Managing Asymmetric Conflict*

Paul Dunne
U.W.E.

María del Carmen García-Alonso
University of Kent

Paul Levine
University of Surrey

Ron Smith
Birckbeck College

February 13, 2004

Abstract

The paper proves that the possibility of differentiation from an incumbent power's technology by a contestant group makes it harder for the incumbent power to want to implement peace by means of effort. In some cases, it may make it altogether unfeasible. The incumbent power is then more likely to adopt a defensive strategy to cope with a defensive type of conflict. This strategy involves less effort than the one that would be made if a defensive strategy was chosen to be induced in the absence of possible differentiation by the incumbent.

*We are grateful to the ESRC for support under grant R00239388. Correspondence to M.C.Garcia-Alonso@kent.ac.uk. Department of Economics, University of Kent, Canterbury, Kent CT2 7NP, UK.

Contents

1 Introduction

Military planning during the Cold War was reassuringly simple. Both sides knew who, how and where they would fight should war come to battle and there were a range of models of the process, developed from Intrilligator (1975).

However, the security perceptions of the most developed countries in the world have changed dramatically during the past few decades. During the Cold War the adversary was another country or set of allied countries with similar technological capabilities. This resulted in a vertical arms race in which each party tried to match, if not supersede the technological frontier set by the enemy, therefore, effort/quality was the main strategic variable (Walker, 1994). Significant resources were devoted to the development of increasingly sophisticated military technologies resulting in an oversized national military industry, protected by allied governments and closed to the market discipline of their civilian counterparts.

The end of the Cold War led to a cooling period in which resources poured into the defence sector were subject to scrutiny and initially decreased. The emphasis was then shifted towards the rationalization of the defence industry, the need for some short of internal discipline that could allow countries to still maintain a viable national defence sector. The exports market then grew in importance as a source of justification for the ever increasing development and production costs that came with every new generation of weapons. Former allies and adversaries were now competitors for an exports market in which importer countries were usually involved in their own regional arms races or internal conflicts. The situation in this period made some exporter countries more than others, change their security perceptions. A potential danger was seen in the exports of technologically advanced weapons to potentially explosive conflict regions. This resulted in the development of a system of export controls and embargoes that would limit the quality and quantity of weapons exported to these countries[1] (this export control system has been unevenly adopted across the different arms exporters, signalling a variety of security concerns).

[1] Models in which Post Cold War security concerns and links with defence industry have been discussed by Garcia-Alonso (1999, 2000) Garcia-Alonso & Levine (2002), Garcia-Alonso & Hartley (2000), Levine Sen & Smith (1994), Levine and Smith (1995, 1997a,b, 2000a,b) Levine Mouzakis and Smith (2000).

During the past few years, a new element within the security perceptions of the main producers and exporters of conventional weapons has appeared. Some of the most recent conflicts in which these powers have taken part involve parties which are not importers of high tech weapons and which use ”technologies” which are very different from those used by their enemies. How and where countries will fight is much more uncertain and the available models are of limited use.

In the traditional arms race and conflict literature (Siqueira (2002) offers a recent example), the ”technology of conflict” is seen as a unidimensional variable. A higher amount of effort, translated into more or more advanced weapons, is the only technological decision conflicting parties take. In reality, the world is not so simple. Technological decisions are more likely to be multidimensional. Not only will effort and/or quality be chosen, but the type of weaponry and the training the opposing forces acquire will need to be chosen. This means that there is much greater scope for a potential asymmetry of technologies across potential combatants.

The objective of this paper is to present a model that captures the main features and interactions involved in a situation where the use of asymmetric conflict technologies can arise and to analyze the impact that asymmetric conflict has on the ability of main arm producers to deter conflict.

Asymmetric Warfare, the way opponents would respond to a dominant military power by fighting it in ways that the incumbent did not expect or prepare against, has become crucial to the security debate in recent times. This concept is not new; attacking in ways that your opponent did not expect has been the basis of strategy since at least Sun Tzu in the fourth century BC (Newman, 2000).

In recent times however, the contrast of conflict strategies among different groups has become ever more stark. In richest traditional weapon producing countries, RMA (Revolution in Military Affairs) is the last keyword to describe the latest technological development of traditional warfare: often called network-centric warfare, is the use of precision-guided weapons; advanced sensors; and communication, command and control networks; to speed up the ‘detect, decide, destroy’ cycle[2]. This rests on developments in electronics and infor-

[2] In military discussions it is often described by abbreviations like C4ISTAR (Command, Control, Communications, Computation (sometimes cryptography) Information (sometimes intelligence) Surveillance, Target Acquisition and Reconnaisance.

mation technology, often of commercial rather than military origin, which have enhanced the effectiveness of military systems. However, RMA is not revolutionary; rather it is a process of incremental technological changes which improves the way that traditional military operations (surveillance, target-acquisition, etc.) are used to combat traditional threats: other military units. Although incremental, this technology is very expensive and governments have been trying, with limited success to cut costs by using commercial practices and components, Lorel et al. (2000). There is also a wider notion of the RMA, which argues that the incremental technological change has produced a qualitative change enabling nations with the appropriate technology to achieve rapid dominance of the battlefield through 'shock and awe'[3] . However, on either notion RMA is the continuation of a traditional vertical arms race, incremental technical progress, putting effort into domination in terms of quantity and quality, rather than a fundamental revolution in ways of fighting, such as that following the introduction of gunpowder (Hirst, 2003). In contrast with the above strategy, the "enemies" that traditional weapon producers have had to face in recent conflicts are characterized by a use of tactics which are radically different not only in degree of technological sophistication but also in their horizontal characterization.

Definitions of Asymmetric Warfare have emphasized asymmetries in technology, i.e. what each side fights with; asymmetries in tactics, i.e. how each side fights; or asymmetries in the stakes, i.e. the costs of defeat to each side. Despite having massive technological and military superiority, the US withdrew from Lebanon in 1983, after a suicide bombing killed 241 troops, and from Somalia in 1993, after a battle in Mogadishu in which television covered the brutal treatment of two American corpses and one injured prisoner. In neither country did the US have large stakes. Indeed, much of the military discussion prior to the coalition attack on Iraq in Spring 2003 centred on the extent to which Iraq could naturalize technological superiority by using guerrilla tactics and urban warfare.

Since the US appears invincible in the traditional military warfare, adversaries have an incentive to resort to other types of warfare: asymmetric warfare, which can exploit other vulnerabilities of the US and its allies. The bombing of the USS Cole in Yemen in 2000 and the attack on the World Trade Centre on September 11th 2001 demonstrated US vulnerability to certain sorts of asymmetric attack. This suggests that while governments

[3] This unfortunate description of the US war style gained currency in the late 1990s, e.g. Ullman (2003).

have been trying to commercialise the military, to cut costs, their opponents have been trying to militarise the commercial, to produce new weapons. Fertiliser and fuel oil can be used as explosives and commercial aircraft can be used as missiles. The emergence of dual-use technology, with both military and civilian uses, is a double-edged sword for governments: procurement costs may well fall, but the weapons can become more readily available to terrorists. Not only are the technologies for weapons of mass destruction (WMD) - nuclear, biological and chemical - inherently dual use, but new information technologies also have potentially military applications, e.g. they can be used to maintain international terrorist networks and to exploit vulnerabilities in the infrastructure of the US and its allies.

Asymmetric conflict is most likely to arise when conflicting powers are themselves asymmetrical in nature. We can see some of the recent conflicts as being fought between incumbent and contestant powers which differ in some crucial elements:

Resource availability: An established power, with a huge lead in available resources, and a contestant power, with limited resources.

Uncertainty: Although uncertainty has always been a component in all conflicts, it has probably become ever more relevant now as it is more difficult for incumbent powers to identify threats and therefore, a potential conflict.

Inertia within the defence industry: Incumbent powers have military capabilities whose characteristics are very similar. The conventional military technologies that were developed during the Cold War continue to be the main focus of further incremental developments. An explanation for this could be the inertia that industry leaders have to evolve along the chosen technological path. Having achieved a technological lead in a certain type of conflict technology, incumbent countries may find it difficult to change it in response to new security challenges. A second factor that may explain persistence of the incumbent's conflict technology may be that it may be seen as the most suitable technology to fight the 'average threat' in a Post Cold War era where potential sources conflicts are not so easily foreseen. However, contestant powers do not share this 'problem', their constrained resources and dependency on traditional weapon imports may actually encourage them to find new ways of fighting, which could become an advantage in certain types of conflicts. Actually, it may be the very availability of resources that established powers enjoy the encouragement for contestants to design new potentially dangerous conflict technologies. Some evidence for this inertia is provided by Trajtenberg (2003) who shows that the shares of US defence R&D expenditures across different categories have not changed since September 11th, with 30% of R&D expenditures still being allocated to big weapon systems.

Costs of conflict: Different adversaries may have different valuations of the costs of conflict and these costs may vary with the type of conflict. An all out war is not perceived as having the same cost as a unilateral strike. These perceptions may also vary across parties as the environmental and human consequences of conflict are not necessarily equally valued.

Such characteristics imply that conflicts and potential conflicts are likely to be very different to those of the Cold War era.[4]

Some of the characteristics of asymmetric warfare can be captured using a simple model, which introduces a variation to the traditional conflict model in order to make it bidimensional and therefore allow for asymmetric conflict technologies. Probabilities of winning conflicts are increasing functions of owns effort and decreasing functions of the opponent's effort. We keep this structure but we introduce an additional parameter representing horizontal technological differentiation from an incumbent power.

Another element that we introduce is that the probability of winning a conflict –the Contest Success Functions– will depend on who attacks and who defends. If only one party attacks (the other acting defensive), the attacking party will have an advantage whenever the technology used for attach is different from the technology that the defending party has. The intuition is that if a party decides to attack with a missile, it will have an advantage (for same given efforts) if the defensive party does not have an advanced missile technology as they are not likely to have antimissile systems, same explanation would apply

[4]Interestingly, this situation is not too dissimilar from the developments in competitive behaviour in the computer industry. For a long time, IBM was the main player in the computing industry with a particular way of doing business and a well established customer base. In a few years, however, the industry changed dramatically, as instead of competing directly with the established leader, entrants opened up new market segments. Later, some of these entrants became challengers to IBM's power and those that prevailed where the ones who won the standardization war, namely Microsoft and Intel (Bresnahan and Greenstein, 1999; Sutton 1998). This final outcome was not in any way certain ex ante, a further characteristic shared with military conflict.

to biowarfare. In addition, asymmetric technology gives the contestant party an advantage if it is on the offensive. However if it is on the defensive, technology differentiation becomes a liability.

We assume there is one incumbent and one contestant power. Their interaction is represented by a multiple stage game. Throughout the game, the contestant's effort and the technological path of the incumbent are considered to be exogenous. The first assumption is based on the contestant being resource constrained, while incumbents are assumed to have more flexibility in the use of their funds. The second assumption is based on the technological inertia of the incumbent. The structure of the game is as follows, in stage 1, the incumbent chooses effort. In stage 2, the contestant chooses the type of technology to be acquired. Technological differentiation from the contestant is represented by a variable which increases as technological differences between the contestant and incumbent are wider and takes a value of zero if they are exactly the same type (even though they might be of different qualities, which is captured by the effort variable). Technology types could range from atomic devices, biological warfare, conventional weapons, internet warfare, etc. Finally, in stage 3, incumbent and contestant simultaneously decide whether to adopt an attack or defense strategy. The probabilities of winning the conflict will be determined by the attack/defend decisions of the two parties. If only one of the two parties adopts the attack strategy, such party will have an advantage with respect to the defensive party if its technology is different to that of the defending party. If both parties adopt an attack strategy, technological differences will cancel each other out and success probabilities will be based solely on effort levels. If nobody attacks, parties will have an exogenously given share of resources, which could also represent an agreed share of resources by means of peaceful negotiation.

A few assumptions are adopted. First, we assume that some of the contested resources are destroyed through conflict, destruction being higher in a conflict in which both parties adopt an attack strategy (in what follows, we will refer to this as mutual attack). Second, we assume that the incumbent's share of resources if no conflict erupts is bigger than the expected resources to be won through conflict.

Already with this simple model a few interesting results can be obtained. Under the model's assumptions and for a small entrenching advantage, a mutual attack strategy becomes a possibility only once we introduce technological differentiation. However, we prove that the contestant will block the mutual attack equilibrium by limiting technological differentiation. Given this, two possibilities remain as candidates for the equilibrium in the last stage of the game: no conflict and a conflict where the incumbent adopts a defensive strategy and the contestant adopts an attack strategy. Through choice of effort, the incumbent may be able to implement any of these two equilibria as the unique equilibrium in the final stage of the game. Interestingly, it may actually be "cheaper" in terms of effort for the incumbent not to induce peace, specially if the no conflict shares of resources and balanced towards the incumbent's side. If the incumbent targets a defensive conflict, when the contestant has the option of differentiation from the incumbent, the outcome is a lower level of effort by the incumbent. However, if the incumbent wanted to implement peace, when the contestant has the option of differentiation, the level of effort will actually be higher. From here we are able to conclude that it is less likely for a incumbent to want to implement no conflict when differentiation becomes a possibility. In this sense, a world where technological differentiation by the contestant is feasible becomes a less safe world as it is more difficult to avoid conflict.

A variation to the above model considers the existence of a fixed cost of technological differentiation. We prove that a reduction such cost will make it less likely for the incumbent to decide or even be able to prevent conflict by means of defence effort.

Our research is linked to the terrorism literature. It has been suggested in this literature (see e.g., Enders and Sandler (2002) and Sandler and Arce (2003)) that when a terrorist group has a choice of targets (those targets being different countries or different objectives within the same country) effort being put into defending one target may actually encourage terrorists to shift to the alternative option. In our paper, the discrete choice of targets becomes a continuous choice of conflict technologies into which the contestant in locked.

The remainder of the paper is organized as follows. Section 2 introduces the main features of the model. Section 3 finds the Subgame Perfect Nash equilibrium of the game with technological differentiation and compares it with the benchmark case in which technological differentiation is not present. Section 4 introduces a fixed cost of technological differentiation. Section 5 concludes the paper.

2 The Model

2.1 Structure of the game

In our model incumbent and contestant "play" a sequential game in which, the incumbent's effort, technological differentiation and the nature of the conflict are determined. Throughout the game, the challenger's effort and the incumbent's technology type are considered exogenous.

The interaction is represented by a three stage game:

Stage 1: The incumbent, a, chooses its effort, e_a with cost $C_a(e_a)$. The challenger's effort e_a is considered exogenous based on them using all their available funds, while the incumbent allocates the country's budget across a range of expenditures and so has more flexibility when it comes to choosing their effort.

Stage 2: The challenger chooses the type of technology to acquire, t the technology used by the incumbent is considered to be exogenous throughout the game.

Stage 3: Incumbent and challenger simultaneously decide whether to attack or defend and get their payoffs.

The game is solved backwards in order to find the Subgame Perfect Equilibrium which will contain the equilibrium effort level of the incumbent and the degree of differentiation by the contestant, which will then result in a unique pure strategy equilibrium in the third stage of the game.

2.2 Contest Success Functions

In the CSF the probability of winning depends on each side's choice to attack or defend and the ratio their efforts plus a variable t to represent the degree of horizontal differentiation of the challenger. This variable increases as the tactical or technological differences between the challenger and the incumbent widens and takes the value zero if they are exactly the same.[5] Technology types could range across atomic devices, biological warfare, conventional weapons, internet warfare, etc.

The form of the CSFs is now described. Note that the first argument in parenthesis bellow refers to the incumbent's strategy and the second refers to the contestant's strategy:

[5]Even though they are the same technologically, they may be of different quality, a feature that will be captured by the effort variable.

$P_a(A,D)$ = probability that incumbent has of winning if they attack and contestant defends.

$P_a(D,A)$ = probability that the incumbent has of winning if the contestant attacks and they defend.

$P_b(A,D)$ = probability that contestant has of winning if it defends and the incumbent attacks.

$P_b(D,A)$ = probability that contestant has of wining if it attacks the incumbent defends.

Similarly define $P_b(D,D)$, $P_a(D,D)$, $P_b(A,A)$ and $P_a(A,A)$. Then the first of our crucial set of assumptions is formalized by the following contest success functions (CSFs):

$$\begin{aligned} P_a(A,D) &= \frac{t+e_a}{\Phi+t+e_b+e_a} \leq 1. \\ P_a(D,A) &= \frac{\Phi+e_a}{\Phi+t+e_b+e_a} \leq 1. \\ P_a(A,A) &= \frac{e_a}{e_b+e_a} \leq 1. \\ P_a(D,D) &= \textit{status quo incumbent share} = s_a = 1-s_b, \end{aligned}$$

where $\Phi \geq 0$ is an exogenous parameter that represents the additional obstacles faced by the contestant given that incumbent is already entrenched (entrenching advantage), $t \geq 0$ is the technology location of the contestant, and e_a and e_b represent the incumbent and contestant's effort respectively. It follows that

$$P_b(D,A) = 1 - P_a(D,A) = \frac{t+e_b}{\Phi+t+e_b+e_a} \leq 1$$

and

$$P_b(A,D) = 1 - P_a(A,D) = \frac{\Phi+e_b}{\Phi+t+e_b+e_a} \leq 1.$$

Note that if $\Phi = 0$:

$$\begin{aligned} P_b(D,A) &\geq P_b(A,A) \geq P_b(A,D). \\ P_a(D,A) &\leq P_a(A,A) \leq P_a(A,D). \end{aligned}$$

where equalities apply if both sides adopt the same technologies.

The above CSFs contain some elements which are familiar in the conflict literature. The properties of the CSFs with respect to each of the party's efforts is standard and does not require much motivation. The "entrenching advantage" has also been used in the conflict literature in situations in which there is a defending and an attacking party, which might decide not to attack. In our model, we introduce two variations, first we allow for both parties to choose to attack or defend. If both parties decide to defend, the outcome is "peace", otherwise, we have a unilateral attack or a mutual attack situation. It seems intuitive to give a symmetrical treatment for the entrenching advantage whenever there is a unilateral conflict, that is, the defensive party will be the one that enjoys the entrenching advantage. However, when there is a mutual attack conflict the two entrenching advantages cancel each other out.

The second novelty of our approach is the introduction of technological differentiation by the contestant. It is interesting to notice that, this will now be a variable to be decided by the contestant only. Its impact on the CSFs will have the reverse effect to that of entrenching. Differentiation will provide an advantage to the attacking party in unilateral conflicts. This can be seen as a "technological surprise" factor, or also, the ability that the attacker can have to attack the weakest point. Similarly, in a mutual attack situation, "technological surprise" advantages cancel out and therefore disappear from the CSF.

For example if one party decides on a missile attack, they will have an advantage (both sides making similar efforts) if the defensive party does not have advanced missile technology, as this means they are unlikely to have the associated technology to produce anti-missile systems. Similarly, with biological warfare, the attacker will have an advantage if the defender does not have biological weapon capability, as they are unlikely to have specific defences against such attack. Asymmetric technology can give a contestant an advantage if they are on the offensive but, if they are on the defensive, technology differentiation can become a liability. However, if both parties attack, the technological advantages will cancel each other out and success probabilities will be solely based on effort levels. If nobody attacks, parties have an exogenously given share of resources, which could also represent an agreed share of resources by means of peaceful negotiation or the status quo share of resources.

2.3 Payoffs

Given stage 3 strategies (S_a, S_b), S_i = (Attack, Defend) by incumbent and contestant respectively, the expected utilities of the incumbent and contestant are:

$$\begin{aligned} EU_a(S_a, S_b) &= P_a(S_a, S_b)\phi_a(S_a, S_b)V - C_a(e_a), \\ EU_b(S_a, S_b) &= P_b(S_a, S_b)\phi_a(S_a, S_b)V - C_b(e_b), \end{aligned}$$

where V is a fixed amount of rent or resources that both incumbent and contestant are aiming to control, $e_i, i = a, b$ are resources devoted to fighting with cost $C(e_i)$. As already noted, the effort of the contestant, e_b, is exogenous throughout. Finally, $(1 - \phi_a(S_a, S_b))V$ represents the cost of being involved in a conflict for the incumbent and similarly $(1 - \phi_b(S_a, S_b))V$ represents the cost of being involved in a conflict for the contestant. If both countries defend, there is no war and these costs are not incurred. A crucial feature in our model is the introduction of a cost of conflict, which is sensitive to conflict type.

We assume that *(a)* the resources destroyed through conflict are higher in a conflict in which both parties adopt an attack strategy (in what follows we will refer to this as mutual attack); *(b)* the incumbent's share of resources if no conflict erupts is greater than the share it would expect to get through conflict; *(c)* both incumbent and challenger are rational players. Assumption *(a)* is very important for the results below. Assumption *(b)* will actually prevent a unilateral attack by the incumbent from being a candidate for equilibrium at the last stage of the game. This assumption seems reasonable when we consider an incumbent power with a big peacetime share of resources that does not, therefore, have a pure economic incentive to go to war.

Also note that we assume that choosing a different technological path from that of the incumbent does not have a secondary cost attached to it. In this sense, we treat differentiation in a way similar to the treatment of location, representing horizontal differentiation, in Hotelling type industrial organisation models.

2.4 The benchmark model without differentiation

In this section, we solve the game for the case when technological differentiation is not possible, we then have a two stage game where incumbent decides effort in the first stage and incumbent and contestant simultaneously decide whether to attack or defend in the second stage. The associated winning probabilities are equivalent to those already described but with $t = 0$.

2.4.1 Simultaneous choice of defend or attack

The payoff matrix at third stage of the game is given by:

$a\backslash b$	$ATTACK$	$DEFEND$
$ATTACK$	$P_a(A,A)\phi_a(A,A)V,\ P_b(A,A)\phi_b(A,A)V$	$P_a(A,D)\phi_a(A,D)V,\ P_b(A,D)\phi_b(A,D)V$
$DEFEND$	$P_a(D,A)\phi_a(D,A)V,\ P_b(D,A)\phi_b(D,A)V$	$s_aV,\ s_bV$

Note that, the cost of effort does not appear in the payoff matrix because they are considered sunk at this stage. We now consider the four possible candidates for Pure Strategy Nash equilibria in the last stage of the game.

NASH EQUILIBRIUM 1 : $(ATTACK, ATTACK)$

$$\begin{aligned} \phi_b(A,A)\,P_b(A,A) &\geq \phi_b(A,D)\,P_b(A,D) \\ \phi_a(A,A)\,P_a(A,A) &\geq \phi_a(D,A)\,P_a(D,A) \end{aligned} \tag{1}$$

If technological differentiation is not possible, this will not be a Nash Equilibrium based on the assumption that the costs of a mutual attack conflict are bigger than those of a unilateral conflict, $\phi_a(A,A) < \phi_a(D,A)$.

NASH EQUILIBRIUM 2: $(DEFEND, DEFEND)$

$$\begin{aligned} s_b &\geq \phi_b(D,A)\,P_b(D,A) \\ s_a &\geq \phi_a(A,D)\,P_a(A,D) \end{aligned} \tag{2}$$

In this case, $(DEFEND, DEFEND)$ will be the a Pure Strategy Nash Equilibrium.

NASH EQUILIBRIUM 3: $(DEFEND, ATTACK)$

$$\begin{aligned} s_b &\leq \phi_b(D,A)\,P_b(D,A) \\ \phi_a(D,A)\,P_a(D,A) &\geq \phi_a(A,A)\,P_a(A,A) \end{aligned} \tag{3}$$

Note that $\phi_a(D,A)\,P_a(D,A) > \phi_a(A,A)\,P_a(A,A)$ due to our assumptions. Therefore, this will be a Pure Strategy Nash Equilibrium as long as $s_b \leq \phi_b(D,A)\,P_b(D,A)$.

NASH EQUILIBRIUM 4: $(ATTACK, DEFEND)$

$$\begin{aligned} s_a &\leq \phi_a(A,D)\,P_a(A,D) \\ \phi_b(A,D)\,P_b(A,D) &\geq \phi_b(A,A)\,P_b(A,A) \end{aligned} \tag{4}$$

We reject this as a candidate for Nash Equilibrium by assuming $s_a > \phi_a(A,D)$.

Altogether, we are left with two candidates for Nash Equilibrium, the choice of effort in the first stage of the game will determine which of those two is part of the Subgame Perfect Nash Equilibrium Strategy of the game.

2.4.2 Incumbent chooses effort

We first define $e_a^{(D,D,t=0)}$ and $e_a^{(D,A,t=0)}$ as the levels of effort the incumbent would put depending on whether (D,A) or (D,D) was the target.

From expression (2), note that for (D,D) to be induced we need e_a to satisfy

$$\begin{aligned} P_b(D,A)\phi_b(D,A) &= \frac{e_b\phi_b(D,A)}{\Phi+e_b+e_a} \leq s_b \\ P_a(A,D)\phi_a(A,D) &= \frac{(\Phi+e_a)\phi_a(A,D)}{\Phi+e_b+e_a} \leq s_a \end{aligned}$$

Assuming that the cost of conflict for the incumbent is bigger that its peace share of resources, $\phi_a(A,D) < s_a$, the incumbent would need to ensure at least $\phi_b(D,A)\,P_b(D,A) \leq s_b$. If $s_b < \phi_b(D,A)$, some effort will be required to ensure that:

$$\frac{e_b\phi_b(D,A)}{\Phi+e_b+e_a} \leq s_b \Longleftrightarrow \frac{e_b(\phi_b(D,A)-s_b)-s_b\Phi}{s_b} \leq e_a.$$

Therefore, the minimum amount of the incumbent's effort required to implement peace is

$$e_a^{(D,D,t=0)} = \frac{e_b(\phi_b(D,A)-s_b)-s_b\Phi}{s_b} \tag{5}$$

If the above effort is implemented, peace will be the unique Nash Equilibrium of the last stage of the game. On the other hand, if the incumbent wanted to induce (D,A),he would put effort that maximizes $EU_a(D,A)$.

$$\frac{dEU_a(D,A)}{de_a} = V\frac{e_b\phi_a(D,A)}{(\Phi+e_b+e_a)^2} - \frac{dC(e_a)}{de_a} = 0. \tag{6}$$

We calculate the equilibrium $e_a^{(D,A,t=0)}$ for $\frac{dC(e_a)}{de_a} = c$ (it can be checked that $EU_a(D,A)$ is concave even with constant marginal costs), where c is a positive constant

$$e_a^{(D,A,t=0)} = \sqrt{V \frac{\phi_a(D,A)\, e_b}{c}} - (\Phi + e_b). \tag{7}$$

The decision of the incumbent of whether to induce (D,A) or (D,D) will depend crucially on the amount of effort required to prevent conflict relative to the other effort. This issue will be discussed this in depth in the following section.

3 Subgame Perfect Equilibrium of the game with differentiation

In this section we obtain the Subgame Perfect Equilibrium of the conflict game when technological differentiation is available.

3.1 Stage 3: Simultaneous choice of defend or attack

Although the CSFs are now different, the same conditions as in the benchmark case need to apply for each possible strategy combination to be a candidate for a Nash Equilibrium of this stage of the game. An important difference arises in terms of possible candidates for Nash Equilibrium, a mutual attack type of conflict now becomes a possibility. This is due to the introduction of technological differentiation on the side of the contestant. Excessive differentiation can now compensate for the incumbent's higher perceived costs of mutual attack conflicts, $\phi_a(D,A) > \phi_a(A,A)$, therefore, making attack the best response to a contestant's attack. In the following subsections, we analyze how this new element in the game will affect the choice of effort and also the choice of conflict type that will be induced using the incumbent's effort.

3.2 Stage 2: Choice of differentiation

In the second stage, the contestant chooses the type of technology it wants to acquire. More specifically, it chooses t, which represents the distance from the type already acquired by the incumbent. This distance represents horizontal technological differentiation. In our analysis, we assume that there is no cost specifically associated to choosing a technology which is different from the incumbent. The only cost would be the cost associated with effort, which is exogenous and sunk at this stage. This assumption gives technological differentiation in our model a similar flavour to that of location in location type models. Also, for the purposes of our analysis technological differentiation is assumed to be unbounded. However, we will see how, despite this, the contestant chooses to limit its degree of differentiation.

We now consider the different candidates for Nash Equilibrium in the last stage and how and which the contestant will want to implement, using differentiation and depending on the incumbent's effort.

Consider the possibility of an equilibrium type 2, (D, D). From expression (2) and the CSFs, this occurs if the combination (e_a, t) satisfies

$$\begin{aligned} P_b(D,A)\phi_b(D,A) &= \frac{(t+e_b)\phi_b(D,A)}{\Phi+t+e_b+e_a} \leq s_b \\ P_a(A,D)\phi_a(A,D) &= \frac{(\Phi+e_a)\phi_a(A,D)}{\Phi+t+e_b+e_a} \leq s_a \end{aligned}$$

It follows that given e_a, decided in stage 1, equilibrium (D, D) occurs iff

$$t \leq \min\left\{\frac{(\Phi+e_a)s_b}{\phi_b(D,A)-s_b} - e_b, \frac{(\Phi+e_b)s_a}{\phi_a(A,D)-s_a} - e_a\right\} = \{t_1(e_a), t_2(e_a)\}\ ;\ t_1' > 0\ \ t_2' < 0 \tag{8}$$

say. For high e_a, the $t < t_2$ is then the binding constraint: high product differentiation invites an attacking strategy by incumbent. For low e_a, $t < t_1$ is the binding constraint: the contestant then chooses low product differentiation as part of a defensive strategy.

We could also consider the possibility of an (A, D) equilibrium. Given e_a, by choosing $t < t_2(e_a)$, the contestant can block this possibility. They will wish to do this if $s_b > P_b(A, D)\phi_b(A, D)$; i.e., if

$$t > \frac{(\Phi+e_b)(\phi_b(A,D)-s_b) - s_b(\Phi+e_a)}{s_b} \tag{9}$$

But if $\phi_b(A, D)$ is low, it is plausible to assume that the right hand side of (9) is negative and the contestant will block equilibrium (A, D) by not exceeding the level of technology differentiation that invites an offensive strategy by the incumbent. Note however that if we assume that $\phi_a(A, D) \leq s_a$, (A, D) would never happen and therefore the contestant would never have to consider blocking it.

Now consider equilibria (D, A) and (A, A). The following analysis shows that the contestant will block equilibrium (A, A) by choosing the maximum possible technological differentiation, $t_3(e_a)$, which just falls short of provoking an offensive strategy (A, A); i.e., the contestant will

$$\underset{\{t\}}{Max} \qquad \phi_b(D,A)\, P_b(D,A)\, V = \phi_b(D,A) \frac{t+e_b}{\Phi+t+e_b+e_a} V$$

$$subject\ to \qquad \phi_a(D,A)\, P_a(D,A) \geq \phi_a(A,A)\, P_a(A,A)$$

Note that, since the contestant's objective function is increasing in t,

$$\frac{\partial P_b(D,A)}{\partial t} = \frac{t(\Phi+t+e_b+e_a)-(t+e_b)}{(\Phi+t+e_b+e_a)^2} = \frac{(\Phi+e_a)}{(\Phi+t+e_b+e_a)^2} > 0,$$

the contestant will simply set the maximum possible technological differentiation, $t = t_3(e_a)$, which is the one that makes the incumbent's constraint bind,

$$\phi_a(D,A)\, P_a(D,A) = \phi_a(A,A)\, P_a(A,A)$$

Substituting above:

$$\phi_a(D,A) \frac{\Phi+e_a}{\Phi+t+e_b+e_a} = \phi_a(A,A) \frac{e_a}{e_a+e_b} \Longleftrightarrow$$

$$t_3(e_a) = \frac{\phi_a(D,A)(e_a+e_b)(\Phi+e_a) - \phi_a(A,A)\, e_a (\Phi+e_b+e_a)}{\phi_a(A,A)\, e_a}. \qquad (10)$$

Note that, given the above, for (D, A) to be an equilibrium, it has to be the case that $\phi_b(A,A)\, P_b(A,A) < \phi_b(D,A)\, P_b(D,A)$ for $t_3(e_a)$, otherwise, it would pay for the contestant to just force the (A, A) equilibrium. Still, $t_3(e_a)$ would have to be done to force (A, A) equilibrium to happen. It can be seen that if $\Phi = 0$, this would not be an issue as long as $\phi_b(A,A) \leq \phi_b(D,A)$, which was our initial conjecture.

For the case when $\Phi > 0$, a sufficient condition for (A, A) not to be a possibility is

$$P_b(A,A) < P_b(D,A)|_{t=t_3(e_a)} \Leftrightarrow \frac{e_b}{e_a+e_b} < \frac{t_3(e_a)+e_b}{\Phi+t_3(e_a)+e_b+e_a} \Leftrightarrow t_3(e_a) > \frac{\Phi e_b}{e_a} = t_4(e_a)$$

say. Let's check that the above holds for the $t_3(e_a)$ we obtained:

$$t_3(e_a) = \frac{\phi_a(D,A)(e_a+e_b)(\Phi+e_a) - \phi_a(A,A)\, e_a(\Phi+e_b+e_a)}{\phi_a(A,A)\, e_a} > \frac{\Phi e_b}{e_a} = t_4(e_a) \Leftrightarrow$$

$$\Phi e_b(\phi_a(D,A) - \phi_a(A,A)) + e_a(\Phi+e_b+e_a)(\phi_a(D,A) - \phi_a(A,A)) > 0.$$

This holds as long as $\phi_a(D,A) > \phi_a(A,A)$, which is our conjecture. Therefore, $t_3(e_a)$ will never be low enough to make the contestant prefer an all out war (in which case technological differentiation would be done with the objective of triggering an mutual attack equilibrium). Note that $t_3(e_a)$ is a convex function of e_a and that in general, the sign of $\dfrac{dt_3(e_a)}{de_a}$ is ambiguous:

$$\frac{dt_3(e_a)}{de_a} = -\frac{\Phi\phi_a(D,A)\,e_b}{t\phi_a(A,A)\,(e_a)^2} + \frac{(\phi_a(D,A) - \phi_a(A,A))}{t\phi_a(A,A)} > 0 \Longleftrightarrow$$

$$-\frac{\Phi\phi_a(D,A)\,e_b}{(e_a)^2} + (\phi_a(D,A) - \phi_a(A,A)) > 0.$$

Therefore, for e_a low enough, increases in effort will decrease technological differentiation. Also, note that if $\phi_a(D,A) = \phi_a(A,A)$, the effect will be clearly negative, while if $\Phi = 0$ the effect is positive as long as our assumption $\phi_a(D,A) > \phi_a(A,A)$ holds. Also note that:

$$\frac{dt_3(e_a)}{d\Phi} > 0.$$

Our findings can be summarized in the following proposition:

Proposition 1. *There are two candidates for unique Pure Strategy Nash Equilibrium in the conflict game (D,D) and (D,A). Given e_a, the contestant can induce (D,D) by choosing $t < \min[t_1(e_a), t_2(e_a)]$ and will always choose to block (A,D). If $t_3(e_a) > t_1(e_a)$, the contestant will induce (D,A) and will block (A,A).*

3.3 Stage 1: Incumbent chooses Effort

3.3.1 Incumbent's effort that implements (D,D) as the unique Pure Strategy Nash Equilibrium in the last stage of the game

Through its choice of effort, the incumbent will consider the implementation of either of the two remaining candidates for equilibrium as the unique equilibrium in the final stage of the game. Interestingly, we will see that it may actually be 'cheaper' in terms of effort for the incumbent to induce a defensive conflict, especially if the no conflict shares of resources are balanced in favour of the incumbent.

If the incumbent wants to stop conflict, it will need to ensure:

$$\begin{aligned} s_b &\geq \phi_b(D,A)\, P_b(D,A)\,, \\ s_a &\geq \phi_a(A,D)\, P_a(A,D)\,, \end{aligned}$$

through its choice of effort.

We define $e_a^{(D,D)}$ as the lowest level of effort that implements equilibrium (D,D). That is, the lowest level of effort that makes $t_3 = t_1$.

First note that if $\phi_b(D,A) < s_b$, then (D,D) would happen for sure and no effort would be necessary on the side of the incumbent to ensure it. We assume the opposite and continue.

Now, $t_1 = t_3$ (see equations (8) and (10)) iff

$$\frac{(\Phi + e_a)s_b}{\phi_b(D,A) - s_b} - e_b = \frac{\phi_a(D,A)(e_a + e_b)(\Phi + e_a) - \phi_a(A,A)\, e_a(\Phi + e_b + e_a)}{\phi_a(A,A)\, e_a}.$$

In general, this results in a nonlinear equation, however, for $\Phi = 0$,

$$e_a^{(D,D)} = \frac{e_b \phi_a(D,A)(\phi_b(D,A) - s_b)}{\phi_b(D,A)\phi_a(A,A) - (\phi_b(D,A) - s_b)\,\phi_a(D,A)}. \tag{11}$$

Note that:

$$\frac{de_a^{(D,D)}}{de_b} = \frac{\phi_a(D,A)(\phi_b(D,A) - s_b)}{\phi_b(D,A)\phi_a(A,A) - (\phi_b(D,A) - s_b)\,\phi_a(D,A)} > 0$$

as long as $e_a^{(D,D)} > 0$. This is quite intuitive as an increase in e_b increases the effort necessary to prevent attack from the contestant as everything else given, an increase in e_b, increases the contestant's probability of winning.

It is also quite intuitive to understand that an increase in the peace share of the contestant is going to make it easier to implement peace using effort.

$$\frac{de_a^{(D,D)}}{ds_b} = \frac{-e_b \phi_a(D,A)\,\phi_b(D,A)\phi_a(A,A)}{(\phi_b(D,A)\phi_a(A,A) - (\phi_b(D,A) - s_b)\,\phi_a(D,A))^2} < 0.$$

It is interesting to observe that, unlike in the case when differentiation was not a possibility, it will not always be feasible for the incumbent to implement (D,D) using effort. The reason is that although the incumbent's effort has a direct negative impact on the contestant's probability of winning, it also has an indirect positive effect through its impact on the degree of differentiation that would be chosen in (D, A).

With $\Phi = 0$, for effort to make contestants prefer no conflict we need

$$s_b \geq \phi_b(D, A) \frac{t_3 + e_b}{t_3 + e_b + e_a},$$

substituting t_3, this can be rewritten as

$$s_b \geq \phi_b(D, A) \left[1 - \frac{e_a \phi_a(A, A)}{(e_a + e_b)\, \phi_a(D, A)}\right].$$

For the above to be feasible under $s_b < \phi_b(D, A)$, we need

$$s_b \geq \phi_b(D, A) \frac{\phi_a(D, A) - \phi_a(A, A)}{\phi_a(D, A)},$$

this condition is also the one that ensures a positive $e_a^{(D,D)}$.

Note that, for instance, if $\phi_a(A, A)$ very small, the above inequality might reverse, meaning that an increase in effort would encourage stronger increases differentiation as, it is less difficult to block (A, A), making (D, A) relatively more attractive.

A smaller $\phi_b(D, A)$, a higher s_b or a smaller rate of difference between $\phi_a(D, A)$ and $\phi_a(A, A)$ will all decrease the contestants incentive to choose (D, A) over (D, D) and therefore, it will be more likely that (D, D) becomes feasible.

Also note that for $\Phi = 0$

$$\begin{aligned} e_a^{(D,D,t=0)} &< e_a^{(D,D)} \Longleftrightarrow \\ \frac{e_b\left(\phi_b(D, A) - s_b\right)}{s_b} &< \frac{e_b \phi_a(D, A)\left(\phi_b(D, A) - s_b\right)}{\phi_b(D, A)\phi_a(A, A) - \left(\phi_b(D, A) - s_b\right)\phi_a(D, A)} \Longleftrightarrow \\ \phi_b(D, A)\left(\phi_a(A, A) - \phi_a(D, A)\right) &< 0. \end{aligned}$$

The above means that if under technological differentiation, (D, D) can be implemented using effort, then this level of effort will be bigger than the level of effort required to implement (D, D) in the absence of technological differentiation. Intuitively this is due to the fact that the contestant has a higher incentive to shift to (D, A) when it can differentiate.

3.3.2 Incumbent's effort that implements (D,A) as the unique Pure Strategy Nash Equilibrium in the last stage of the game

We denote the level of effort that the incumbent would make if it wanted to target (D, A) as $e_a^{(D,A)}$. This effort can be obtained from the following maximization problem

$$\underset{\{e_a\}}{Max} \qquad EU_a\,(D, A) = \phi_a\,(D, A)\, P_a\,(D, A)\, V - C\,(e_a)\,,$$

substituting for $P_a\,(D, A)$ we get

$$\underset{\{e_a\}}{Max} \qquad EU_a = \phi_a\,(D, A)\,\frac{\Phi + e_a}{\Phi + t_3(e_a) + e_b + e_a} V - C\,(e_a)\,.$$

We then obtain the First Order Condition:

$$\begin{aligned} \frac{dEU_a}{de_a} &= \phi_a\,(D, A)\, V \left(\frac{t_3(e_a) + e_b}{(\Phi + t_3(e_a) + e_b + e_a)^2} + \frac{-\,(\Phi + e_a)\,\dfrac{dt_3(e_a)}{de_a}}{(\Phi + t_3(e_a) + e_b + e_a)^2} \right) - \\ -\frac{dC\,(e_a)}{de_a} &= 0. \end{aligned}$$

The above tells us that, as long as $\dfrac{dt_3(e_a)}{de_a} > 0$, the fact that the incumbent is forward looking reduces the optimal amount of effort when (D, A) is the equilibrium targeted.

Substituting for $t_3(e_a)$ in the expected utility equation, we get:

$$EU_a(D, A) = V\frac{\phi_a\,(A, A)\, e_a}{(e_a + e_b)} - C\,(e_a)\,.$$

The above is equivalent to the expected incumbent's utility if (A, A) was the expected equilibrium. This is due to the fact that the incumbent advances that in the following stage the contestant will chose the degree of technological differentiation that just makes the incumbent prefer (D, A) to (A, A).

The first order condition simplifies to:

$$\frac{dEU_a\,(D, A)}{de_a} = V\frac{\phi_a\,(A, A)\, e_b}{(e_a + e_b)^2} - \frac{dC\,(e_a)}{de_a} = 0.$$

Proposition 2. *If the incumbent targets (D, A), when the contestant has the option of technologically differentiating from incumbent, the outcome is a lower level of effort by the incumbent as long as "entrenching" advantages Φ, are sufficiently small and $\phi_a\,(D, A) >$*

$\phi_a(A, A)$.

Proof

In the case where there technological differentiation is not a possibility, the equivalent first order condition for incumbent effort was:

$$\frac{dEU_a(D, A)}{de_a} = V\frac{\phi_a(D, A)\, e_b}{(\Phi + e_b + e_a)^2} - \frac{dC(e_a)}{de_a} = 0.$$

Therefore a condition for the optimal effort of the incumbent when (D, A) is targeted to be lower than in the absence of technological differentiation

$$\frac{\phi_a(A, A)}{(e_a + e_b)^2} < \frac{\phi_a(D, A)}{(\Phi + e_b + e_a)^2}.$$

Note that if $\Phi = 0$, the above condition would be equivalent to $\phi_a(A, A) < \phi_a(D, A)$. However, if $\phi_a(A, A) = \phi_a(D, A)$, the inequality will clearly reverse, in such case, the level of effort by the incumbent, without technological differentiation by the contestant, would actually be lower□

Finally, we calculate the equilibrium $e_a^{(D,A)}$ for $\frac{dC(e_a)}{de_a} = c$, substituting this in the first order condition we obtain:

$$e_a^{(D,A)} = \sqrt{V\frac{\phi_a(A, A)\, e_b}{c}} - e_b. \tag{12}$$

Note that:

$$\frac{de_a^{(D,A)}}{dV} > 0.$$

$$\frac{de_a^{(D,A)}}{dc} < 0.$$

$$\frac{de_a^{(D,A)}}{de_b} = \frac{\sqrt{V\frac{\phi_a(A, A)}{c}}}{2\sqrt{e_b}} - 1 > 0 \Leftrightarrow \sqrt{\frac{V\phi_a(A, A)}{c}} > 2\sqrt{e_b}.$$

An increase in V increases the incentive to put effort in order to increase probability of winning in a (D, A) type of conflict. An increase in marginal cost of effort naturally decreases incentive to put effort though. The impact of e_b on $e_a^{(D,A)}$ is ambiguous.

3.3.3 The decision of the incumbent on whether to induce (D, D) or (D, A)

The incumbent will induce no conflict, (D, D), iff $EU_a\left(e_a^{(D,D)}\right) > EU_a\left(e_a^{(D,A)}\right)$.

We have three main cases under the assumption that $s_a > \phi_a(D, A)$.This assumption important here, it effectively excludes (A, D) as a possible equilibrium without having to analyze the contestant's blocking of such equilibrium, this will help the comparison of technological differentiation and no differentiation case and it will ensure that all equilibria presented will be unique, therefore, however, note also that (A, A) was in principle a possible equilibrium but the contestant blocks it by choosing t_3, which just falls short of making incumbent prefer it to (D, A):

1. (D, D) feasible for any level of the incumbent's effort. This is the simplest case. The incumbent will then choose $e_a^{(D,D)} = 0$ and the contestant will be indifferent between any level of technological differentiation that ensures (D, D), although if there was any cost associated to differentiating, zero differentiation would be the optimal decision for the contestant. This case will arise when $\phi_b(D, A) < s_b$.

2. $e_a^{(D,A)} > e_a^{(D,D)}$. Given our assumption that for given effort, the incumbent prefers no war to (D, A), $e_a^{(D,D)}$will be implemented and again the contestant will be indifferent between any level of technological differentiation that ensures (D, D).

3. $e_a^{(D,A)} < e_a^{(D,D)}$. Note that this implies that $t_3\left(e_a^{(D,A)}\right) > t_1\left(e_a^{(D,A)}\right)$ since otherwise, by definition of $e_a^{(D,D)}$(lowest level of effort that ensures $t_3 = t_1$), we would have a contradiction. In this case, if

$$EU_a\left(e_a^{(D,A)}\right) > EU_a\left(e_a^{(D,D)}\right) \Leftrightarrow$$
$$\Leftrightarrow \phi_a(D, A) \frac{\Phi + e_a^{(D,A)}}{\Phi + t_3(e_a^{(D,A)}) + e_b + e_a^{(D,A)}} V - C\left(e_a^{(D,A)}\right) > s_a V - C\left(e_a^{(D,D)}\right),$$

then (D, A) will be the equilibrium. Note that, since costs are increasing in effort, the above condition requires that $e_a^{(D,A)} < e_a^{(D,D)}$ as a necessary, albeit not sufficient, condition.

Finally, based on our results in this section, we can now compare it with the benchmark (no differentiation possible) case, the following result becomes apparent

Proposition 3. *For sufficiently small "entrenching" advantages Φ, a decision by the* *incumbent to induce* (D, D)*, that is, to prevent conflict, will become less likely to be adopted by the incumbent, even if feasible, if differentiation by the contestant is a possibility.*

Simply recall that it takes more effort to persuade the contestant not to attack when they can technologically differentiate, which gives them an advantage in (D, A) situation, $e_a^{(D,D,t=0)} < e_a^{(D,D)}$. Besides, when the contestant can technologically differentiate it actually decreases the incentive of incumbent to put effort down to just above the incentive to put effort in a (A, A) type of conflict and therefore, $e_a^{(D,A)} < e_a^{(D,A,t=0)}$. This makes it less likely that the government chooses to implement peace when technological differentiation is possible.

4 The model with fixed costs of technological differentiation

In the previous two sections, we analyzed the optimal levels of effort with and without the option of technological differentiation. This section aims to integrate the two analysis by introducing a fixed cost of technological innovation. In this way, we will be able to have (D, A) outcome with or without technological differentiation t. For simplicity, we will restrict ourselves to the case of no entrenching advantage $\Phi = 0$.

We assume there is a cost F of choosing a technology which is different from that of the incumbent (this is parallel to the set up costs in location type models). We now proceed to solve the game backwards using the results from previous sections.

The last stage of the game remains the same as those in previous sections, as at that point F is sunk and differentiation, if developed is by then a given parameter.

In the second stage of the game, the contestant chooses the degree of technological differentiation t. In previous sections in the paper, we have obtained the optimal degree of technological differentiation, should the contestant decide to differentiate t_3. Still, now the contestant might decide not to differentiate at all, even if a (D, A) type of third stage equilibrium is expected, if the fixed costs of differentiating are sufficiently high.

Note that, for the contestant, the variable payoff of differentiating is higher than that of not differentiating from the incumbent's technology (for a given level of the incumbent's effort): $V\phi_b(D, A)\, P_b(D, A)|_{t=t_3} > V\phi_b(D, A)\, P_b(D, A)|_{t=0}$. Therefore, for the contestant to not want to differentiate (even in advance of a (D, A) equilibrium in the third stage), fixed differentiation costs F, will need to be bigger than F^*, which is defined as:

$$F^{*}=V\phi_{b}\left(D,A\right)\,P_{b}\left(D,A\right)|_{t=t_{3}}-V\phi_{b}\left(D,A\right)\,P_{b}\left(D,A\right)|_{t=0}=$$

$$=V\phi_{b}(D,A)\left(\frac{(t_{3}+e_{b})}{t_{3}+e_{b}+e_{a}}-\frac{e_{b}}{e_{b}+e_{a}}\right)=$$

$$F^{*}=\frac{V\phi_{b}(D,A)e_{a}}{(e_{a}+e_{b})}\frac{\phi_{a}\left(D,A\right)-\phi_{a}\left(A,A\right)}{\phi_{a}\left(D,A\right)}.$$

Note that an increase in the incumbent's effort e_a, will increase the level of fixed costs of technological differentiation F, needed for the incumbent not to want to differentiate:

$$\frac{\partial F^{*}}{\partial e_{a}}=\frac{e_{b}}{\left(e_{a}+e_{b}\right)^{2}}V\phi_{b}(D,A)\frac{\phi_{a}\left(D,A\right)-\phi_{a}\left(A,A\right)}{\phi_{a}\left(D,A\right)}>0.$$

In order to ensure that the contestant decides to target equilibrium (D,D), which amounts to preventing conflict, the incumbent must put an effort that ensures that the contestant does not have an incentive to attack when the incumbent defends. Such condition will have to be ensured for both the differentiation and non differentiation case. The condition is presented below:

$$Vs_{b}\geq\max\left\{V\phi_{b}\left(D,A\right)\,P_{b}\left(D,A\right)|_{t=t_{3}}-F,\qquad V\phi_{b}\left(D,A\right)\,P_{b}\left(D,A\right)|_{t=0}\right\}.$$

The above condition states that the payoff for the contestant, if it does not attack (and neither does the incumbent), needs to be higher than the maximum of the payoffs made by the contestant whether it differentiates or not at the stage where differentiation is chosen[6].

Recalling that

$$t_{3}=\left(e_{a}+e_{b}\right)\frac{\phi_{a}\left(D,A\right)-\phi_{a}\left(A,A\right)}{\phi_{a}\left(A,A\right)},$$

the previous condition can be rewritten in the following way:

[6]Note that if $\left[V\phi_{b}\left(D,A\right)\,P_{b}\left(D,A\right)|_{t=t_{3}}-F\right]>\left[V\phi_{b}\left(D,A\right)\,P_{b}\left(D,A\right)|_{t=0}\right]$, the contestant will differentiate only if $Vs_{b}<\left[V\phi_{b}\left(D,A\right)\,P_{b}\left(D,A\right)|_{t=t_{3}}-F\right]$. Otherwise, they will not differentiate and therefore, there will be no conflict, even if $Vs_{b}<V\phi_{b}\left(D,A\right)\,P_{b}\left(D,A\right)|_{t=t_{3}}$.

$$Vs_b \geq \max\left\{V\phi_b(D,A)\frac{\phi_a(D,A)(e_a+e_b)-\phi_a(A,A)e_a}{\phi_a(D,A)(e_a+e_b)} - F, \qquad V\phi_b(D,A)\frac{e_b}{e_b+e_a}\right\}. \tag{13}$$

The above condition can be easily represented graphically[7] once the following two points are considered:

- First, the difference between the first and second term in the condition above is increasing in the incumbent's effort e_a, and zero at $F=0$.

$$\frac{\partial\left(V\phi_b(D,A)\frac{\phi_a(D,A)-\phi_a(A,A)}{\phi_a(D,A)}\frac{e_a}{(e_a+e_b)}\right)}{\partial e_a} =$$

$$= V\phi_b(D,A)\frac{\phi_a(D,A)-\phi_a(A,A)}{\phi_a(D,A)}\frac{e_b}{(e_a+e_b)^2} > 0.$$

- Second, terms 1 and 2 in the R.H.S. of equation (13) intersect at the following level of the incumbent's effort

$$e_a^I = \frac{Fe_b}{V\phi_b(D,A)\frac{\phi_a(D,A)-\phi_a(A,A)}{\phi_a(D,A)} - F}.$$

Which, as noted, is zero for zero fixed cost and increases with F. Note that if $F > V\phi_b(D,A)\frac{\phi_a(D,A)-\phi_a(A,A)}{\phi_a(D,A)}$, not differentiating would always be better for the contestant[8].

[7] For the shake of clarity, we will use linear functions to represent terms 1 and 2 of the R.H.S. of the conflict preventing condition. In reality, the are convex in the incumbent's effort, this does not affect the comparative statics.

[8] Whether the contestant differentiates or not, his expected utility at the above level of effort by the incumbent is:

$$V\phi_b(D,A)\frac{e_b}{e_b+e_a^I} = V\phi_b(D,A)\frac{e_b}{e_b+\frac{Fe_b}{V\phi_b(D,A)\frac{\phi_a(D,A)-\phi_a(A,A)}{\phi_a(D,A)}-F}} =$$

$$= \frac{V\phi_b(D,A)\left[\phi_a(D,A)-\phi_a(A,A)\right]-\phi_a(D,A)F}{\phi_a(D,A)-\phi_a(A,A)}.$$

The above is used in Figure 1 to make a graphical representation of terms 1 and 2 in the R.H.S. of equation (13) as a function of the incumbent's effort. In turn, Figure 1 also represents the minimum level of effort the incumbent must exert in order to prevent conflict $e_a^{(D,D)}$ for a given level of peace share of wealth for the contestant Vs_b.

In addition, Figure 1 captures the impact of an increase in the fixed differentiation costs on the minimum amount of effort that prevents conflict. More specifically, Figure 1 represents the impact on the optimal level of conflict preventing effort of a change in the fixed cost of differentiating from $F = 0$ to $F = F^*\left(e_a^I\right)$.

Two points must be noted for the understanding of this comparative statics:

- First, increases in F, starting at $F = 0$, will cause downward shifts to function represented by the first term of the condition for non conflict, equation (13). This will cause a reduction in the minimum amount of effort that prevents conflict $e_a^{(D,D)}$, until the shift reaches the intersection between the other two functions: the second term of the R.H.S. of the condition and Vs_b. This will happen at effort level e_a^I already defined. This intersection will happen at a level of fixed cost of effort, $F^*\left(e_a^I\right)$, defined by

$$V\phi_b\left(D,A\right)\frac{e_b}{e_b+e_a^I} = Vs_b,$$

where, substituting for e_a^I we get

$$Vs_b = \frac{V\phi_b\left(D,A\right)\left[\phi_a\left(D,A\right)-\phi_a\left(A,A\right)\right]-\phi_a\left(D,A\right)F}{\phi_a\left(D,A\right)-\phi_a\left(A,A\right)} \Longleftrightarrow$$

$$F^*\left(e_a^I\right) = \frac{\left(\phi_a\left(D,A\right)-\phi_a\left(A,A\right)\right)V}{\phi_a\left(D,A\right)}\left[\phi_b\left(D,A\right)-s_b\right].$$

As already stated, any increase in F within the $\left[0, F^*\left(e_a^I\right)\right]$ interval will cause the conflict preventing effort $e_a^{(D,D)}$, to decrease. The reason is that the prevailing no conflict inducing condition in that interval will be:

$$Vs_b \geq V\phi_b(D,A)\frac{\phi_a\left(D,A\right)\left(e_a+e_b\right)-\phi_a\left(A,A\right)e_a}{\phi_a\left(D,A\right)\left(e_a+e_b\right)} - F,$$

Therefore, the effort that ensures no conflict can be written as[9]

[9] Note that the previous equation is equivalent to:

$$e_a^{(D,D)} = \frac{e_b \phi_a(D,A)(V\phi_b(D,A) - Vs_b - F)}{\phi_b(D,A)\phi_a(A,A)V - (\phi_b(D,A) - s_b)\phi_a(D,A)V + F\phi_a(D,A)}$$

which is clearly decreasing in F.

Note also that at $F = F^*\left(e_a^I\right)$, the above effort coincides with $e_a^{(D,D,t=0)}$ (already obtained in previous sections). That is, increases in F decrease the optimal level of conflict preventing effort until this reaches the level of conflict preventing effort one would obtain if technological differentiation was not possible.

- Second, note that if $F > F^*\left(e_a^I\right)$, the relevant conflict preventing effort will still be the effort obtained for the section where technological differentiation was not possible,

$$e_a^{(D,D,t=0)} = \frac{e_b(\phi_b(D,A) - s_b)}{s_b}.$$

In other words, increases in F beyond $F^*\left(e_a^I\right)$ will not affect the level of conflict preventing effort.

We have analyzed the condition on the incumbent's effort that would ensure no conflict. We will now discuss whether or not the incumbent will want to prevent conflict and how changes in the fixed cost of technological differentiation will affect the incumbent's willingness to prevent conflict.

First, we must analyze the impact of changes in F on the optimal effort the incumbent would implement should he decide not to target conflict prevention. In finding such optimal effort, we must bear in mind that the incumbent's payoff function for (D, A) as target is likely to be discontinuous. This situation is represented in Figures 2 and 3. Both Figures represent the incumbent's expected payoff functions when the (D, A) equilibrium

---

$$\frac{Vs_b + F}{V\phi_b(D,A)}\phi_a(D,A)(e_a + e_b) - \phi_a(D,A)(e_a + e_b) + \phi_a(A,A)e_a \geq 0,$$

itself equivalent to:

$$e_a\left(\frac{Vs_b + F}{V\phi_b(D,A)}\phi_a(D,A) - \phi_a(D,A) + \phi_a(A,A)\right) + e_b\left(\frac{Vs_b + F}{V\phi_b(D,A)}\phi_a(D,A) - \phi_a(D,A)\right) \geq 0.$$

is expected for the differentiation and non differentiation cases (already analyzed in previous sections (thinner curves)). However, the relevant expected payoff function is now represented by the thicker discontinuous curves. The discontinuity point in the function depends on the incumbent's level of effort at which terms 1 and 2 in condition (13) intersect. As already discussed, such intersection happens at effort level e_a^I. Higher levels of effort will make the contestant prefer to differentiate and therefore the technological differentiation payoff function becomes the relevant one. Note that in cases where F is big enough, technological differentiation will not be observed even if conflict erupts, Figure 2 represents such situation. However, if the fixed cost of effort F is sufficiently low the optimal level of conflict effort $e_a^{(D,A)}$ will not prevent differentiation from taking place, Figure 3 represents this situation. Finally note that decreases in F will either decrease or have no impact on the optimal conflict effort $e_a^{(D,A)}$ (see Figure 4).

We can therefore conclude that a decrease in the fixed cost of technological differentiation to the contestant will decrease the scope for the incumbent to want to prevent conflict. The reason is that an increase in the fixed cost of differentiation will increase the level of conflict preventing effort but, it will decrease the optimal level of conflict effort thereby making it relatively more expensive to prevent conflict and therefore making it less likely for the incumbent to want to do so, even if at all feasible.

In other words, as the switch to alternative technologies becomes cheaper, conflict prevention by means of traditional military effort will become less feasible and less likely to be targeted by the incumbent.

The decision of the incumbent on whether to induce (D, D) or (D, A) was analyzed in more detail in a previous section, the same analysis applies here. Basically, if $e_a^{(D,A)} \geq e_a^{(D,D)}$, we know that the incumbent will implement the no conflict equilibrium, while if $e_a^{(D,A)} < e_a^{(D,D)}$, conflict will happen if $EU_a\left(e_a^{(D,A)}\right) > EU_a\left(e_a^{(D,D)}\right)$.

5 Conclusion

In this paper, we have presented a simple multiple stage model that captures some of the issues and difficulties involved in managing asymmetric conflict.

We have proven that a mutual attack strategy becomes a possibility only once we introduce technological differentiation. However, the challenger will block mutual attack equilibrium by limiting technological differentiation. The only conflict type where technological differentiation becomes an advantage when one party unilaterally attacks, therefore, the reason for limiting differentiation to avoid mutual attack becomes apparent (note that if the costs of conflict where the same across conflict types. There are two possible candidates for the equilibrium in the last stage of the game: no conflict and a conflict in which the incumbent adopts a defensive strategy and challengers adopt an attack strategy.

If the incumbent targets a defensive conflict, when the challenger has the option of technological differentiation from the incumbent, then the outcome is a lower level of effort by the incumbent; effort actually encourages higher differentiation[10] and hence, the incentives for effort are reduced. However, if the incumbent wants to prevent conflict when the challenger has the option of differentiation, the level of effort required will actually be higher as that type of conflict is more attractive for the challenger when differentiation is an option. This leads to the conclusion that the incumbent is less likely to want to prevent conflict when technological differentiation becomes a possibility. A world where technological differentiation by the challenger is feasible then becomes a less safe world, as it is more difficult to avoid conflict, at least by means of military effort. Crucially, if the challenger's status quo share of resources is very small and differentiation is feasible, inducing peace by use of conventional military effort will become impossible.

In the paper, we have also introduced a fixed cost of technological differentiation in order to analyze the impact of a reduction in the cost of choosing an alternative to the traditional military technology on the chances that the incumbent may want to implement conflict, the result is similar to that already described. A reduction such cost will make it less likely for the incumbent to decide or even be able to prevent conflict by means of defence effort.

We have treated the status quo shares as exogenous, in other games the incumbent may be able to influence them and may have incentives to try to improve the challenger's status quo share or other non-military incentives such as those discussed by Frey and Luechinger (2003).

[10] This result can be linked to the terrorism literature. It has been suggested in this literature (see e.g., Sandler and Enders (2002) and Sandler and Arce (2003)) that when a terrorist group has a choice of targets (those targets being different countries or different objectives within the same country) effort being put into defending one target may actually encourage terrorists to shift to the alternative option.

One concern that we did not pursue was that there may be an incentive for the incumbent to 'follow' the technological differentiation of the challenger in order to eliminate their technological advantage. But in reality there are many potential challengers and this raises the question of which challenger an incumbent would try to follow. With technological or tactical differentiation one cannot simply aim to counteract the average of your enemies. It would also lead one to consider whether it was feasible to 'follow' everybody. It would make sense though to at least keep track of what goes on in the national civilian industry that could become an unexpected weapon (dual use issues: computers, biowarfare. . .), this would at least limit the maximum technological differentiation available to contestants or the number of possible technological paths. One would, however, have to be careful not to push contestants into potentially more dangerous strategies. The terrorism literature discusses this problem in terms of terrorist objectives rather than technologies (an uneven increase in the defence of one country or location within a country will encourage terrorists into attacking other countries or objectives).

The model presented in the paper does not consider the introduction of incomplete information in any of the variables. A number of papers within the terrorism literature have used models of incomplete information. Lapan and Sandler (1993) and Overgaard (1994) present an attack by a terrorist group as a signal of the terrorist effort. The introduction of such type of asymmetric information in our model could be an interesting future line of research.

Our model has potential applications in the industrial organization literature. It becomes a bit more difficult to see how the final stage of the game may relate to the issues refer to in the computing industry, it is still possible to see a link though[11]. We could think of the incumbent as an established firm with a deep pocket. The contestant may then be seen as a new entrant which may choose indirect entry with a product horizontally different from the incumbent's, targeting a consumer base with a potential for future growth. The actual conflict can the be seen as an advertising war, which may be a damaging mutually aggressive advertising campaign in which both firms are trying to persuade consumers that the other product is not worth buying or defensive, in which one of two firms just try to defend the status quo market share.

[11] For a description of market structure characteristics of the computer industry see Bresnahan and Greenstein (1999).

References

[1] Bresnahan, T. F. and Greenstein, S. (1999) Technological Competition and the Structure of the Computer Industry, *Journal of Industrial Economics*, **47**, 1, 1-40.

[2] Enders, W. and Sandler, T. (2002) What do we know about the Substitution Effect in Transnational Terrorism? MIMEO, University of Southern California.

[3] Frey, Bruno S. and Luechinger, Simon (2003) How to fight terrorism: alternatives to deterrence, *Defence and Peace Economics*, August 2003, p237-250.

[4] García-Alonso, M.C. and Levine, P. (2002) Domestic Procurement, Subsidies and the Arms Trade. Chapter 7 in Brauer, J. and Dunne, P. (eds.) (2002) Arming the South: The Economics of Military Expenditures, Arms Production and Trade in Developing Countries. Palgrave.

[5] García-Alonso, M.C. (1999) Price Competition in a Model of Arms Trade, *Defence and Peace Economics* **10**, 3, 273-303

[6] García-Alonso, M.C. (2000). The Role of Technology Security in a Model of Horizontal Differentiation, *International Journal of Industrial Economics* **18**, 5, 747-773.

[7] García-Alonso, M.C. and Hartley, K. (2000). Export Controls, Market Structure and International Coordination, *Defence and Peace Economics*, 11, 5, September.

[8] Hirst, Paul (2003) War and Power in the 21st Century, Polity Press.

[9] Intilligator, Michael (1975) Strategic Considerations in the Richardson Model of the Arms Race. *Journal of Political Economy*, pp. 339-54?

[10] Lapan, H. E. and Sandler, T. (1993) Terrorism and Signalling. *European Journal of Political Economy*, **9**, 3, 383-397.

[11] Levine, P., Sen, S. and Smith, R. (1994) A Model of the International Arms Market, *Defence and Peace Economics*, 5, 1-18.

[12] Levine, P. and Smith, R. (1995) The Arms Trade and Arms Control, *Economic Journal*, 105, 471-484.

[13] Levine, P. and Smith, R. (1997a). Regional Arms Races and the Arms Trade, *Journal of Economic Dynamics and Control*, 21, 631-654.

[14] Levine, P. and Smith, R. (1997b). The Arms Trade, *Economic Policy*, October, 337-370.

[15] Levine, P., Mouzakis, F. and Smith, R. (2000). Arms Export Controls and Emerging Domestic Producers, *Defence and Peace Economics*, 11, 505-531.

[16] Levine, P. and Smith, R. (2000a) The Arms Trade Game: from Laissez-faire to a Common Defence Policy, *Oxford Economic Papers*, 52, 357-380.

[17] Levine, P. and Smith, R. (2000b) Arms Export Controls and Proliferation, *Journal of Conflict Resolution*, 44, 885-895.

[18] Lorel, Mark, Lowell, Julia , Kennedy, Michael and Levaus, P. (2000) Cheaper, Faster, Better; Commercial Approaches to Weapons acquisition, Rand.

[19] Newman, N.J. (2000) Asymmetric threats to British Military Intervention Operations, Whitehall Paper 49, Royal United Services Institute.

[20] Overgaard, P. B. (1994) Terrorist attacks as a signal of resources. *Journal of Conflict Resolution*, **38**, 3, 452-478.

[21] Sandler, T. and Arce, M. D. G. (2003) Terrorism and Game Theory. MIMEO, University of Southern California.

[22] Sandler, Todd and Enders, Walter (2002) An Economic Perspective on Transnational Terrorism, MIMEO, University of Southern California, Los Angeles.

[23] Siqueira, K. (2002) Conflict and Third Party Intervention. MIMEO, Clarkson University, New York.

[24] Sutton, J. (2001) Technology and Market Structure: Theory and Market History. The MIT Press.

[25] Trajtenberg, M. (2003) Defense R&D Policy in the Antiterrorist Era. NBER working paper number 9725.

[26] Walker, M. (1994) The Cold War. Vintage: London.

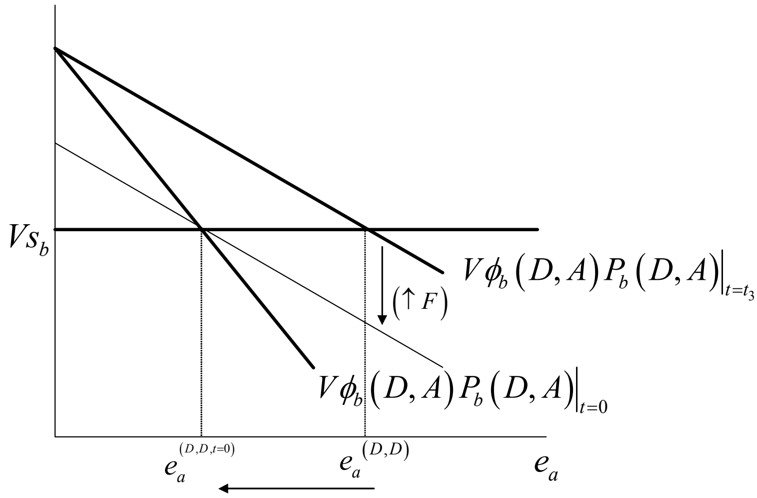

Figure 1: Impact on the conflict preventing level of effort of an increase in fixed costs of differentiation (from $F = 0$ to $F = F^*(e_a^I)$).

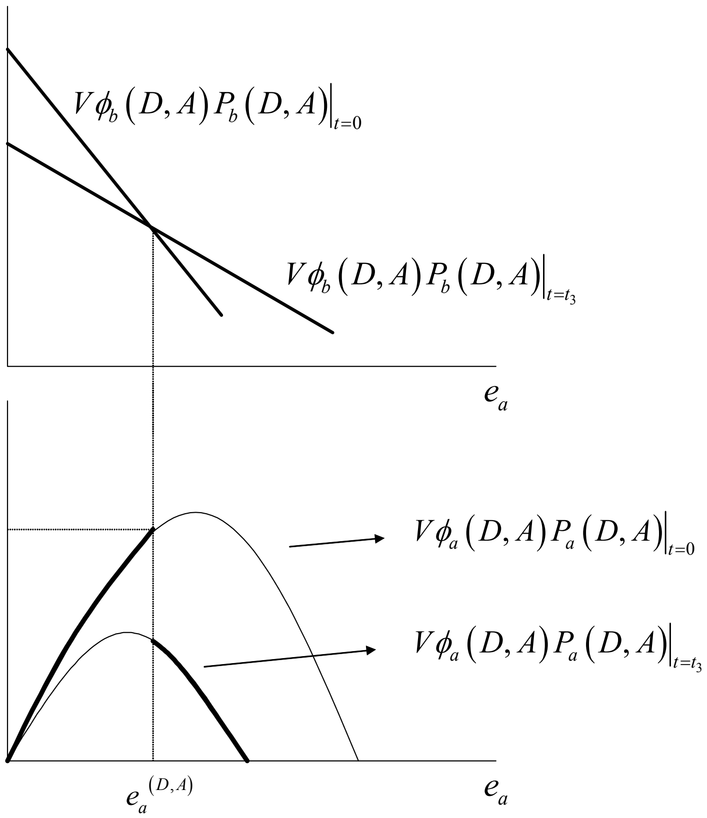

Figure 2: Thicker discontinuous curve represents the expected payoff for the incumbent for high F if defensive conflict is expected.

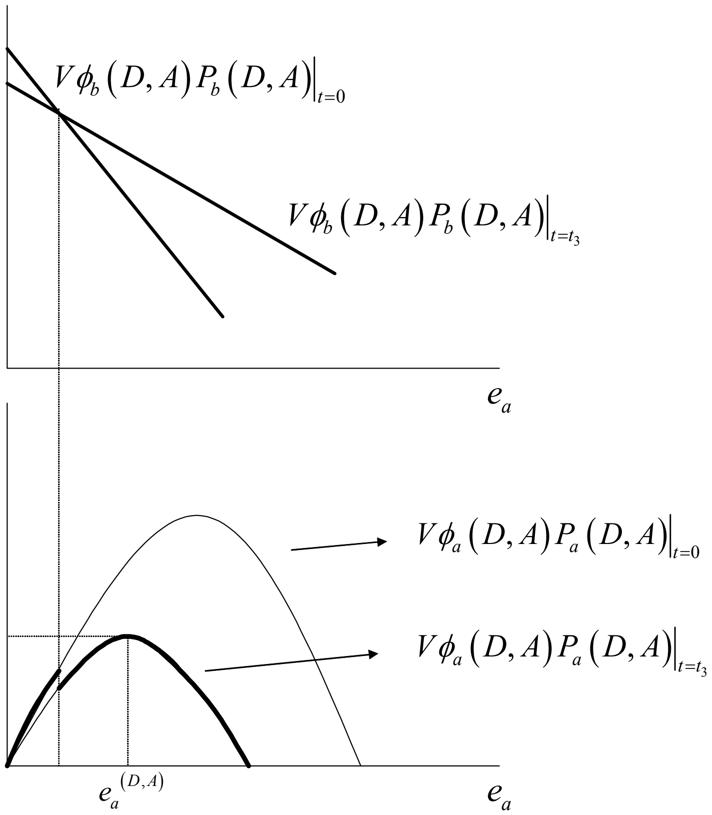

Figure 3: Thicker discontinuous curve represents the expected payoff for the incumbent for low F if defensive conflict is expected.

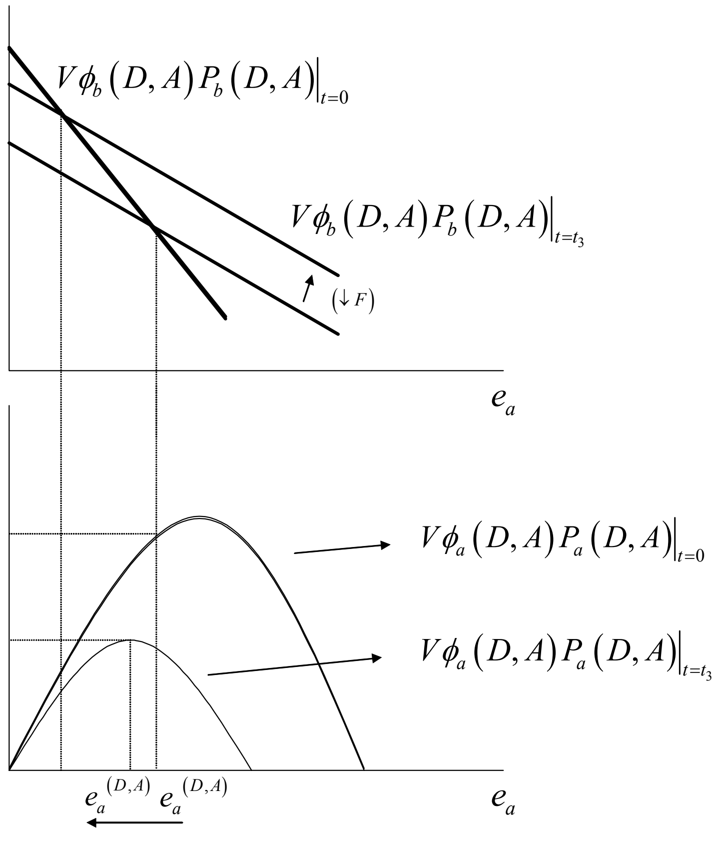

Figure 4: Impact on $e_a^{(D,A)}$ of a decrease in F .