments acting independently only care about competition between domestic firms. The second effect of an external market is to encourage too much investment in quality relative to the cooperative outcome. This is because the provision of quality has a beggar-thy-neighbour character. When governments raise quality unilaterally this increases market share. In equilibrium however the benefit to competitiveness disappears and countries are left with too much quality compared with that chosen cooperatively. A high investment into this quality raises fixed costs and reduces firm number further. The presence of an external market then tends to reduce firm number (i.e., raise concentration) and encourages excessive investment into quality. A high taste-for-variety has the opposite effect, raising firm numbers which, because there is a trade-off between quality and variety, reduces quality.

JEL Classification: F12, H56, L10

Keywords: Revolution in Military Affairs, military procurement, defence industry structure, arms trade

1 Introduction

In this paper, we construct a integrated model of military procurement, technological change, international trade and regional conflict. This work builds of previous work by the authors[1] and develops it in a number of directions. First, we draw upon an important recent literature[2] concerned with endogenous property rights, where agents allocate resources to attack or defence as well as to production and trade. The resulting distribution of property reflects agents ability to protect their resources from others or steal resources from others. In this literature the analogy to the production function is the conflict success function (CSF): the inputs are the fighting efforts of the two sides and the outputs are their relative degree of success: proportion of the stakes won, in the continuously divisible case or probability of winning, in an all or nothing case. Our use of CSFs to model the allocation of resources for for military goods provides rigorous micro-foundations for the concept of security, a feature that was lacking in earlier work.

The second development of work is aimed at addressing a number of important aspects of the ways in which new technologies and globalization are transforming fighting and the defence industry, the 'Revolution in Military Affairs'. Increasingly, the military is characterized by process innovation and fixed costs of R&D are escalating. Production is becoming concentrated in a smaller number of firms and a smaller number of producer countries. Trade in arms between producer countries is increasing with the greater willingness to forgo the security of domestic procurement. Governments are 'commercializing the military' by switching into COTS dual-use technology.

To address these issues associated with the RMA, we construct a partial equilibrium model of military procurement with two-way international trade in a world where many of the recipients of this trade are non-producers engaged in regional arms races. Governments in producer countries buy products from the domestic firms and import from the rest of the world; governments of non-producer countries do not have domestic production and therefore, cover their procurement needs through imports. Producer country governments have an interest in maintaining a domestic sector and therefore, pay domestic firms the

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[1]Garcia-Alonso, 1999, 2000; Garcia-Alonso and Levine, 1997; Levine et al, 1994; Levine and Smith, 1995, 1997a, 1997b, 2000a, 2000b; Levine et al, 2000.

[2]See, for example, Anderton (2000), Hirchleifer (2000), Skaperdas (1991), Garfinkel and Skerpadas (2000).

price that ensures their existence, i.e., the price that makes them meet the participation constraint. However, firms are price setters in the exports market.

A basic feature of our model is that governments choose the number of firms that compose the domestic procurement sector, whose existence they will guarantee. They also choose the 'quality' of level of technology. High fixed production costs associated with the latter in particular, imply that firms' existence depends on them being government providers and therefore, the government is actually choosing the number of domestic firms. In order to explain why governments want to keep several firms as domestic suppliers within the same sector, we use a 'taste for variety model'. This type of model has been traditionally used in the monopolistic competition literature starting with Dixit and Stiglitz (1977). Our paper however, uses a more general form of the Dixit-Stiglitz utility function, discussed in Benassy (1996), which allows for the taste for variety and the elasticity of substitution between the differentiated military goods to be independent.

The rest of the paper is organized as follows. Section 2 provides the basic set-up and the sequence of moves in the procurement game with governments and firms as players. There are three stages to the procurement and trade game. First, given their military expenditure (assumed to be exogenously fixed), producer governments choose the number of domestic suppliers to support, and the amount and quality of goods to procure from each. They also formulate a subgame perfect plan to import goods at the asking, possibly out-of-equilibrium prices, which they will implement at stage 3 of the game. At stage 2 given domestic procurement decisions, firms produce and take part in a Bertrand price-setting game in the export market. At stage 3 all governments make their import decisions given possibly out-of-equilibrium prices. In addition, importing government engaged in regional arms races, choose they levels of military expenditure. In a subgame perfect equilibrium of the entire game, sections 3, 4 and 5 solve for these stages starting at stage 3.

Section 5 characterizes the equilibrium which in general can accommodate various asymmetries between both producers and non-producers. Section 6 provides analytical results for the symmetric form of this equilibrium. Section 7 compares this equilibrium with the cooperative arrangement. Section 8 illustrates the results using numerical solutions and section 9 provides some concluding remarks.

2 The Set-up

2.1 The Model

We model an international market for a public service good, consisting of ℓ producing and importing countries and r non-producers who only import. The total budget in each country available for this particular public service good is given. Producer country 1 produces differentiated goods $j = 1, 2, \cdots, n_1$, country 2 produces goods $j = n_1 + 1, n_1 + 2, \cdots, n_1 + n_2$ etc, so there are $\sum_{i=1}^{\ell} n_i = N$, say, goods in total. Governments procure from domestic firms (if they exist) and overseas firms who enter or exit the market freely.

As well as horizontal differentiation there is vertical differentiation: each good can also be produced in an unlimited number of vertically differentiated varieties or 'qualities'. If quality increases by a proportion λ, say, then one unit of the good provides λ more services. The *maximum* quality of good j in country $i = 1, 2, \cdots, \ell$ is q_{ij} which is the quality of the domestically procured good. We assume that each firm can produce a lower quality good at the same cost and we allow for the possibility that there is an *arms export regime* in place that restricts the quality of the imported good by country i of variety j to $u_{ij} = \gamma_{ik} q_{kj}$, where q_{kj} is the quality of the domestically procured good by country k of variety $j = n_{k-1} + 1, n_{k-1} + 2, \cdots, n_{k-1} + n_k$. The parameter $\gamma_{ik} \leq 1$ captures the extent of the arms export constraint by the exporting country k on the importing country i. We take this regime to be exogenously imposed on the military authority making the procurement decisions and we do not go into details of how this regime can be sustained.[3]

It makes for a simpler presentation if we focus on decisions in producer country 1. Government 1 procures $d_{1j}, j = 1, 2, \cdots, n_1$ domestically produced goods with quality q_{1j} and $m_{1j}, j = n_1 + 1, n_1 + 2, \cdots, N$ imported goods with quality u_{1j}. Military strength takes the form of a generalized Dixit-Stigliz CES utility function of the form

$$S_1 = [w_1 n_1 + (1 - w_1)(N - n_1)]^{\nu} \left[w_1 \sum_{j=1}^{n_1} (q_{1j} d_{1j})^{\alpha} + (1 - w_1) \sum_{j=n_1+1}^{N} (u_{1j} m_{1j})^{\alpha} \right]^{\frac{1}{\alpha}} ; \ \alpha \in [0, 1), \ \nu > 0$$

(1)

---

[3]At suitable stages of the game we impose symmetry between firms in the same country. Then each country produces the same quality of each variety and with $j = k$, the export control regime imposed by country k on country i is expressed by country i importing quality $u_{ik} = \gamma_{ik} q_{kk} = \gamma_{ik} q_k$, rewriting $q_{kk} = q_k$.

In (1) the weights w_1 and $1-w_1$, with $w_1 \in [\frac{1}{2}, 1]$, express possible preferences for domestic rather than imported procurement in country 1.[4] If we put $\nu = 0$ and $w = \frac{1}{2}$, (1) reduces to the familiar Dixit-Stiglitz utility function used in the new trade and endogenous growth literatures. But as Benassy (1996) points out, this form of utility is restricted in that it implies an on-to-one correspondence between the taste for variety and the elasticity of substitution.

To see the significance of this generalized form of the Dixit-Stiglitz utility function, suppose there are two producer countries. Define a function $v_1(n_1, n_2)$ to represent the proportional capability gain from spreading a certain amount of quality-adjusted output $(n_1 + n_2)y$, say, between all $n_1 + n_2$ varieties rather than concentrating a proportion w_1 on one variety in country 1 and a proportion $1 - w_1$ on one imported variety; i.e.,

$$
\begin{aligned}
v_1(n_1, n_2) &= \frac{[w_1 n_1 + (1 - w_1)n_2]^\nu \left[w_1 \sum_{j=1}^{n_1} y^\alpha + (1 - w_1) \sum_{j=n_1+1}^{N} y^\alpha \right]^{\frac{1}{\alpha}}}{(n_1 + n_2)y} \\
&= \frac{[w_1 n_1 + (1 - w_1)n_2]^{\nu + \frac{1}{\alpha}}}{[w_1^{1+\alpha} + (1 - w_1)^{1+\alpha}]^{\frac{1}{\alpha}}(n_1 + n_2)}
\end{aligned}
$$

Suppose that the total number of varieties $N = n_1 + n_2$ increases keeping the proportion $\frac{n_1}{N} = \kappa$ fixed. Then putting $n_1 = \kappa N$ and $n_2 = (1 - \kappa)N$, $v_1 = v_1(N) = $ constant $\times$ $N^{(\nu + \frac{1}{\alpha} - 1)}$. We now define the *taste for variety* by the elasticity $\frac{Ndv_1}{v_1 dN} = \tau$ say given by

$$
\tau = \frac{Ndv_1}{v_1 dN} = \nu + \frac{1}{\alpha} - 1
$$

The significance of the extra term in (1) is now apparent. If $\nu = 0$, then the taste for variety $\tau = \frac{1}{\alpha} - 1 = \frac{1}{\sigma - 1}$ which is determined solely by the elasticity of substitution σ. Thus this formulation establishes an arbitrary link between different characteristics: taste for variety and elasticity of substitution, the latter also determining the market power since the mark-up on marginal costs in the export market is $\frac{1}{\alpha}$. Introducing the extra term breaks this link and has important consequences for the subsequent analysis.

Governments in producer countries procure from domestic and foreign firms, possibly at different prices. Let p_{1j} be the price of the procured domestic good and P_j be the world

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[4]Note that(1) can be given an 'iceberg' technology interpretation by writing it as $U_1 = [w_1 n_1 + (1 - w_1)(N - n_1)]^\nu \left[\sum_{j=1}^{n_1} (q_{1j} d_{1j})^\alpha + \sum_{j=n_1+1}^{N} (T_1 u_{1j} m_{1j})^\alpha \right]^{\frac{1}{\alpha}}$, where $T_1 = \left(\frac{1-w_1}{w_1} \right)^{\frac{1}{\alpha}}$ is the fraction of the original good that actually arrives, the rest 'melting away' on route. However we do not distinguish between trade between close and distant countries, so we do not pursue this spatial interpretation.

market price of the traded good of variety j produced by firms in all producing countries $j = 1, 2, \cdots, N$. Then the budget constraint for government in producer country 1 is:

$$\sum_{j=1}^{n_1} p_{1j} d_{1j} + \sum_{j=n_1+1}^{N} P_j m_{1j} = G_1 \tag{2}$$

where G_i is total procurement expenditure in country i.

For the non-producing country $i = \ell + 1, \ell + 2, \cdots, \ell + r$ military strength is given by

$$S_i = N^\nu \left[\sum_{j=1}^{N} (u_{ij} m_{ij})^\alpha \right]^{\frac{1}{\alpha}} \tag{3}$$

where u_{ij} is the quality allowed to importing non-producing country i. Their budget constraint is:

$$\sum_{j=1}^{N} P_j m_{ij} = G_i \tag{4}$$

The model is completed by specifying the following cost structure for the firm. Firm j produces d_j units of variety j for its domestic government at a procurement price p_j and exports x_j units at a international market price P_j. The cost of producing total output $y_j = d_j + x_j$ for firm j in country i, at maximum quality q_j, is assumed to be

$$C(y_j, q_j) = F_i + f_i q_j^{\beta_i} + c_i y_j = H_i(q_j) + c_i y_i \tag{5}$$

say. The first term in (5) we associate with fixed capital costs and R&D, and the final term constitutes variable costs. It follows that the profit of this firm is

$$\pi_j = p_j d_j + P_j x_j - C_i(y_j, q_j) \tag{6}$$

and since there is free entry and exit, we must impose the participation constraint $\pi_j \geq 0$ on the procurement decision.

2.2   Sequencing of Events

We first consider the optimal decisions of a single government taking the decisions of other governments as given. The sequencing of events is as follows:

1. Domestic Procurement by Producers. Given military expenditure, the government in producer country 1 sets and procures domestic goods of quantity d_{1j} and quality q_{1j} at price p_{1j}, for $j = 1, 2, \cdots, n_1$. It also formulates a time-consistent plan

to import goods m_{1j} of quality u_{1j}, for $j = n_1 + 1, n_1 + 2, \cdots, N$ at the world market equilibrium price P_j. All decisions are subject to a budget constraint and a non-negative profit participation constraint for domestic firms. The procurement price may be greater or less than the international market price. Firms already participating in the international market will always accept domestic procurement as long as the procurement price exceeds the marginal cost. In general, the world market price can depend on procurement decisions at this stage, but for large N (assumed in the analysis) we have monopolistic competition with the price (set in stage 2 below) given by $P_j = P = \frac{c}{\alpha}$ which depends only on the marginal cost c and the elasticity parameter α.

2. Monopolistic Competition between Firms. With a commitment to producing d_{1j}, in a Bertrand price-setting equilibrium of this stage of the game, firms in producer country 1 set world prices P_j and export quantity x_{1j} of quality u_{ij} to countries $i = 2, \cdots, \ell + r$. Note that decisions on quality are decided at stage 1 by the procuring governments.

3. Military Spending by Non-Producers and Demand for Imports by all Countries. Given the world market price P_j and quality u_{ij}, and military expenditure, governments in both producer and non-producer countries $i = 1, 2, \cdots, \ell + r$ procure imports of good, m_{ij}, $j = 1, 2, \cdots, N$ of quality u_{ij}, where $i \neq j$ for producer countries $i = 1, 2, \cdots, \ell$. Non-producers anticipating these decisions allocate resources between consumption and military expenditure.

To solve for the equilibrium[5] we proceed by backward induction starting at stage 3.

3 Military Spending by Non-Producers and Demand for Imports

3.1 Non-Producers

Consider non-producers in regions $i = \ell + 1, \ell + 2, \cdots, \ell + r$ as pairs of risk-neutral, countries, A and B say, involved in a regional conflict. They have an given, GDP Y_{iA}, Y_{iB} which can be devoted to military expenditure G_{iA}, G_{iB} or other forms of consumption

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[5]Note that in the absence of procurement considerations and with $\nu = 0$, the trade equilibrium corresponds exactly to the standard 'new-trade' model (see, for example, Krugman 1979). Then stage 1 of our model is the free-entry process.

expenditure C_{iA}, C_{iB}. Choose the price of the consumption good as the numeraire. The budget constraint is therefore

$$Y_{ik} = G_{ik} + C_{ik}; \ k = A, B \tag{7}$$

Consider now a war in region i leaving $\phi_i(C_{iA} + C_{iB})$ available for consumption. The parameter $\phi_i \in [0, 1]$ captures the destructive effect of a war in region i. Suppose that the prize for winning is some proportion of total consumption, $\theta_i \phi_i(C_{iA} + C_{iB})$. Then assuming the two countries maximize expected consumption the expected utility, following a war for country $k = A$ is given by

$$U_{iA}(S_{iA}, S_{iB}, G_{iA}, G_{iB}) = [p_{iA}(S_{iA}, S_{iB})\theta_i + (1 - p_{iA}(S_{iA}, S_{iB}))(1 - \theta_i)]\phi_i(C_{iA} + C_{iB}) \tag{8}$$

In (8), $p_i(S_{iA}, S_{iB})$ is a *Contest Success Function* (CSF) used extensively in the conflict literature. Dropping the regional subscript i in the rest of this subsection, a general form of the CSF , discussed in Skeperdas (1996), takes the form

$$p_A = \frac{f(S_A)}{f(S_A) + f(S_B)}; \ f' > 0 \tag{9}$$

$$p_B = \frac{f(S_B)}{f(S_A) + f(S_B)} = 1 - p_A \tag{10}$$

Two forms of the CSF, discussed in Hirchleifer (2000) at some length, are the *ratio form* and the *difference form*. These take the forms respectively

$$p_A = \frac{(b_A S_A)^m}{(b_A S_A)^m + (b_B S_B)^m} \tag{11}$$

$$p_B = \frac{\exp(kb_A S_A)}{\exp(kb_A S_A) + \exp(kb_B S_B)} = \frac{1}{1 + \exp(k(b_B S_B - b_A S_A))} \tag{12}$$

In both these forms b_i, $i = A, B$ is a measure of the effectiveness of the same military capability in the hands of country i; m in (11) or k in (12) are *decisiveness parameters* scaling the degree to which a side's greater military strength translates into enhanced battle success. As we shall see the form of CSF is quite crucial in determining the effect of variety and quality on the choice of military expenditure.

At stage 3, given the price P_j, and the number of differentiated goods, in a regional Nash equilibrium, the importing government in non-producing producer country A chooses both total expenditure G_A and a composition of imports $m_{Aj}, j = 1, 2, \cdots, N$ to maximize U_A given by (8) subject to its budget constraint (7), given the corresponding decision G_B

of its rival . We decompose this optimization problem into two parts. First maximize the military strengths S_A, S_B that can be achieved with a given expenditures G_A, G_B. Let $S_A^*(G_A), S_B^*(G_B)$ be these maximized levels of military security. Then country A maximizes the utility $U_{iA}(S_{iA}^*(G_A), S_{iB}^*(G_B), G_{iA}, G_{iB})$ with respect to G_A given its budget constraint and G_B, and country B acts similarly.

To carry out the first part of this optimization define a Lagrangian for non-producer country A

$$S_A - \lambda \left(\sum_{j=1}^{N} P_j m_{Aj} - G_A \right)$$

where $\lambda \geq 0$ is a Lagrange multiplier. Then the first-order conditions are:

$$\frac{1}{\alpha} \left[\sum_{j=1}^{N} (u_{Aj} m_{Aj})^\alpha \right]^{\frac{1}{\alpha}-1} \alpha u_{Aj}^\alpha m_{Aj}^{\alpha-1} = \lambda P_j \; ; \; j = 1, 2, \cdots N \tag{13}$$

Now divide the *jth* of (13) equation by the *kth* to give

$$\left(\frac{u_{Aj} m_{Aj}}{u_{Ak} m_{Ak}} \right)^{\alpha-1} = \frac{u_{Ak} P_j}{u_{Aj} P_k}$$

Substituting back into the budget constraint (3) we get

$$\sum_{k=1}^{N} \left(\frac{P_k}{u_{Ak}} \right) u_{Aj} m_{Aj} \left(\frac{u_{Aj} P_k}{u_{Ak} P_j} \right)^{-\frac{1}{1-\alpha}} = \sum_{k=1}^{N} \left(\frac{P_k}{u_{Ak}} \right)^{1-\sigma} \left(\frac{P_j}{u_{Aj}} \right)^{\sigma} u_{Aj} m_{ij} = G_A$$

where $\sigma = \frac{1}{1-\alpha} > 1$. This results in the quality-adjusted demand by government A for good $j = 1, 2, \cdots, N$ given by

$$u_{Aj} m_{Aj} \;\; = \;\; \frac{G_A}{\left(\frac{P_j}{u_{Aj}} \right)^{\sigma} \sum_{k=1}^{N} \left(\frac{P_k}{u_{Ak}} \right)^{1-\sigma}} \tag{14}$$

To interpret and manipulate (14) it is convenient to define

$$\hat{P}_A = \left[\sum_{k=1}^{N} \left(\frac{P_k}{u_{Ak}} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{15}$$

Then $\hat{P}_A$ is the quality-adjusted form of the familiar price index of imported goods, facing non-producer country A, used in the product differentiation literature (see, for example, Beath and Katsoulacas (1991), chapter 3). Now (14) and (15) can be written

$$u_{Aj} m_{Aj} = \frac{G_A}{\hat{P}_A^{1-\sigma}} \left(\frac{P_j}{u_{Aj}} \right)^{-\sigma} \tag{16}$$

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The importance of (16) is that given $\hat{P}$, the elasticity of quality-adjusted demand for variety j on the world market with respect to the quality-adjusted price is constant with elasticity $-\sigma$. Substituting (16) into (3) and using $\sigma = \frac{1}{1-\alpha}$ we have

$$S_A^* = \frac{N^\nu G_A}{\hat{P}_A^{1-\sigma}} \left[\sum_{j=1}^{N} \frac{P_j}{u_{Aj}}^{-\alpha\sigma} \right]^{\frac{1}{\alpha}} = \frac{N^\nu G_A}{\hat{P}_A^{1-\sigma}} \hat{P}_A^{\frac{1-\sigma}{\alpha}} = \frac{N^\nu G_A}{\hat{P}_A} \tag{17}$$

Hence in terms of optimal security, the budget constraint of country A takes the convenient form

$$C_A + G_A = C_A + N^{-\nu} \hat{P}_A S_A^* = Y_A \tag{18}$$

Country A now maximizes

$$U_A(S_A^*, S_B^*, G_A, G_B) = [p_A(S_A^*, S_B^*)\theta + (1 - p_A(S_A^*, S_B^*))(1 - \theta)]\phi(C_A + C_B) \tag{19}$$

with respect to G_A, subject to (18) and a corresponding budget constraint for country B, given G_B.

The first order conditions defining the Nash equilibrium are

$$(2\theta - 1)\frac{\partial p_A}{\partial S_A^*} = \frac{[p_A(S_A^*, S_B^*)\theta + (1 - p_A(S_A^*, S_B^*))(1 - \theta)]N^{-\nu}\hat{P}_A}{Y_A + Y_B - N^{-\nu}\hat{P}_A S_A^* - N^{-\nu}\hat{P}_B S_B^*} \tag{20}$$

$$(2\theta - 1)\frac{\partial p_B}{\partial S_B^*} = \frac{[p_B(S_A^*, S_B^*)\theta + (1 - p_B(S_A^*, S_B^*))(1 - \theta)]N^{-\nu}\hat{P}_B}{Y_A + Y_B - N^{-\nu}\hat{P}_A S_A^* - N^{-\nu}\hat{P}_B S_B^*} \tag{21}$$

With $S_A^* = \frac{N^\nu G_A}{\hat{P}_A}$ and $S_B^* = \frac{N^\nu G_B}{\hat{P}_B}$, (20) and (21) define two *reaction functions* in G_A and G_B. Their intersection is a Nash equilibrium; in general this is asymmetrical with asymmetries arising from differences in GDP Y_A and Y_B and differences in the prices facing each country. The latter can arise if export control regimes differ between the two countries. However for the most part we focus on symmetrical equilibria. Then putting $p_A = p_B = \frac{1}{2}$, $S_A = S_B = S$, say, $\hat{P}_A = \hat{P}_B = \hat{P}$, $Y_A = Y_B = Y^{np}$ and $G_A = G_B = G^{np}$, for all non-producers the, right-hand-side of the reaction functions become $\frac{N^{-\nu}\hat{P}}{4(Y^{np} - N^{-\nu}\hat{P}S)}$. The left-hand-side however depends on the form of the CSF. For the ratio form in a symmetric equilibrium $\frac{\partial p_A}{\partial S_A} = \frac{m}{4S}$; for the difference form $\frac{\partial p_A}{\partial S_A} = \frac{kb}{4}$ where we have put $b_A = b_B = b$. Hence substituting into (20) or (21) we arrive at the symmetric Nash equilibria for the two cases

$$\text{Ratio form of CSF} \quad : \quad G^{np} = \frac{(2\theta - 1)mY^{np}}{(1 + (2\theta - 1)m} \tag{22}$$

$$\text{Difference form of CSF} \quad : \quad G^{np} = Y^{np} - \frac{N^{-\nu}\hat{P}}{(2\theta - 1)kb} \tag{23}$$

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Thus an internal maximum $G > 0$ requires $\theta > \frac{1}{2}$ in both cases whilst for the difference form we must impose a further condition $\hat{P} < (2\theta - 1)bY^{np}$.

With the ratio form of the CSF we now see that military expenditure is independent of both the price index and the total number of varieties. This is a familiar property of the standard Dixit-Stigliz monopolistic competition model where the utility function is a Cobb-Douglas function of the numeraire good and the composite quantity index of differentiated goods (military strength in the context of our model). However with the difference form the optimal choice of government spending declines as the price index $\hat{P}$ increases, or the total number of varieties, N decreases. Since in a market symmetric equilibrium $\hat{P} = \frac{PN^{-\frac{1}{(\sigma-1)}}}{\gamma q}$, and the price P is constant (as we shall see), it follows from (23) that

$$\frac{\partial G^{np}}{\partial N} = \left(\nu + \frac{1}{\sigma - 1}\right)\frac{N^{-\nu-1}\hat{P}}{(2\theta - 1)kb} > 0 \tag{24}$$

$$\frac{\partial G^{np}}{\partial q} = \frac{N^{-\nu}\hat{P}}{q(2\theta - 1)kb} > 0 \tag{25}$$

Since in the symmetric Nash equilibrium, *any* military expenditure is inefficient in this set-up, this means that the welfare of the non-producers actually *increases* if quality decreases and/or the arms control regime is strengthened (then γ and therefore $\hat{P}$ falls) and/or N decreases. This remarkable result is a consequence of the unique character of military goods–they bring security to the purchaser but insecurity to a threatened rivals. We summarize our result as:

Proposition 1

With the ratio form of the CSF, the military expenditure of non-producers is independent of the number and quality of differentiated goods. However with the difference form, military expenditure falls and welfare increases as the number and quality decreases and/or the export regime is strengthened.

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3.2 Producers

As for non-producers we import demand for any good $j = 1, 2, \cdots, N$ can similarly be written as

$$
\begin{aligned}
u_{ij}m_{ij} &= \frac{[G_i - \sum_{j=n_{i-1}+1}^{n_{i-1}+n_i} p_{ij}d_{ij}]}{\left(\frac{P_j}{u_{ij}}\right)^\sigma \sum_{k \neq [N_{i-1},N_i]}^N \left(\frac{P_k}{u_{ik}}\right)^{1-\sigma}} \; ; j \neq N_{i-1}+1, \, N_{i-1}+2, \cdots, N_{i-1}+n_i \\
&= 0 \; ; \qquad\quad j = N_{i-1}+1, \, N_{i-1}+2, \cdots, N_{i-1}+n_i
\end{aligned}
\tag{26}
$$

where we have defined $N_i = n_1 + n_2 + \cdots + n_i$ for $i \geq 1$ (in which case $N_1 = n_1$ and $N_\ell = N$), country $i = 1, 2, \cdots, \ell$ produces varieties $j = N_{i-1}+1, N_{i-1}+2, \cdots, N_{i-1}+n_i = N_i$ and imports m_{ij} units of variety $j = 1, 2, \cdots, N_{i-1}, N_i+1, N_i+2, \cdots, N$ (defining $N_0 = 0$). Again we can define an price index of imported goods for producer countries as

$$
\hat{P}_i = \left[\sum_{k \neq [N_{i-1}, N_i]}^N \left(\frac{P_k}{u_{ik}}\right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \; ; \; i = 1, 2, \cdots, \ell
\tag{27}
$$

4 Monopolistic Competition between Firms

Turning to stage 2 of the game, in producer country 1 firm $j = 1, 2, \cdots, n_1$ profit at stage 2 is given by

$$
\pi_{1j} = (p_{1j} - c_1)d_{1j} + (P_j - c_1)x_{1j} - H_1(q_{1j}) \, ; \, j = 1, 2, \cdots, n_1
\tag{28}
$$

where exports to producers and non-producers are given by

$$
x_{1j} = \sum_{i=2}^{\ell+r} m_{ij} = \sum_{i=1}^{\ell} \frac{[G_i - \sum_{j=n_{i-1}+1}^{n_{i-1}+n_i} p_{ij}d_{ij}]}{P_j^\sigma \hat{P}_i^{1-\sigma}} + \sum_{i=\ell+1}^{\ell+r} \frac{G_i u_{i1}^{\sigma-1}}{P_j^\sigma \hat{P}^{1-\sigma}}
\tag{29}
$$

The first term in (29) consists of exports to other producing countries and depends on the procurement decisions already taken at stage 1 and on all prices set at stage 2 of the game. The second term consists of exports to non-producing countries and depend on the all prices set by firms at stage 2 of the game.

For producers let $\Gamma_i = G_i - \sum_{j=N_{i-1}+1}^{N_{i-1}+n_i} p_{ij}d_{ij}$ be the part of the government budget devoted to imports. Define $\Gamma_i = G_i$ and $\hat{P}_i = \hat{P}$ for non-producers. Then maximizing profits given by (28) with respect to P_j, gives the first-order conditions

$$
(P_j - c_1)\frac{\partial x_{1j}}{\partial P_j} + x_{1j} = 0
\tag{30}
$$

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where from (29)

$$\frac{\partial x_{1j}}{\partial P_j} = -\frac{\sigma x_{1j}}{P_j} - \underbrace{P_j^{-\sigma} \sum_{i=2}^{\ell+r} \frac{\Gamma_i u_{i1}^{\sigma-1}}{\hat{P}_i^{2(1-\sigma)}} \frac{\partial(\hat{P}_i^{1-\sigma})}{\partial P_j}}_{\text{strategic interaction term}} \tag{31}$$

In working out the effect of a change in the price of variety firm j considers two effects: the first term takes the total price index of imports facing other countries $\hat{P}_i$; $i = 2, 3, \cdots, \ell+r$ as given. The second *strategic* term considers the effect on each of these price indices of the firms export price. If N is large, which we assume in this paper, then there are so many firms that we can ignore this strategic effect.[6] Then substituting (31) back into (30), the first order condition becomes

$$\left[-\frac{\sigma(P_j - c_1)}{P_j} + 1 \right] x_{1j} = 0 \, ; \, j = 1, 2, \cdots, n_1 \tag{32}$$

Hence using (29) we obtain from (32) the *Lerner Index* for any variety $j \in [1, n_1]$ in country 1 as

$$L_1 = \frac{P_1 - c_1}{P_1} = \frac{1}{\sigma}$$

This is the familiar monopolistic competition result. The price of every good exported from country 1 is a constant mark-up on marginal cost: $P_1 = \frac{c_1}{\alpha}$. Similarly for country i, the price is given by $P_i = \frac{c_i}{\alpha}$

5 Domestic Procurement by Producers

We now complete the equilibrium by evaluating the optimal decisions of the government in country 1 at stage 1 of the game. Producer countries $i = 1, 2, \cdots, \ell$ face no one rival or enemy and devote resources to military expenditure. They prepare defensively and offensively to provide an 'insurance' in an uncertain world against a range of possible security needs. We do not attempt to model their expenditure decisions and take the military expenditure of producers as exogenous.

Each government in the producer countries then maximizes military strength for a given government spending. The government when choosing the procurement price, p_1, relaxes or tightens the firms' participation constraint and, in effect, chooses the number

---

[6]However as the number becomes small this strategic term becomes significant - see Garia-Alonso and Levine (2003) for analysis of this case.

of domestic firms. Imposing symmetry between identical domestic firms, we have that $d_{1j} = d_1$ and $q_{1j} = q_1$ for all domestic varieties. Similarly, given the symmetry between all firms in countries $i = 2, 3, \cdots, \ell$ in the international market, government 1 will choose the same amount of imports of each variety from that country, m_{1i} say, of quality u_{1i}.[7] We examine a complete information Nash equilibrium of stage 1 of the game, and a subgame perfect equilibrium of the whole game, where for country 1, independent decision variables are d_1, q_1 and n_1.[8]

The optimization problem of the government in country 1 is to maximize military strength given by

$$S_1 = [w_1 n_1 + (1 - w_1)(N - n_1)]^\nu \left[w_1 n_1 (q_1 d_1)^\alpha + (1 - w_1) \sum_{i=2}^{\ell} n_i (u_{1i} m_{1i})^\alpha \right]^{\frac{1}{\alpha}} \qquad (33)$$

with respect to the independent choice variables d_1, q_1, and n_1, given the world prices $P_i = P = \frac{c_i}{\alpha}$; $i = 2, 3, \cdots, \ell$ of each variety from country i, the corresponding decisions of other countries, and two sets of constraints. These are the budget constraint (BC_1) and the representative domestic firm's participation constraint (PC_1) given by

$$BC_1 \quad : \quad p_1 n_1 d_1 + \sum_{i=2}^{\ell} P_i n_i m_{1i} = G_1$$

$$PC_1 \quad : \quad \pi_1 = (p_1 - c_1) d_1 + (P_1 - c_1) x_1 - F_1 - f_1 q_1^{\beta_1} \geq 0$$

and the corresponding constraints in the other countries. Clearly the PC constraint must bind so the procurement price is given by

$$p_1 = c_1 + \frac{F_1 + f_1 q_1^{\beta_1} - (P_1 - c_1) x_1}{d_1} = c_1 + \frac{H_1(q_1) - R_1(x_1)}{d_1} \qquad (34)$$

---

[7]Note that at stage 1 we have imposed symmetry between firms and have therefore defined m_{ij} and u_{ij} as the quantity and quality respectively imported by country i of **any good** from country j, $i, j = 1, 2, \cdots, \ell$. By contrast at stage 3 j referred to the **variety** available on the world market, $j = 1, 2, \cdots, N$.

[8]Any two from three possible decision variables, d_1, p_1 and n_1 can be assumed, but will lead to different Nash equilibria at stage 1. Our particular choice, d_1, and n_1 is made partly, for analytical convenience, but can be also justified by the need to **observe** decision variables in a more realistic incomplete information setting, where the process of dynamic adjustment towards the equilibrium, for example of a Cournot-type, needs to be addressed. It is plausible to assume that the domestic procurement decision, d_i, and the number of firms supported, n_i, $i = 1, 2, \cdots, \ell$ are more readily observed that the procurement price, p_i, $i = 1, 2, \cdots, \ell$, which involves a possibly hidden subsidy.

where we have written export net revenue $(P_1 - c_1)x_1 = R_1(x_1)$ and we recall that total fixed production costs in country 1 are denoted by $H_1(q_1)$. It is useful to note from (35) that exports x_1 of each variety from country 1 can be written in terms of decision variables as

$$
\begin{aligned}
x_1 &= \sum_{i=2}^{\ell+r} m_{i1} = \sum_{i=2}^{\ell} m_{i1} + \sum_{i=\ell+1}^{\ell+r} \frac{G_i u_{i1}^{\sigma-1}}{P_1^\sigma (n_1 P_1^{1-\sigma} u_{i1}^{\sigma-1} + n_2 P_2^{1-\sigma} u_{i2}^{\sigma-1} + \cdots + n_\ell P_\ell^{1-\sigma} u_{i\ell}^{\sigma-1})} \\
&= x_1^p + x_1^{np}
\end{aligned}
\tag{35}
$$

where u_{ij} is the quality of each good produced by country j and imported by country i and we have written the total exports of each firm in country 1 as the sum of exports to other producers, x_1^p and to non-producers, x_1^{np}. At stage 1, country 1 only commits to a *total* import budget $P_i n_i m_{1i}$ (determined by the BC_1 as a residual given other decisions by all countries) and *not* to any procurement contract with any foreign firm. The import decision $m_{1j}, j = 1, 2, \cdots, \ell$ for any particular variety is decided at stage 3, given this total budget, and given out-of-equilibrium (at stage 1) prices P_j charged by firms.

Since we are assuming a Nash equilibrium in independent decision variables d_1, q_1 and n_1 for country 1, we can eliminate the the procurement price, p_1, using the PC_1 constraint. The BC_1 constraint now becomes

$$
BC_1 : n_1(c_1 d_1 + H_1(q_1) - R_1(x_1)) + \sum_{i=2}^{\ell} P_i n_i m_{1i} = G_1
\tag{36}
$$

and the government now maximizes S_1 given by (33) with respect to d_1, q_1, and n_1, given (36), and the corresponding budget constraints and independent decision variables of other governments. To carry out this constrained optimization, define a Lagrangian

$$
\begin{aligned}
\mathcal{L}_1 = S_1 \quad &- \quad \lambda_1 [n_1(c_1 d_1 + H_1(q_1) - R_1(x_1)) + \sum_{i=2}^{\ell} P_i n_i m_{1i} - G_1] \\
&- \quad \sum_{i=2}^{\ell} \mu_i [n_i(c_i d_i + H_i(q_i) - R_i(x_i)) + \sum_{j=1, j\neq i}^{\ell} P_j n_j m_{ij} - G_i]
\end{aligned}
$$

where $\lambda_1 \geq 0$ is a Lagrange multiplier country 1 assigns to its own budget constraint, and $\mu_{1i} \geq 0, \ i = 2, 3, \cdots, \ell$ are Lagrange multipliers assigned to the other countries' budget constraints. Then country 1 maximizes $\mathcal{L}_1$ with respect to independent decision variables d_1, q_1, n_1, and with respect to endogenous variables $\{m_{ij}, \lambda_i, \mu_i\}, i, j = 1, 2, \cdots, \ell, j \neq i$, given the independent decision variables of the other countries $\{d_i, q_i, n_i\}, i = 2, 3, \cdots, \ell$.

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This optimization problem is greatly simplified as a result of the following Lemma:

Lemma

In a subgame perfect equilibrium (SPE) of the game $\mu_{1i} = 0$, $i = 2, 3, \cdots, \ell$.

Proof

The first-order condition with respect to m_{1i} is given by

$$\frac{\partial S_1}{\partial m_{1i}} = S_1^{1-\alpha}[w_1 n_1 + (1-w_1)(N-n_1)]^{\alpha\nu}(1-w_1)n_i u_{1i}^\alpha m_{1i}^{\alpha-1} = \lambda_1 P_i n_i - \sum_{j=2}^{\ell} \mu_j n_j \frac{\partial R_j}{\partial m_{1j}} \; ; \; i > 1 \tag{37}$$

In (37) we have that export net revenue is given by $R_i = (P_i - c_i)x_i = (P_i - c_i)(x_i^p + x_i^{np})$. From the counterpart of (35) for country i, we have $x_i^p = \sum_{j \neq i}^{\ell} m_{ji}$. Hence $\frac{\partial R_i}{\partial m_{1i}} = (P_i - c_i)$. Then dividing the $i = r$ equation by the $i = s$ equation, the relative demand by country for imported goods from countries $i = r, s$ is given by

$$\frac{m_{1r}}{m_{1s}} = \left(\frac{u_{1r}}{u_{1s}}\right)^{\sigma-1} \left[\frac{\lambda_1 P_s - \frac{1}{n_s}\sum_{j=2}^{\ell}\mu_{1j}n_j(P_j - c_j)}{\lambda_1 P_r - \frac{1}{n_r}\sum_{j=2}^{\ell}\mu_{1j}n_j(P_j - c_j)}\right]^\sigma \tag{38}$$

However from (26) at stage 3 of the game, the relative demand for two imported goods from countries $i = r, s$ is given by

$$\frac{m_{1r}}{m_{1s}} = \left(\frac{u_{1r}}{u_{is}}\right)^{\sigma-1} \left(\frac{P_s}{P_r}\right)^\sigma \tag{39}$$

where prices are now out of equilibrium as defined in stages 2 and 1. In a SPE we must have agreement with the *anticipated* decision on imports given by (38) and the *actual* decision taken at stage 3 given by (39). This requires $\sum_{j=2}^{\ell} \mu_{1j}n_j(P_j - c_j) = 0$. Since $\mu_{1i} \geq 0$ it follows that $\mu_{1i} = 0$ for all $i = 2, 3, \cdots, \ell$. $\qquad\square$

If at stage 1 the governments could commit to both domestic and overseas contracts, then imports of the later would satisfy the first-order condition (37) with $\mu_{1i} > 0$. According to (37), the marginal benefit (the left-hand-side) equals the marginal budgetary cost. The first term of the latter, on the right-hand-side, equals the shadow price of BC_1 multiplied by the procurement price. The second term equals the sum of the shadow price of BC_i, $i > 1$ multiplied by the marginal revenue gain to each foreign country from exporting to country 1. Exports to country 1 relax these budget constraints and bring benefit to that country through allowing for more imports. Taking this into consideration lowers the effective cost of imports and there increases their volume.

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Having made this commitment to importing more than it would in the absence of these strategic considerations, at stage 3 country 1 has a given import budget $G_1 - p_1 n_1 d_1$. If it were to re-optimise given world market prices, it would choose imports given by (26) and therefore set $\mu_{1i} = 0$; $i > 1$. The ex ante optimal contract at stage 1 is no longer optimal ex post at stage 3. The equilibrium is not subgame perfect in other words. The subgame perfection condition imposes $\mu_i = 0$, $i > 2$ and implies that at stage 1 country 1 ignores the budget constraints of other countries.

With $\mu_i = 0$, $i > 2$, the remaining first-order conditions for an internal solution (where $n_1 \geq 0$ and $d_1 \geq 0$ and are not binding, but BC_1 does bind) are then

$$d_1 \quad : \quad \frac{\partial U_1}{\partial d_1} = S_1^{1-\alpha}[w_1 n_1 + (1-w_1)(N-n_1)]^{\alpha\nu} w_1 q_1^\alpha d_1^{\alpha-1} = \lambda c \tag{40}$$

$$n_1 \quad : \quad \frac{\partial S_1}{\partial n_1} = \frac{S_1^{1-\alpha}}{\alpha} w_1 q_1^\alpha d_1^\alpha [w_1 n_1 + (1-w_1)(N-n_1)]^{\alpha\nu} + \nu w_1 S_1 [w_1 n_1 + (1-w_1)(N-n_1)]^{-1}$$

$$= \lambda(cd_1 + H_1(q_1) - R(x_1) - n_1 \frac{\partial R_1}{\partial n_1}) \tag{41}$$

$$q_1 \quad : \quad \frac{\partial S_1}{\partial q_1} = S_1^{1-\alpha}[w_1 n_1 + (1-w_1)(N-n_1)]^{\alpha\nu} w_1 q_1^{\alpha-1} d_1^\alpha = \lambda \left(\frac{\partial H_1}{\partial q_1} - \frac{\partial R_1}{\partial q_1} \right) \tag{42}$$

These 3 equations and (37) plus the constraint BC_1 solve for the decision variables n_1, d_1, q_1, and for endogenous variables m_{1j} and λ_1.

To complete the equilibrium we need to evaluate the responses of net export revenue and quality, $\frac{\partial R_1}{\partial n_1}$ and $\frac{\partial R_1}{\partial q_1}$ respectively. First from (35) write exports to non-producers as

$$x_1^{np} = \frac{1}{P_1} \sum_{i=\ell+1}^{\ell+r} \frac{G_i}{\sum_{k=1}^\ell n_k (\tilde{P}_{1k}^i)^{\sigma-1}} \tag{43}$$

where we have defined the quality-adjusted price of good 1 relative to k, both exported to non-producer i.

$$\tilde{P}_{1k}^i = \frac{P_1 u_{ik}}{P_k u_{i1}} \tag{44}$$

According to (43), the value of exports to non-producers by each firm in country 1, $P_1 x_1^{np}$, depends positively on the 'competitiveness' of its good, $\frac{1}{\tilde{P}_{1k}^i}$ and expenditure by non-producers, and negatively on the numbers of competitors, n_k; $k = 1, 2, \cdots, \ell$. Differenti-

ating (43) we then have

$$
\frac{\partial R_1}{\partial n_1} = (P_1 - c_1)\frac{\partial x_1^{np}}{\partial n_1}
$$

$$
= -L_1 \sum_{i=\ell+1}^{\ell+r} \frac{G_i}{\left[\sum_{k=1}^{\ell} n_k(\tilde{P}_{1k}^i)^{\sigma-1}\right]^2} + L_1 \sum_{i=\ell+1}^{\ell+r} \frac{\frac{\partial G_i}{\partial n_1}}{\left[\sum_{k=1}^{\ell} n_k(\tilde{P}_{1k}^i)^{\sigma-1}\right]} \tag{45}
$$

$$
\frac{\partial R_1}{\partial q_1} = (P_1 - c_1)\frac{\partial x_1^{np}}{\partial q_1}
$$

$$
= \frac{L_1(\sigma-1)}{q_1} \sum_{i=\ell+1}^{\ell+r} \frac{G_i \sum_{k=2}^{\ell} n_k(\tilde{P}_{1k}^i)^{\sigma-1}}{\left[\sum_{k=1}^{\ell} n_k(\tilde{P}_{1k}^i)^{\sigma-1}\right]^2} + L_1 \sum_{i=\ell+1}^{\ell+r} \frac{\frac{\partial G_i}{\partial q_1}}{\left[\sum_{k=1}^{\ell} n_k(\tilde{P}_{1k}^i)^{\sigma-1}\right]} \tag{46}
$$

where, in (46) we have used the expression for the export control regime on quality exported by country to country j given by $u_{j1} = \gamma_{j1}q_1$. Equations (45) and (46) are crucial for the results that follow in the paper. We have seen from proposition 1 that if we choose a difference form of the CSF then $\frac{\partial G_i}{\partial n_1} > 0$ and $\frac{\partial G_i}{\partial q_1} > 0$. Thus from (45) there are two opposing effects on the net revenue per firm in country 1 from increasing the number of firms, given q_1, d_1 and the corresponding decisions n_i, q_i and d_i, $i > 1$ of other countries. For a given expenditure by non-producers an increase in n_1 spreads a given demand over more competing domestic firms. Since costs include a fixed component, average costs increase and revenue falls. This is the first negative term in (45). However with the difference form of the CSF, the second term is positive because an increase in n_1 increases the total variety available, N, and boosts demand from the external market. The sign of $\frac{\partial R_1}{\partial n_1}$ depends on which of these opposite forces dominates. By contrast, there is no ambiguity with respect to the effect on net revenue of a unilateral increase in quality. Given expenditure by non-producers, a unilateral increase in q_1 increases competitiveness and raises demand. It also raises expenditure and so both terms in (46) are positive.

We can now characterize a Nash equilibrium of stage 1 of the game. Using $H_1(q_1) = F_1 + f_1 q_1^\beta$ and dividing (41), and (42) by (40), in turn, we can eliminate the shadow price λ to obtain

$$
d_1 = \frac{\left[H(q_1) - R_1(x_1) - n_1\frac{\partial R_1}{\partial n_1}\right]}{P_1\left[1 - \alpha + \frac{\alpha\nu}{[w_1 n_1 + (1-w_1)(N-n_1)]^{1+\alpha\nu}}\left(\frac{U_1}{d_1}\right)^\alpha\right]} \tag{47}
$$

$$
m_{1i} = d_1\left(\frac{c_1(1-w_1)}{P_i w_1}\right)^\sigma \left(\frac{u_{1i}}{q_1}\right)^{\sigma-1} = \phi_1 d_1 \,;\; i > 1 \tag{48}
$$

$$
c_1 d_1 = \beta_1 f_1 q_1^{\beta_1} - q_1\frac{\partial R_1}{\partial q_1} \tag{49}
$$

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The budget constraint BC_1, given by (36) and the expression for net revenue

$$R_1(x_1) = (P_1 - c_1)x_1 = (P_1 - c_1)[\sum_{i=2}^{\ell} m_{i1} + x_1^{np}] \qquad (50)$$

with x_1^{np} given by (43), completes the solution for the single economy given the decisions on d_i, $\{m_{ij}\}$, q_i and n_i by the other countries $i > 1$. Combining these equations with analogous ones for the remaining $\ell - 1$ producer countries completes the Nash equilibrium at stage 1 of the game and the Subgame Perfect Nash equilibrium of the whole game. In our set-up asymmetries between producer countries can arise from differences in costs $\{c_i, F_i, f_i, \beta_i\}$, expenditures $\{G_i\}$, the domestic bias parameter $\{w_i\}$ and the nature of the arms export control regime imposed by country j on country i, $\{\gamma_{ij}\}$. For instance regarding the latter, if there are say 4 countries consisting of two alliances each with two countries, then a possible choice of γ_{ij} is the matrix Γ where

$$\Gamma = \begin{pmatrix} 1 & 1 & \gamma & \gamma \\ 1 & 1 & \gamma & \gamma \\ \gamma & \gamma & 1 & 1 \\ \gamma & \gamma & 1 & 1 \end{pmatrix}$$

where $0 \leq \gamma < 1$. Given the domination of one producer country in the world– the US – these asymmetries between producer countries are clearly of practical importance. However a non-symmetric non-cooperative equilibrium can only be solved by numerical solution. By contrast, a symmetric equilibrium is tractable and provides some valuable insights into the procurement and export arrangements in the EU which can be thought of as three approximately equally-sized size countries –the UK, Germany, and France – procuring and exporting arms. For the remainder of the paper we concentrate on the symmetric equilibrium.

6 A Symmetric Non-Cooperative Equilibrium

We solve for a symmetric non-cooperative equilibrium in which all producer countries and all non-producing countries are identical in every respect. Then $w_i = w$, $c_i = c$, $F_i = F$, $f_i = f$, $\beta_i = \beta$, $\gamma_{ij} = \gamma$, $G_i = G^p$ say, for producers $(i = 1, 2, \cdots, \ell)$ and $G_i = G^{np}$ for non-producers $(i = \ell + 1, \ell + 2, \cdots, \ell + r)$. Then $P_1 = P_2 = \cdots = P = \frac{c}{\alpha}$,

$d_1 = d_2 = \cdots = d$, $n_1 = n_2 = \cdots = n$ etc, $N = \ell n$, $\phi_i = \phi = \left(\frac{\alpha(1-w)}{w}\right)^\sigma \gamma^{\sigma-1}$ and

$\frac{U_i}{d_i} = \frac{U}{d} = n^{\nu + \frac{1}{\alpha}}[w + (1-w)(\ell-1)]^\nu[w + (1-w)(\ell-1)\phi^\alpha]^{\frac{1}{\alpha}}$ for producing countries. In addition from (45) and (46) we have

$$\frac{\partial R_1}{\partial n_1} = \frac{\partial R_2}{\partial n_2} = \cdots = -\frac{Lr}{N}\left[\frac{G^{np}}{N} - \frac{\partial G^{np}}{\partial N}\right] \tag{51}$$

$$\frac{\partial R_1}{\partial q_1} = \frac{\partial R_2}{\partial q_2} = \cdots = \frac{Lr}{N}\left[\frac{(\sigma-1)(\ell-1)G^{np}}{q\ell} + \frac{\partial G^{np}}{\partial q}\right] \tag{52}$$

The first-order condition (47) now becomes

$$d = \frac{(H(q) - R(x) + \Theta_1)}{P(1 - \alpha + \Theta_2)} \tag{53}$$

where we have defined

$$\Theta_1 \equiv \frac{Lr}{\ell}\left[\frac{G^{np}}{N} - \frac{\partial G^{np}}{\partial N}\right]$$

$$\Theta_2 \equiv \frac{\alpha\nu[w + (1-w)(\ell-1)\phi^\alpha]}{[w + (1-w)(\ell-1)]}$$

and we have used (24). Substituting for $H(q) - R(x)$ from (53) into (34) we arrive at the procurement price in the non-cooperative symmetric equilibrium

$$p = P(1 + \Theta_2) - \frac{\Theta_1}{d} \tag{54}$$

Hence for a 'traditional' Dixit-Stigliz utility function where $\nu = \Theta_2 = 0$ and in the limit as the external market becomes small (but still of sufficient size to determine the world market price), $\Theta_1 \to 0$ and we have that $p = P$; i.e., the procurement price equals the market price. Generally however either $p > P$, in which case the procurement process involves a subsidy, or $p < P$ implying that the government taxes away part of the monopolistic profits. A high taste for variety ν encourages the former and whilst the effect of large external market depends on the sign of Θ_1. It is clear from (54) that $\Theta_1 > 0$ iff the elasticity $\frac{N\partial G^{np}}{G^{np}\partial N} < 1$. If the CSFs are of the ratio form, this elasticity is zero and $\Theta_1 > 0$ unambiguously. For the difference form of the CSFs Θ_1 can be negative if the elasticity is sufficiently high. As we shall see for our calibration the elasticity with the difference form of the CSF *is* extremely high so $\Theta_1 < 1$ and the procurement price unambiguously involves a subsidy. The intuition behind this external effect is that increasing the number of differentiated goods, each produced by a single firm, reduces the net export revenue to the external market per firm and tightens the participation constraint. In a non-cooperative

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equilibrium each government takes into account only their own contribution to the world supply of differentiated goods and, through reducing the procurement price, lowers its optimal number of domestic firms as the external market becomes more important, provided that the effect on demand is not large. For the ratio form of the CSF this demand effect is zero but for the ratio effect it may be extremely large. We summarize this result as:

Proposition 2: The Procurement Price

In a symmetric, non-cooperative equilibrium without strategic pricing by firms, the procurement price may be above or below the world market price. A high taste for variety encourages the former and provided the elasticity $\frac{N\partial G^{np}}{G^{np}\partial N} < 1$, which is always true if CSFs are of the ratio form, a large external market encourages the latter.

In general the full solution to the symmetric non-cooperative equilibrium requires numerical solutions which are provided later in the paper. However we can derive explicit expressions for the total number of firms in the case where CSFs are of ratio form and $\frac{\partial G^{np}}{\partial N} = 0$. To do this, first put $R(x) = (P - c)x = (P - c)((\ell - 1)\phi d + \frac{rG^{np}}{NP})$. Then (53) becomes

$$d = \frac{\left[H(q) - \frac{rG^{np}}{\sigma \ell n}\left(1 - \frac{1}{\ell}\right)\right]}{P\left[(1 - \alpha)(1 + (\ell - 1)\phi) + \Theta_2\right]} \tag{55}$$

Writing the symmetric budget constraint as

$$nd(p + P(\ell - 1)\phi) = G^p \tag{56}$$

and using (54) some algebra leads to

$$d = \frac{G^p + \frac{rG^{np}}{\sigma \ell^2}}{nP\left(1 + (\ell - 1)\phi + \Theta_2\right)} \tag{57}$$

Equation (55) says that given quality, the producer countries respond to a increases in external demand per variety. $\frac{rG^{np}}{\ell n}$ by reducing the size of the firm. Thus the right-hand-side of (55) is upward-sloping in n. Equation (57) reflects the trade-off between size and number arising from the budget constraint and the right-hand-side of is downward-sloping in n. Hence given quality, we can solve for the equilibrium number of differentiated goods (equals the number of firms), n, and hence the total world number $N = \ell n$.

To complete the non-cooperative equilibrium we need to determine quality, using (49).

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Using (52), (49) becomes

$$cd = \beta f q^\beta - \frac{Lr(\sigma - 1)(\ell - 1)G^{np}}{\ell N} \tag{58}$$

Hence combining (57) and (58) and substituting $\Theta_1 = \frac{LrG^{np}}{\ell N}$ and $L = \frac{1}{\sigma}$, quality can be expressed in terms of the firm number per country, n, as:

$$\beta f q^\beta = \frac{1}{n}\left[(\sigma - 1)(\ell - 1)\frac{rG^{np}}{\sigma \ell^2} + \frac{c\left(G^p + \frac{rG^{np}}{\sigma \ell^2}\right)}{P\left(1 + (\ell - 1)\phi + \Theta_2\right)}\right] \tag{59}$$

Thus there is a *trade-off* between firm number and quality. According to (60), for a given quality, firm number is decreasing with that quality. According to (59), for a given number, quality is decreasing with that firm number. The equilibrium levels of n and q are determined by the intersection of these two downward-sloping curves, given government expenditures and the other parameters of the model.

We express the following result for N in terms of the total world expenditure $G = \ell G^p + rG^{np}$ and the relative size of the external market of non-producers $\frac{rG^{np}}{G}$:

$$N = \frac{G}{\beta F}\left[\theta - \frac{rG^{np}}{G}\left(\theta\left(1 - \frac{1}{\sigma \ell}\right) - \frac{1}{\sigma \ell}(\ell - 1)(\beta - \sigma + 1)\right)\right] \tag{60}$$

where we have defined

$$\theta = \frac{\beta(1 - \alpha)(1 + (\ell - 1)\phi) + \beta\Theta_2 - \alpha}{1 + (\ell - 1)\phi + \Theta_2} \in ((1 - \alpha), 1)$$

Again we can examine the special case of a 'traditional' Dixit-Stigliz utility function where $\nu = \Theta_2 = 0$, there is no investment in quality ($\beta \to \infty$) and the limit as the external market becomes small. Then $\theta = \beta(1 - \alpha)$ and we have that $N = \frac{G(1-\alpha)}{F}$, a familiar result for a closed economy monopolistic competition model.

Unambiguous results for the effect on N of changes in ν, w and the relative size of the external market, $\frac{rG^{np}}{G}$ can be obtained if we confine ourselves to the case where $\beta \to \infty$ and investment into R&D disappears. Then

$$\theta \quad \to \quad \frac{\beta[(1 - \alpha)(1 + (\ell - 1)\phi) + \Theta_2]}{1 + (\ell - 1)\phi + \Theta_2} \equiv \beta\Lambda \tag{61}$$

$$N \quad \to \quad \frac{G}{F}\left[\Lambda - \frac{rG^{np}}{G}\left(\Lambda\left(1 - \frac{1}{\sigma \ell}\right) - \frac{1}{\sigma \ell}(\ell - 1)\right)\right] \tag{62}$$

From (60) and the definition of Θ_1 given after (53) we can now examine the effect on the world number of firms of changes in the taste for variety parameter ν, the preference

for domestic supply parameter $w \in [\frac{1}{2}, 1]$ and the relative size of the external market $\frac{rG^{np}}{G}$. First note that $\Lambda \in [1 - \alpha, 1]$ as ν increases from 0 to ∞. Furthermore, from (60), N is increasing in Λ if $1 > \frac{rG^{np}}{G}\left(1 - \frac{1}{\sigma\ell}\right)$. Since $\frac{rG^{np}}{G} < 1$, $\sigma > 1$ and $\ell \geq 1$ this condition is satisfied. Hence it follows that N is an increasing function of ν and we arrive at the expected result that an increase in the taste for variety in producer countries increases the number of differentiated goods.

Next consider an increase in the domestic procurement bias parameter, w. In the range $w \in [\frac{1}{2}, 1]$, ϕ falls from α^σ to 0 and Θ_2 goes from $\frac{\alpha\nu[1+(\ell-1)\alpha^{\sigma\alpha}]}{\ell}$ to $\alpha\nu$. Since $\alpha^{\alpha\sigma} < 1$, $\frac{1+(\ell-1)\alpha^{\sigma\alpha}}{\ell} < 1$ and therefore this represents an increase in Θ_2 and therefore Λ. A strengthening of the arms control regime, modelled as a decrease in $\gamma \in [0, 1]$ has exactly the same effect as an increase in w. We have already shown that N is an increasing function of θ. It follows that as producer countries become less concerned with domestic supply and/or relax the arms control regime, Θ_2 falls and therefore the equilibrium number of firms, N, falls.

Finally from (60), N decreases with the relative size of the external market, $\frac{rG^{np}}{G}$, if the following condition is satisfied:

$$\Lambda > \frac{1}{\Lambda\ell}(\theta + \ell - 1) \tag{63}$$

Since $\theta < 1$, the right-hand side of (63) is an increasing function of ℓ and at $\ell = \infty$ equals $\frac{1}{\sigma}$. But $\theta > 1 - \alpha$. Hence (63) holds.

A willingness to procure from abroad, export more arms and the growing relative size of the international market are three features one may plausibly associate with *military globalization*. In that sense we may conclude that globalization is associated with a *decrease* in the number of firms in the world market. Summarizing our results:

Proposition 3: The Number of Firms

In the case of the ratio form of the CSFs and high β, in a symmetric, noncooperative equilibrium, the number of firms increases as the taste for variety by producer countries increases. More openness of economies in the form of a reduction in preferences of producer countries for domestic supply, a relaxation of arms controls and an increase in the relative size of the external market results in a decrease in the number of firms.

7 Cooperation Between Producers

Returning to $\beta < \infty$ and staying with the case of the ratio form of the CSFs, we now examine a symmetric cooperative agreement at stage 1 where there are no export controls between producers and a common export control regime with respect to non-producers. Then $u_{ij} = q_j$ for producers $i = 1, 2, \cdots, \ell$, and $u_{ij} = \gamma q_j$ for non-producers $i = \ell + 1, \ell + 2, \cdots, \ell + r$. ℓ identical producers would then choose $d_1 = d_2 = \cdots = d_\ell = d$, $n_1 = n_2 = \cdots = n_\ell = n$, $q_1 = q_2 = \cdots = q_\ell = n$ and commit, at stage 1, to $m_1 = m_2 = \cdots = m_\ell = m$ to maximize $U_1 = U_2 = \cdots = U_\ell = U$ where

$$U = [w + (1-w)(\ell-1)]n^{\nu+\frac{1}{\alpha}}q[wd^\alpha + (1-w)(\ell-1)m^\alpha]^{\frac{1}{\alpha}} \tag{64}$$

subject to budget constraints $BC_1 = BC_2 = \cdots = BC$ and participation constraints $PC_1 = PC_2 = \cdots = PC$ where

$$BC \quad : \quad n[pd + P(\ell-1)m] = G^p$$

$$PC \quad : \quad \pi = (p-c)d + R(x) - H(q) = 0$$

In PC the net revenue is given by

$$R(x) = (P-c)x = (P-c)(x^p + x^{np}) = (P-c)\left[(\ell-1)m + \frac{rG^{np}}{\ell nP}\right] \tag{65}$$

Using (65) we can consolidate the BC and PC constraints as

$$n[c(d + (\ell-1)m) + H(q)] = L\frac{rG^{np}}{\ell} + G^p \tag{66}$$

Hence the optimal procurement decision for the producers together is found by maximizing $n^{\nu+\frac{1}{\alpha}}q[wd^\alpha + (1-w)(\ell-1)m^\alpha]^{\frac{1}{\alpha}}$ with respect to n, d and m subject to the consolidated constraint (66).

To carry out this optimization define a Langrangian

$$n^{\nu+\frac{1}{\alpha}}[wd^\alpha + (1-w)(\ell-1)m^\alpha]^{\frac{1}{\alpha}}q - \lambda\left[n[c(d + (\ell-1)m) + H(q)] - L\frac{rG^{np}}{\ell} - G^p\right]$$

where $\lambda \geq 0$ is a Lagrangian multiplier. The first-order conditions are:

$$n \quad : \quad (\nu + \frac{1}{\alpha})n^{(\nu+\frac{1}{\alpha}-1)}[wd^\alpha + (1-w)(\ell-1)m^\alpha]^{\frac{1}{\alpha}}q = \lambda\left[c(d + (\ell-1)m) + H(q) - Lr\frac{\partial G^{np}}{\partial N}\right]$$

$$d \quad : \quad n^{(\nu+\frac{1}{\alpha})}[wd^\alpha + (1-w)(\ell-1)m^\alpha]^{\frac{1}{\alpha}-1}wd^{\alpha-1}q = \lambda nc$$

$$m \quad : \quad n^{(\nu+\frac{1}{\alpha})}[wd^\alpha + (1-w)(\ell-1)m^\alpha]^{\frac{1}{\alpha}-1}(1-w)(\ell-1)m^{\alpha-1}q = \lambda nc(\ell-1)$$

$$q \quad : \quad n^{(\nu+\frac{1}{\alpha})}[wd^\alpha + (1-w)(\ell-1)m^\alpha]^{\frac{1}{\alpha}} = \lambda\left[nH'(q) - \frac{Lr}{\ell}\frac{\partial G^{np}}{\partial q}\right]$$

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Dividing the first, the third and the fourth first-order condition by the second we arrive at:

$$m = \left(\frac{1-w}{w}\right)^{\sigma} d = \bar{\phi} d \text{ say,} \tag{67}$$

$$cd\left[\left(\nu + \frac{1}{\alpha}\right)(w + (1-w)(\ell-1)\bar{\phi}^{\alpha} - w(1 + (\ell-1)\bar{\phi})\right] = w\left[H(q) - \frac{Lr}{\ell}\frac{\partial G^{np}}{\partial N}\right] \tag{68}$$

$$cd = q\left[\beta f q^{\beta-1} - \frac{Lr}{N}\frac{\partial G^{np}}{\partial q}\right]\frac{wd^{\alpha}}{wd^{\alpha} + (1-w)(\ell-1)m^{\alpha}} \tag{69}$$

Equations (67), (68), (68) together with the constraint (66) characterize the optimal co-operative procurement agreement. Equations (68) and (69) can be simplified somewhat by noting that $[w + (1-w)(\ell-1)\bar{\phi}^{\alpha}] = w\left[1 + \left(\frac{1-w}{w}\right)(\ell-1)\bar{\phi}^{\alpha}\right] = w\left[1 + \left(\frac{1-w}{w}\right)^{\frac{1}{1-\alpha}}(\ell-1)\right] = w[1 + \bar{\phi}(\ell-1)]$. Then putting $\frac{\partial G^{np}}{\partial N} = 0$ for the case of the ratio form of the CSFs and substituting into (68), a little algebra results in

$$N = \frac{(1-\alpha+\alpha\nu)G\left[1 - \frac{rG^{np}}{G}\left(1 - \frac{1}{\sigma\ell}\right)\right]}{H(q)(1+\alpha\nu)} \tag{70}$$

whilst (67) and (69) now give

$$cd = \frac{\beta f q^{\beta}}{(1 + \bar{\phi}(\ell-1))} \tag{71}$$

We can compare this result with the corresponding non-cooperative equilibrium given by (60). Putting $\ell = 1$ in the latter expression we find, as expected, that the non-cooperative equilibrium and the cooperative arrangement are the same if there is only one country. A more interesting result follows from (70). The right-hand-side is *independent of the domestic production bias parameter*, w. Since imports $m = \bar{\phi} d$ where $\bar{\phi} = \left(\frac{1-w}{w}\right)^{\sigma}$, an increase in w has no effect on the total number of firms (varieties) in the cooperative arrangement and only affects the trade between producers.[9] Note that this contrasts with the non-cooperative arrangement where an decrease in w leads to a decrease in the total number of firms (see proposition 2). As with the non-cooperative equilibrium, however, since $L < 1$, from (70) we can see that an increase in the relative size of the external market leads to a lower total number of firms under cooperation, and comparing (70) with

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[9]Compare the trade equation in the non-cooperative equilibrium, where $m = \phi d$ and $\phi = \left(\frac{c(1-w)}{Pw}\right)^{\sigma}$. With cooperation, trade is valued not at the world market price, but at the marginal cost, resulting in *more* trade.

(60), cooperation enhances this 'external effect' on the total firm number. To summarize:

Proposition 4: Optimal Cooperative Procurement

In the case of the ratio form of the CSFs, in the optimal cooperative procurement arrangement, the total number of firms is independent of the preferences of producer countries for domestic supply. As with the non-cooperative equilibrium, given quality an increase in the relative size of the external market leads to a lower total number of firms under cooperation, which enhances this 'external effect' on the total firm number.

8   Numerical Results

We now turn to numerical solutions of the non-cooperative and cooperative outcomes. We address two sets of issues. First, we investigate the determinants of the number of weapons systems procured and thus industrial structure and attempt to explain the post-Cold War increase in concentration. We analyse a symmetric equilibrium of equal firms and producers; this can be thought of as the EU producers trading in isolation from the US and other major producers . As discussed in Dunne et al (2003), the number of firms has fallen far more than the decrease in world military expenditure, so in the model three mechanisms that can increase concentration are investigated. First, a rise in R&D costs as a proportion of output, second, a decrease in the bias for domestic production and third, an increase in the relative size of the external market of non-producers. This last mechanism could reflect, for instance, a shift in the military expenditure of non-producers from low-tech, possibly domestically produced weapons, to modern major weapons system. The second issue investigated is the effect of this increase in concentration and the rising share of R&D on the security and military expenditure of arms importing countries. The third issue involves the gains from the coordination of procurement decisions between producers.

8.1   Calibration of Parameter Values

By choice of units we can put $c = f = 1$. We exclude arms export regimes for now and put $\gamma = 1$. We examine a symmetric equilibrium of three countries (say, the UK, Germany and

France in a EU setting) so $\ell = 3$. We can calibrate the parameter α as follows. From the binding participation constraint we have that revenue equals total costs, $P(d + x) = Py$ where we recall that d =domestic procurement, x =exports and $y = d + x$=output, all per firm. In equilibrium the procurement price equals the international market price $P = \frac{c}{\alpha}$ where c =marginal cost (equals average production cost given our assumption of constant returns to scale). Thus we have

$$Py = \frac{c}{\alpha}y = \text{Total Costs(TC)} = F + fq^{\beta} + cy \tag{72}$$

where q is is quality. In (72) let us associate the second quality component of total costs with R&D, the third with variable cost leaving F as fixed capital cost. Denote the shares of fixed, R&D and variable cost in total cost as γ_F, γ_R and γ_V. Thus

$$\frac{cy}{\frac{cy}{\alpha}} = \frac{\text{variable costs}}{\text{total cost}} = \alpha = \gamma_V \tag{73}$$

From Dunne et al (2002), a reasonable value for $\gamma_V = 0.5$ for Europe which is therefore our chosen value for α.

In the rest of the calibration we restrict ourselves to the case of where the CSF of the non-producers is of the ratio form. Then G^{np} is constant. Define $\Phi = \frac{rG^{np}}{\ell G^p}$ to be the ratio of military expenditure in producer and non-producer countries. A reasonable value for this parameter is $\Phi = 0.5$. In our baseline calibration we assume no home bias in military procurement so $w = 0.5$.

The remaining parameters to be calibrated are $[F, \nu, \beta] = \Xi$, say. Given Ξ we can compute the non-cooperative equilibrium. Suppose that we have data for three outputs: firm number per country $n = \hat{n}$, R&D expenditure by firms as a proportion of output , $\hat{RD}$ and the subsidy as a proportion of the world price $s = \frac{p - P}{P} = \hat{s}$. In our baseline calibration we choose $\hat{n} = 35$, $\hat{RD} = 22\%$ and $\hat{s} = 7\%$. From the non-cooperative equilibrium we have a solution $n = n(\Xi)$, $RD = RD(\Xi)$ and $s = s(\Xi)$. Then Ξ can be calibrated as the solution to:

$$\hat{n} = n(\Xi)$$
$$\hat{RD} = RD(\Xi)$$
$$\hat{s} = s(\Xi)$$

The result of this exercise is a model calibrated in a non-cooperative equilibrium to be consistent with stylized facts regarding firm number, R&D expenditure and the level of

subsidies given to the defence industry. Clearly this procedure can be extended to other parameters such as α if we had more stylized facts. Our baseline calibration is summarized in the following table

Parameter	Value	Method and Source
c	1	normalization
f	1	normalization
γ	1	assumption (no arms exports)
α	0.5	calibration (data from Dunne et al, 2002)
ℓ	3	assumption (EU context)
w	0.5	assumption (no home bias)
Φ	0.5	data from Dunne et al (2002)
$\hat{n}$	35	data from Dunne et al (2002)
$\hat{RD}$	22%	data from Dunne et al (2002)
$\hat{s}$	7%	data from Dunne et al (2002)
F	0.00027	calibrated using non-cooperative equilibrium
β	1.5	calibrated using non-cooperative equilibrium
ν	1	calibrated using non-cooperative equilibrium

Table 1. Parameter Values

8.2 Changes in the Taste Parameter ν

Our first simulation allows ν to vary keeping Φ and β at their baseline values. Unless otherwise indicates, the CSF is of the ration form so that G^{np} is constant. Figures 1 and 2 how output from the model for this particular choice of parameters. Figure 1 shows the number of firms per country grow as variety per se is valued more. R&D expenditure as a % of output remains constant. Figure 2 shows the procurement price increasing from a level below the world market price at $\nu = 0$ (implying a tax) to the point where at $\nu = 1$, there is a modest subsidy with a procurement price around 7% above the world market price paid by importers confirming the result of our calibration in table 1.

With ν set at $\nu = 1$, we now examine the non-cooperative and cooperative outcomes as three parameters change: β the R&D investment parameter, w the domestic bias

parameter and Φ, the proportion of world demand for military goods coming from the external market of non-producers vary in turn from baseline values.

8.3   Process Innovation

In our next experiment we examine the effect of more process innovation and escalating fixed costs associated with R&D investment. In our model this effect is captured by a fall the the parameter β. In figures 3 and 5 we see that with a high value β, R&D as a proportion to total output is low, and comparing the non-cooperative equilibrium with the optimal cooperative arrangement the number of firms is too high. As β falls, $R\&D$ investments rises until at $\beta = 1.5$ we can reproduce data on R&D as a proportion of output which suggests figures of 20-25% (see Dunne et al (2002)). The firm number falls substantially seeing a total of around 160 at the low R&D end to around 100 at $\beta = 1.5$, at the latter high-investment end an absence of cooperation sees an insufficient number of firms and an excessive production of quality. The beggar-thy-neighbour aspect of quality in the external market drives this result. When countries order high-tech, high quality specifications for domestic procurement, acting independently they improve the competitiveness of their exports to the external market. In a Nash equilibrium however these gains are wiped out: R&D expenditure is high but there is no improvement in competitiveness. A subsidy (seen in figure 4 where the procurement price exceeds the world price is then required and figure 6 shows that the gains from cooperation between producer countries (to those countries) rises substantially with more process innovation.[10]

In the previous simulations we have assumed the ratio form of the CSF for non-producers; in figure 7 we compare results for the ratio and difference forms as β varies in the close vicinity of $\beta = 1.5$. As we have shown in section 3.1, military expenditure of importers as a percentage of GDP is constant with a ratio CSF whilst with the difference CSF, it increases if the number of weapons systems, N, or the quality rise. As β rises, N increases but quality falls. The former dominates so in figure 7 the military expenditure of importers rises with β. Clearly, with the difference form of the CSF, importers military spending is very sensitive to this small change and we find this is the case for small

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[10]Let U^C and U^{NC} be the utilities under cooperation and non-cooperation, respectively. Utility loss is then defined as $\frac{U^C - U^{NC}}{U^C} \times 100$.

changes to other parameter values as well. Large fluctuations in military expenditure are not observed in the world and this suggests that the difference form of the CSF function is not appropriate, at least in its pure form. In a discussion of the choice of CSF function, Hirshleifer (2000) suggests a model consisting of a combination of the ratio and difference forms. With a low weight on the difference form, this will allow for some response of importers' military expenditure to changes in the number of weapons systems and quality. The remaining simulations use the ratio form of the CSF.

8.4   Changes in Domestic Procurement Bias

In our third experiment we allow the domestic procurement bias parameter, w to increase from $w = 0.5$ to $w = 1$ at which point producing countries are self-sufficient, and only exporting to non-producers.

In figure 8 the number of firms per country in the non-cooperative equilibrium first falls and then rises rises with w. As countries become more self-sufficient they internalize the benefits of variety arising from $\nu > 0$ and choose to support more domestic firms. On the other hand from figure 12, they also internalize the benefits of investment into quality, raising fixed costs and tending to reduce the number of firms. The net effect is the ∪-curve. Under cooperation firm number is independent of w as predicted by proposition 4. In Figure 8, for higher values of w, both firm number and quality increases in the noncooperative equilibrium and consequently, from figure 9 the subsidy (procurement price minus the world price) also increases. From proposition there are two affects at work here: taste for variety $\nu > 0$ tends to encourage subsidy whilst the external market effect encourages the opposite (a tax on export profits). With our parameter values in the non-cooperative equilibrium the former effect dominates for higher values of w.

Figure 10 shows total output per firm in the non-cooperative equilibrium broken down into exports to non-producers and producers and domestic procurement. As w increases exports to producers fall and initially output is diverted to domestic procurement. With the increase in the number of firms, the total size of each firm falls and all three components eventually fall for higher values of w. The utility loss to producers from failing to cooperative are shown in figure 11. Considering the welfare of producers only, in the absence of cooperation are 'too few' firms and they produce 'too much quality'. The latter

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is shown in figure 12 which plots R&D as a proportion of the total output of the firm.

8.5 Changes in the Composition of World Demand

In our final experiment we allow the proportion of world demand from non-producers, $\Phi = \frac{rG^n}{\ell G^p}$ increase from $\Phi = 0.5$ towards unity. Figures 13 to 17 show the numerical results. Figure 13 shows that the the subsequent fall in the firm number under both non-cooperation and cooperation as Φ rises as as before there are too few firms and too much quality in the absence of cooperation. From figure 14 these changes in industry structure are brought about by initially a subsidy under non-cooperative giving way to a tax at higher values of Φ. The optimal (cooperative) procurement price for the producers, by contrast, involves a substantial tax throughout the full range of Φ. All these results are consistent with the results of propositions 2 to 4.

A falling number of firms as Φ rises is associated with a rise in the size of each firm. Figure 15 shows this happening and a switch of output from domestic procurement and internal trade to the external market. Figure 16 shows that the gains to cooperation between producers rise substantially as the external market becomes more important. This is largely the result of excessive investment into quality as figure 17 shows, but a close examination of figure 13 reveals that the difference in firm number between the cooperative and non-cooperative outcome also rises contributing to this welfare deficiency.

9 Conclusions

In order to examine a number of developments associated with the RMA, this paper has constructed a partial equilibrium model of military procurement with two-way international trade in a world where many of the recipients of this trade are non-producers engaged in regional arms races. We have studied the consequences for industry structure, procurement subsidies and the welfare of producers of: increased process innovation and the consequent rise in the fixed costs of R&D; increased trade between producers owing to the willingness to forgo the security benefits of domestic procurement; and a growing external market of non-producers.

Our comparison of the non-cooperative and cooperative outcomes has highlighted two sources of inefficiencies from the position of the producers. Adopting a generalized Dixit-

Stiglitz utility function to model military strength, the existence of a separate taste-for-variety parameter (ν) means governments acting independently choose to support too few firms and generate too little variety. Competition for the external market generates two effects. First export revenue from exports rises as the total number of firms falls. This occurs because firms then compete less intensively and can spread their fixed costs over a larger market share. As the external market increases, governments then choose to support *less* firms. Under non-cooperation, however, this reduction in firm number is too little compared with the optimum because governments acting independently only care about competition between domestic firms.

The second effect of an external market is to encourage too much investment in quality relative to the cooperative outcome. This is because the provision of quality has a beggar-thy-neighbour character. When governments raise quality unilaterally this increases market share. In equilibrium however the benefit to competitiveness disappears and countries are left with too much quality compared with that chosen cooperatively. A high investment into this quality raises fixed costs and reduces firm number further. The presence of an external market then tends to reduce firm number (i.e., raise concentration) and encourages excessive investment into quality. A high taste-for-variety has the opposite effect, raising firm numbers which, because there is a trade-off between quality and variety, reduces quality.

A number of important issues have not been addressed, the most important being the effect of these changes in industry structure and process innovation on the military expenditure and welfare of non-producers. In the ratio form of the CSF used in our main simulations there are no such affects, but with the difference form of CSFs, the military expenditure of non-producers rises as the number and quality of military goods increase. We have seen that with a pure difference form, the elasticity of military expenditure of non-producers with respect to the total number of varieties on offer is implausibly high; the same is true of the elasticity with respect to quality. However some linear combination of the ratio and difference form could be reconciled with observed variations in expenditure. Since cooperation reduces the level of quality (but can raise the total number of goods, unless the taste-for-variety parameter is small) the possibility emerges that if the quality effect dominates, a common defence policy in the EU can *be mutually beneficial to both*

producers and recipients of arms. This will be investigated further in future work.

References

Anderton, C. H. (2000). Ratio and logistic conflict technologies. *Journal of Conflict Resolution*, **44**(6), 823–838.

Beath, J. and Katsoulacos, Y. (1991). *The Economic Theory of Product Differentiation.* Cambridge University Press.

Benassy, J.-P. (1996). Taste for variety and optimum production patterns in monopolistic competition. *Economic Letters*, **52**, 41–47.

Dunne, J. P., Garcia-Alonso, M., Levine, P., and Smith, R. P. (2002). Concentration in the international arms industry. *Mimeo, Presented at the ECAAR Panel Session American Economic Association/ASSA Annual Meetings, Atlanta, January 2002.*

Dunne, J. P., Garcia-Alonso, M., Levine, P., and Smith, R. P. (2003). The Economics of the Revolution in Miltary Affairs. *Mimeo, Prepared for the 38th Economic Policy Panel, Rome 17-18 October 2003.*

Garcia-Alonso, M. (1999). Price competition in a model of the arms trade. *Defence and Peace Economics*, **10**(3), 273–303.

Garcia-Alonso, M. (2000). The role of technology in a model of horizontal differentiation. *International Journal of Industrial Economics*, **18**(5), 747–773.

Garcia-Alonso, M. and Levine, P. (2003). Strategic procurement, openness and market structure. *Mimeo.*

Garfinkel, M. R. and Skaperdas, S. (2000). Conflict without misperceptions or incomplete informamtion. *Journal of Conflict Resolution*, **44**(6), 793–807.

Hirshleifer, J. (2000). The macrotechnology of conflict. *Journal of Conflict Resolution*, **44**(6), 773–792.

Krugman, P. R. (1979). Increasing returns, monopolistic competition, and international trade. *Journal of International Economics*, **9**(4), 469–479.

Levine, P. and Smith, R. (2000). The arms trade game: from laissez-faire to a common defence policy. *Oxford Economic Papers*, **52**, 357–380.

Levine, P., Mouzakis, F., and Smith, R. (2000). Arms export controls and emerging domestic producers. *Defence and Peace Economics*, **11**, 505–531.

Skaperdas, S. (1991). Conflict and attitudes towards risk. *American Economic Review*, **81**, 160–164.
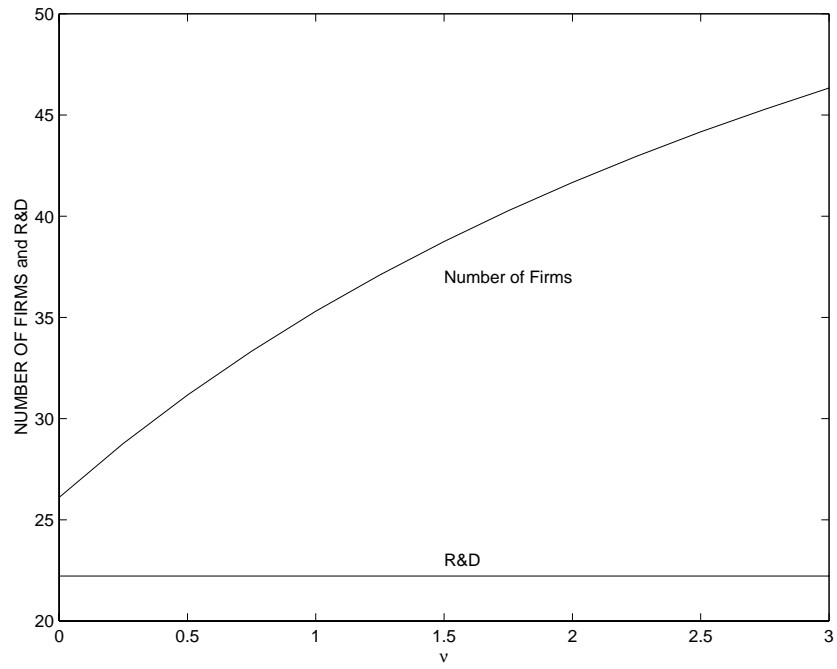
Figure 1: **Number of Firms per Country and R&D Expenditure as % of Output as ν increases.**
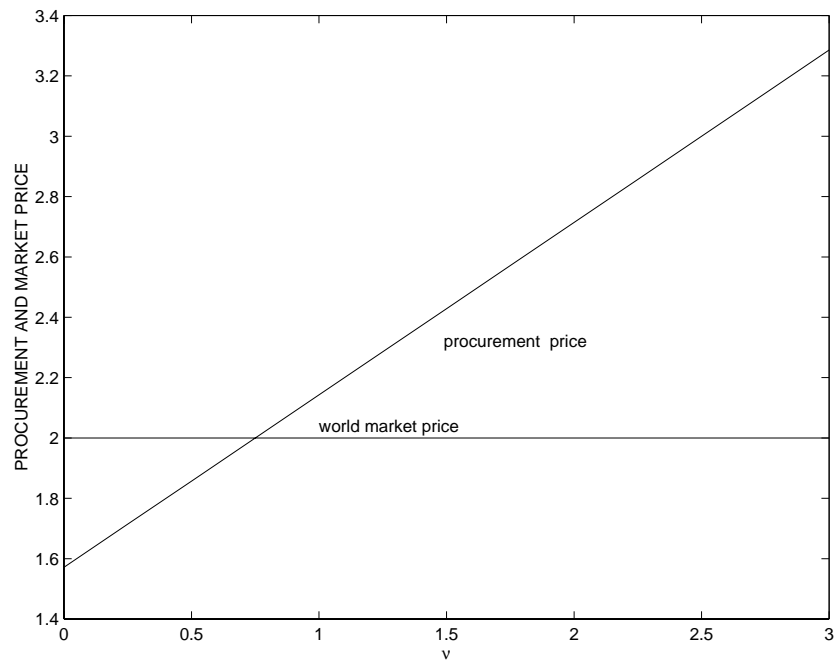
Figure 2: **Procurement and World Market Prices as ν increases.**

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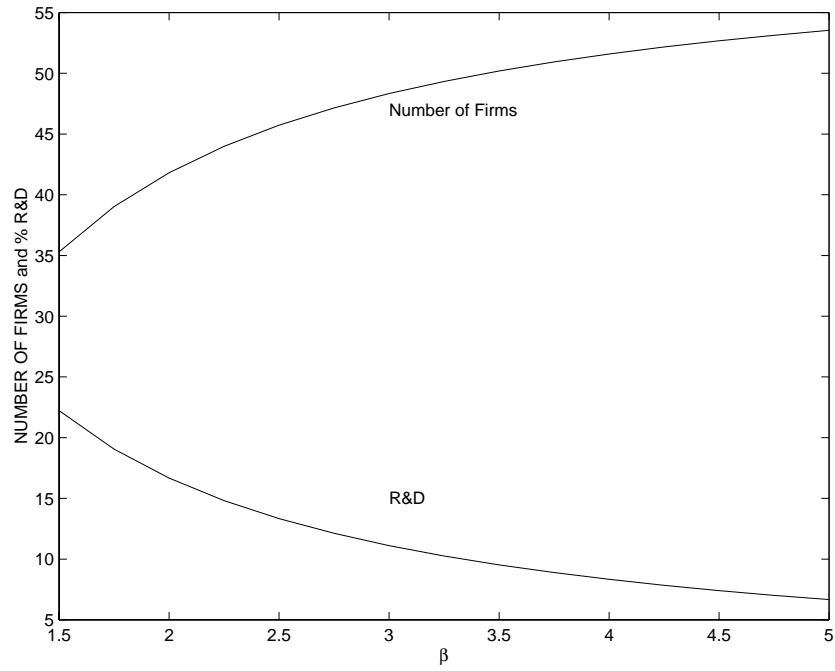
Figure 3: **Number of Firms per Country as β increases: Non-Cooperation compared with Cooperation.**
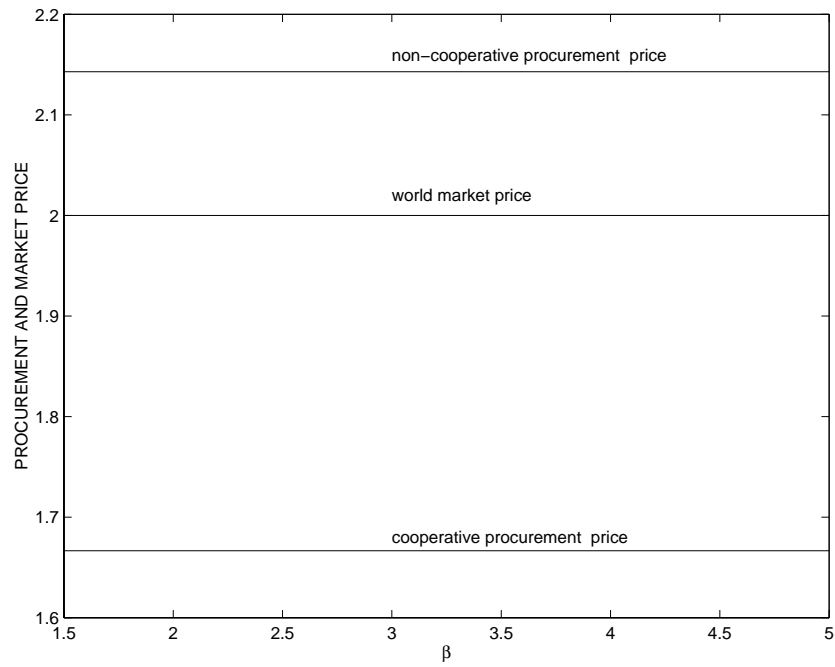
Figure 4: **Non-Cooperative and Cooperative Procurement and World Market Prices as β increases.**
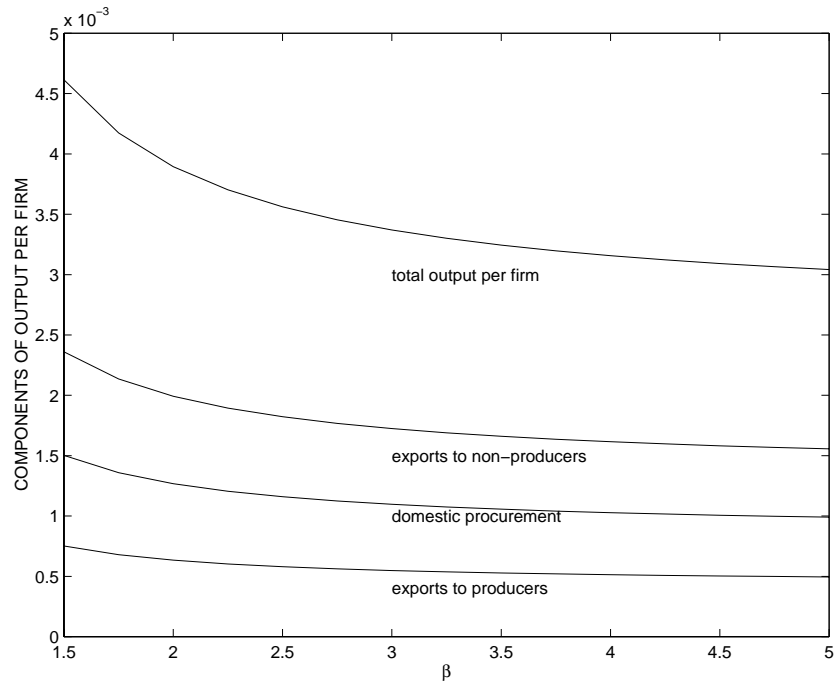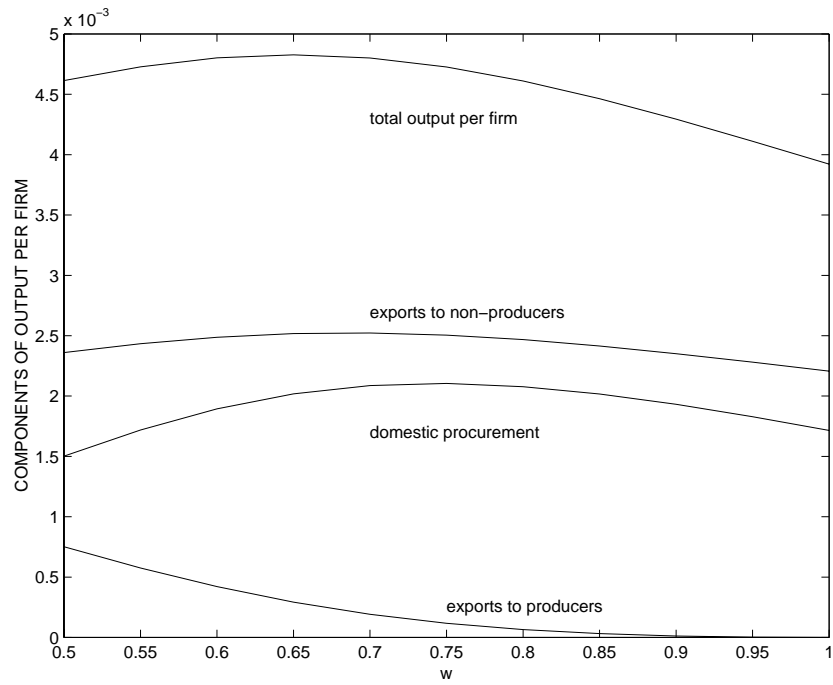
Figure 5: **R&D Expenditure as a Proportion of Output as β increases.**
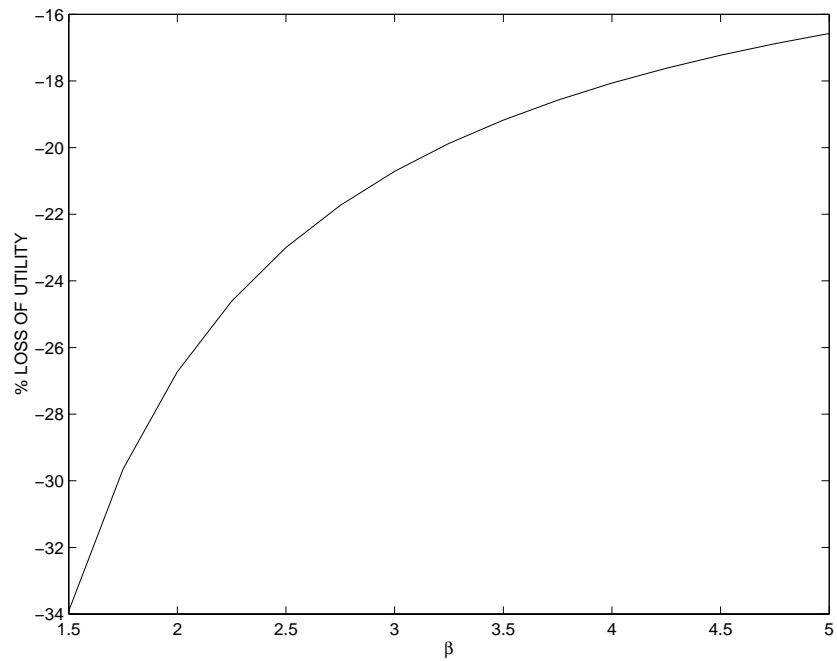
Figure 6: **Loss of Utility: Cooperation compared with Non-Cooperation as β increases.**
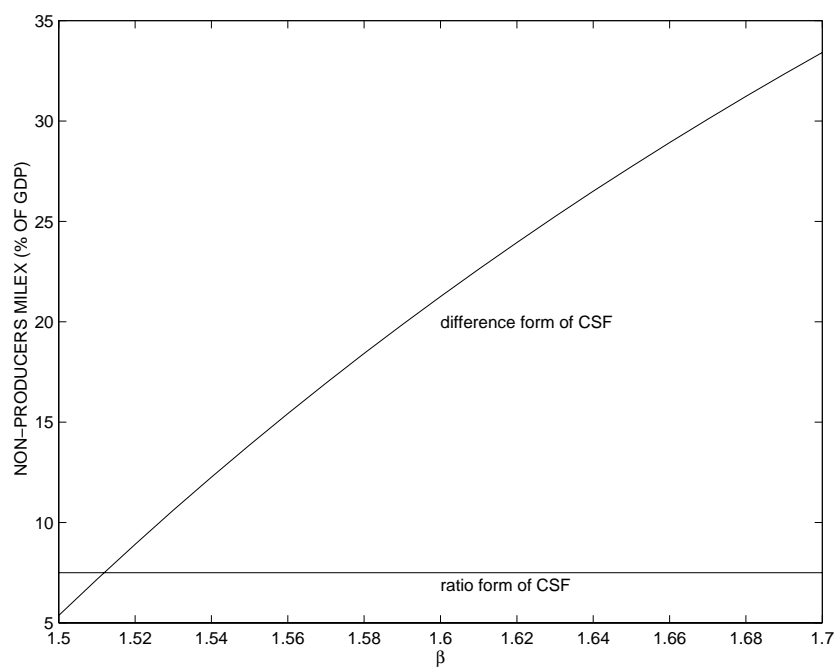
Figure 7: **Non-Producers' Military expenditure as a % of GDP' as β increases. Ratio and Difference Form of CSF Compared.**
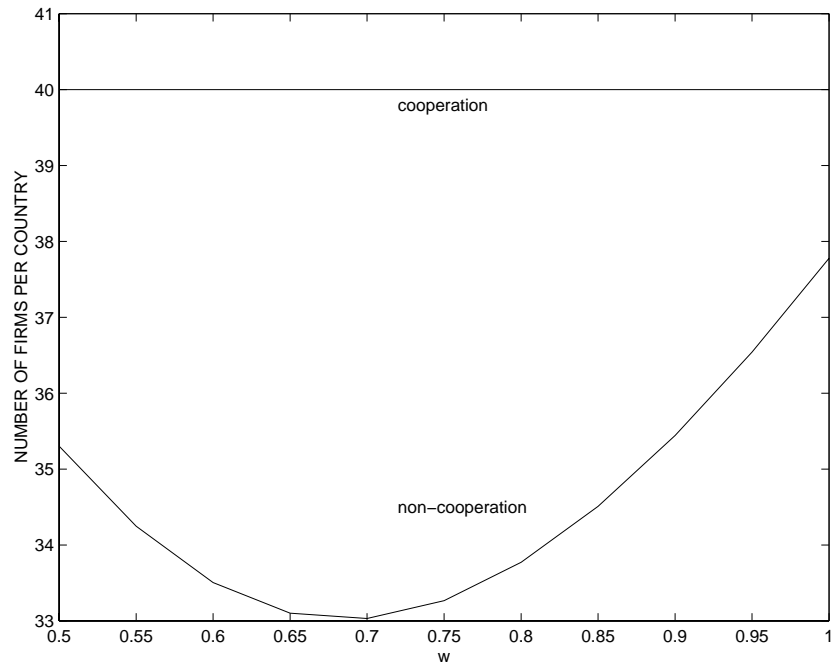
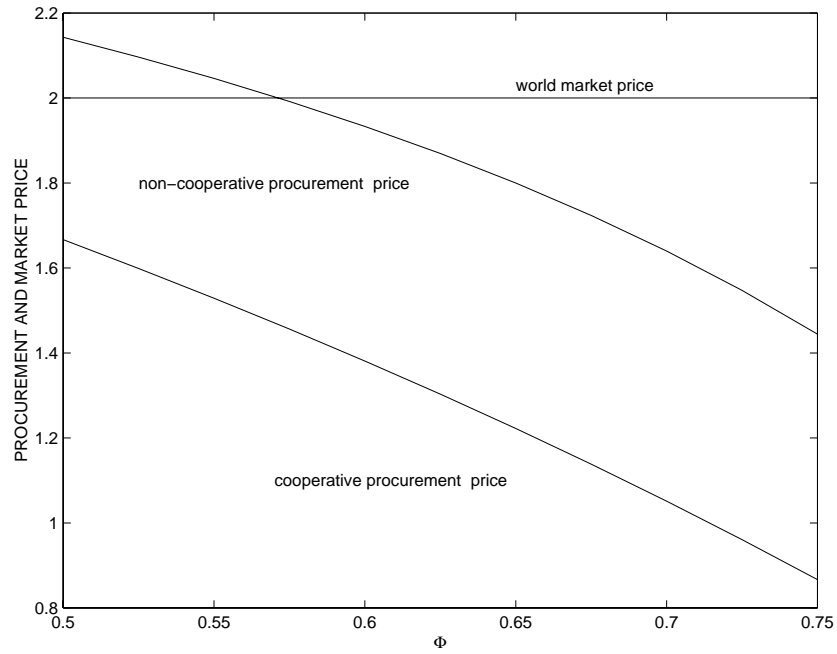Figure 8: **Number of Firms per Country as w increases: Non-Cooperation compared with Cooperation.**
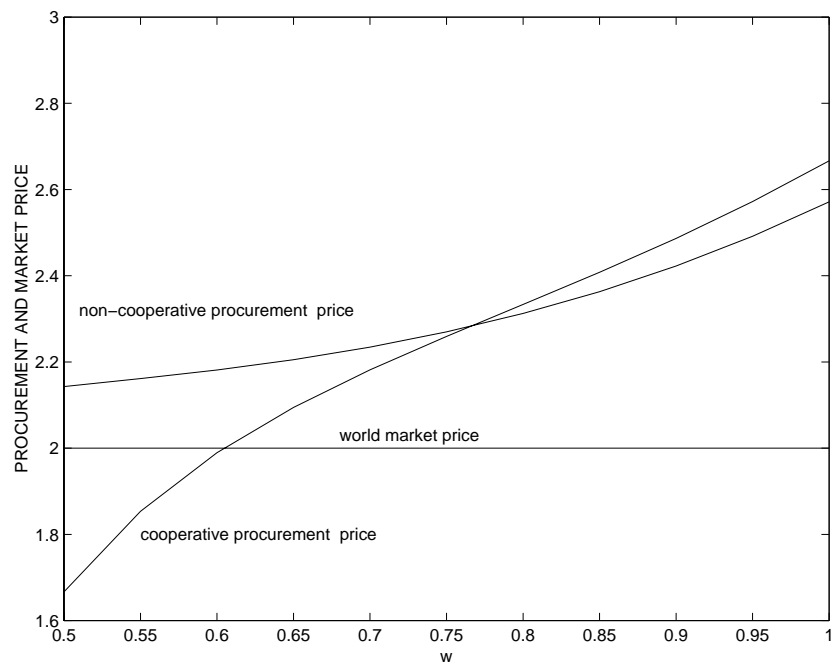
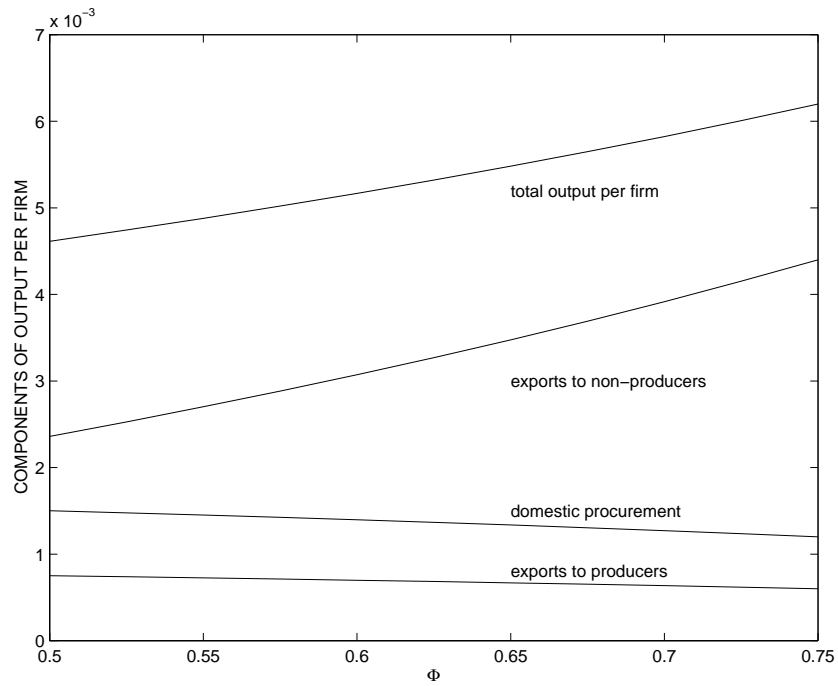Figure 9: **Non-Cooperative and Cooperative Procurement and World Market Prices as w increases.**

Figure 10: **Components of Output per Firm in the Non-Cooperative Equilibrium as w increases.**
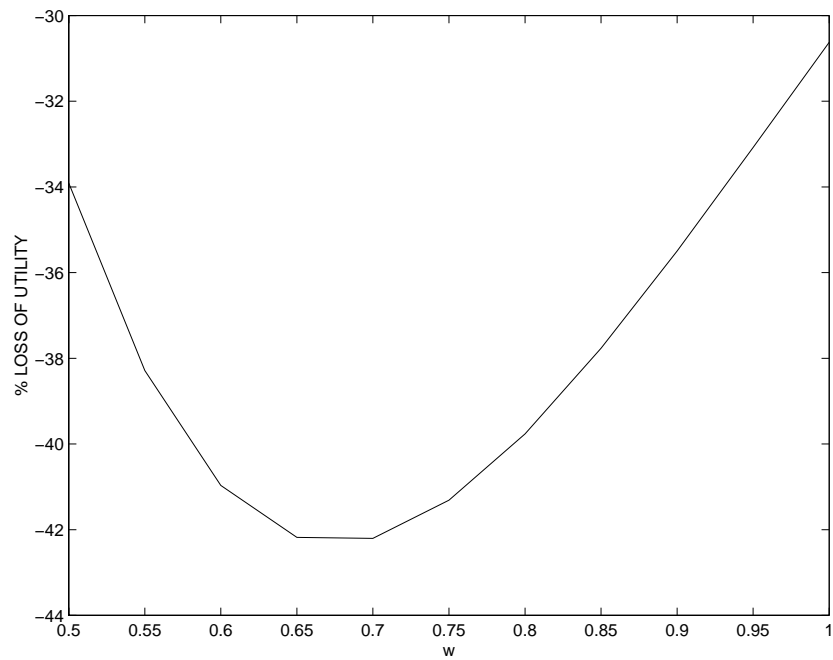
Figure 11: **Loss of Utility: Cooperation compared with Non-Cooperation as w increases.**

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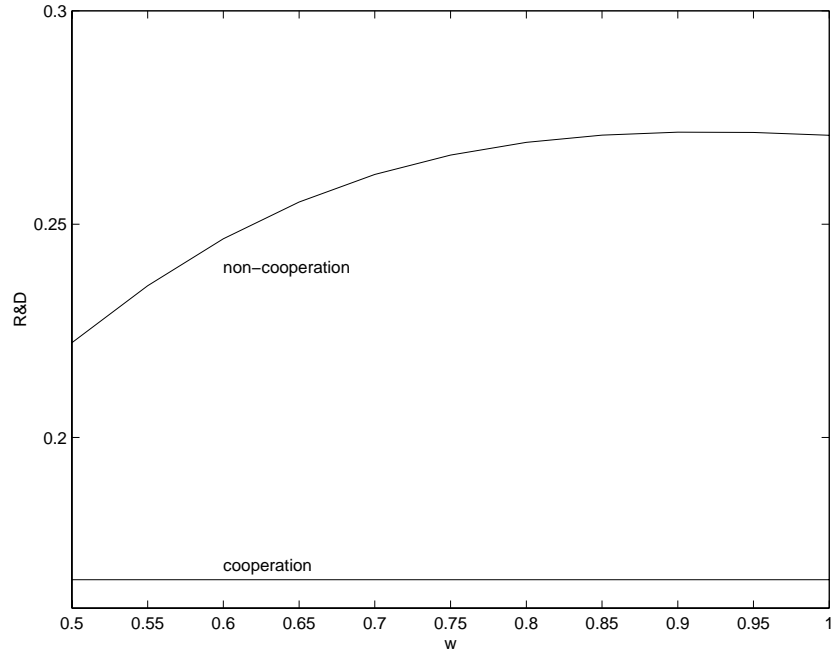
Figure 12: **R&D Expenditure as a Proportion of Output as w increases:**
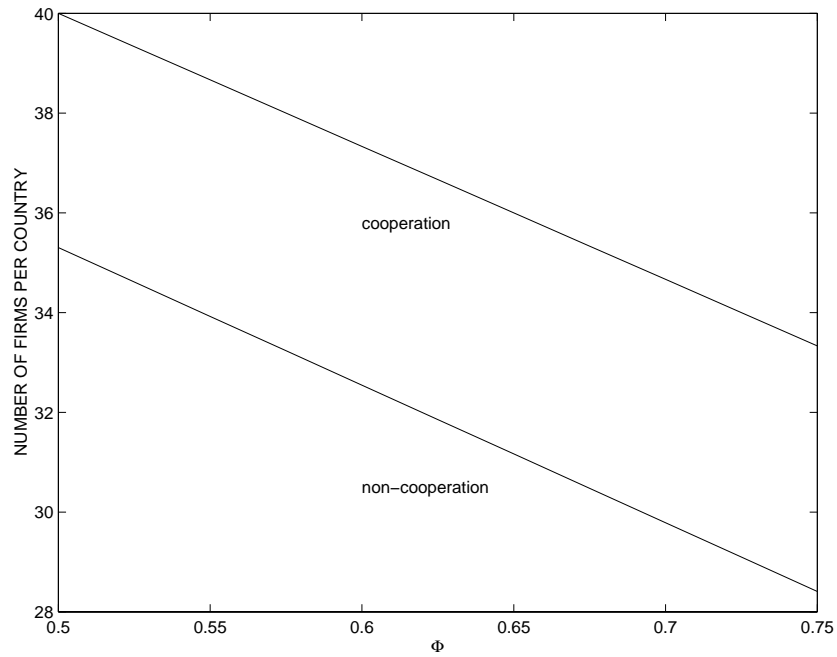
Figure 13: **Number of Firms per Country as Φ increases: Non-Cooperation compared with Cooperation.**

Figure 14: **Non-Cooperative and Cooperative Procurement and World Market Prices as Φ increases.**

Figure 15: **Components of Output per Firm in the Non-Cooperative Equilibrium as Φ increases.**
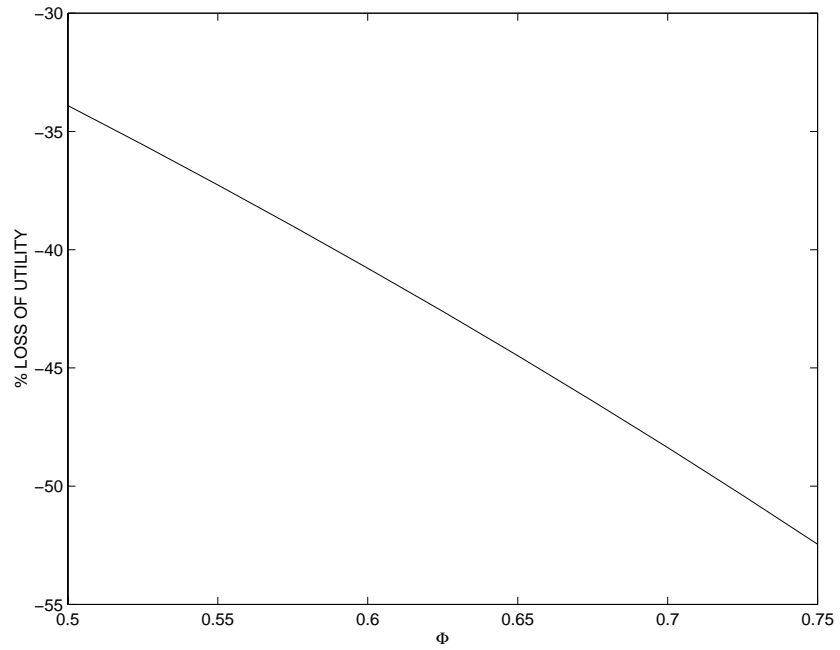
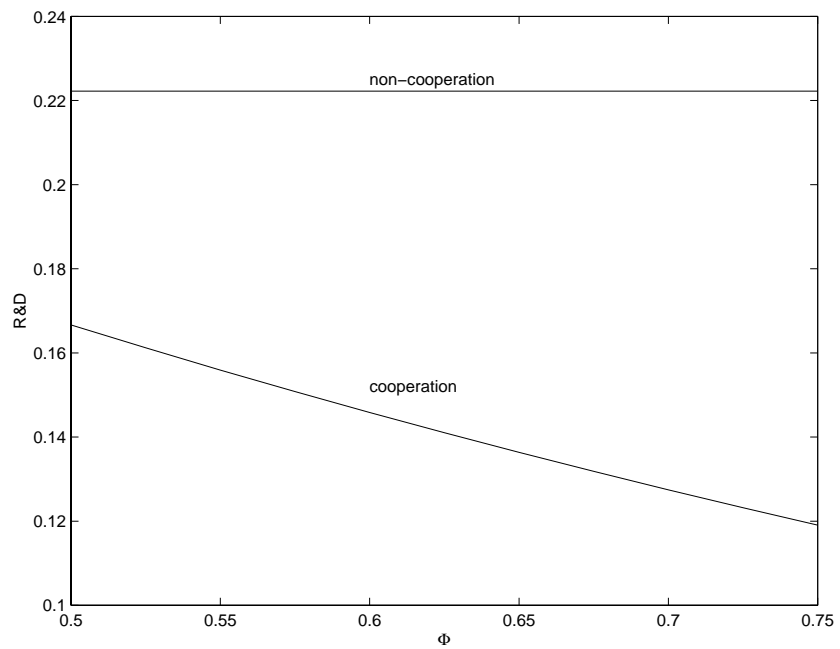Figure 16: **Loss of Utility: Cooperation compared with Non-Cooperation as Φ increases.**

Figure 17: **R&D Expenditure as a Proportion of Output as Φ increases:**

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