BUE

Faculty of Business Admin, Economics and Political Science

Department of Economics

Module handbook: **20007/08**

Module Code: ECON01H02a

Module Title: **Econometric Modelling a**

School of Economics

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1. Introduction

The general aims of this module are to provide students with an appreciation of the use of econometrics as a tool for studying economics, to make them familiar with the main tools of econometrics at an advanced level and to introduce students to computer based methods of econometric investigation.

The module is taught by Paul Dunne and Yasmine Mohamed

Website: Details of the course including datasets will be posted on the course website at: http://carecon.org.uk/BUE/Econometrics

2. Learning Outcomes

By the end of the module student should have a knowledge and understanding of econometric analysis, have a knowledge and understanding of the value and limitations of econometric techniques, develop a critical perspective on the use of econometric analysis, be proficient in using an advanced econometric package (Microfit and or Eviews) and be able to independently replicate econometric analyses of economic data.

3. Organisation of the Module

Lectures and workshops will complement each other. Workshops will be computer based exercises on Microfit and will allow students to work through the topics covered in the lectures. Eviews can also be used.

Teaching and Learning Methods

The basic framework is a two hour lecture and a one hour seminar, though there is some flexibility. As a general rule, the lecture session will be one of exposition and the workshops will be computer based and give the opportunity for questions, discussion and practise. However, there will be departures from this framework depending upon the part of the syllabus being covered.

Most of the exercises are computer based and it is important that students work independently outside of the workshops on the exercises. Students should also frequently visit the course website. Aside from providing updates of the course, reading lists, lecture notes and other resources, it also operates as a means of communication between lecturers and students and it is important that students check it out regularly.

4. Module Programme and lecture details

Lect. no.	w/c date	Title
1	21.09.08	Introduction to Econometrics: Research and Econometric Methodology
2	28.09.08	The Simple Regression Model: Assumptions of the classical linear model
	5.10.08	Holiday –no lecture
4	12.10.08	Multiple Regression: Goodness of Fit; Interpretation;
5	19.10.08	Statistical Inference and Hypothesis Testing
6	26.10.08	Violation of the Assumptions of the Classical Regression Model
7	2.11.08	Autocorrelation and Dynamic Models
8	9.11.08	Time Series Analysis
9	16.11.08	Heteroskedasticity and Limited Dependent Variable Models
10	23.11.08	Multicollinearity, Prediction and Structural Stability
11	30.11.08	Developments in Time Series Analysis
	7.12.07	Holiday –no lecture or class
	15.12.07	Revision session

5. Workshops

Wk 2: Introduction to Microfit. MM Ch 1 – 3, 8 - 9. Microfit Exercise 1.
Wk3: Microfit exercises. MM Ch 1 – 4, 10.
Wk4: Microfit exercises. MM Ch 10.
Wk5: Microfit exercises. MM Ch 10.
Wk6: Microfit exercises. MM Ch 10.
Wk7: Microfit exercises. MM Ch 11.
Wk8: Microfit exercises. MM Ch 11.
Wk9: Microfit exercises. MM Ch 14.
Wk10: Microfit exercises. MM Ch 14.
Wk11: Microfit exercises. MM Ch 17
Wk12: Microfit exercises. MM Ch 15 – 16.

6. Source material

Principal Texts

There are many textbooks, which can be used to follow the course. It is worth having a look at them and finding the one that best suits your individual requirements. Those who have done an undergraduate econometric course might consider following the topics in one of the more advanced texts.

NB Earlier editions of the books below are fine –but not if more than 10 years old. So don't go and spend lots of money unnecessarily.

You should review basic Maths and Stats in the textbooks.
See: Thomas Ch 1 to 5, Salvatore Ch 1 to 5, Gujarati Ch 1 to 5 and Appendix A and Dougherty Ch 1 to 3. All texts are referenced below.

For those needing an easier start the following provide relatively straightforward expositions:

- Stewart, J. (1991) 'Understanding Econometrics', Philip Allan.
  This is a useful starting point with more words per equation than any other econometrics text.

- Gujarati, D. N. (2003) 'Essential of Econometrics', McGraw Hill.
  Another simple introduction.

- Studenmund, A. H. (2006), "Using Econometrics", Pearson. An innovative approach, suitable for those completely new to econometrics and those with some experience.

- R Carter Hill, William E Griffiths, George G Judge (2001) 'Undergraduate Econometrics', Second edition, John Wiley and Sons. Very good introduction and good discussion.

- Salvatore (2001) 'Statistics and Econometrics', Schaum's Outlines, Second Edition, McGraw Hill. Provides lots of worked examples

What you will need to master by the end:

- Gujarati, D. N. (2005) 'Basic Econometrics', McGraw Hill, Fourth edition.
  A useful introductory text with a detailed discussion.

- Dougherty (2006) 'Introduction to Econometrics', Oxford University Press, Third edition.
  Useful introduction.

- Maddala, G. S. (2001) 'Introduction to Econometrics', Third Edition, John Wiley.
  This is more up to date than most texts only uses matrix algebra in appendices.

- Kennedy, P. (2008) 'A Guide to Econometrics', Blackwell.
  Is very useful. It lacks notation and technical detail but explains concepts well and is a useful accompaniment to a more formal text

Software Manuals

Pesaran, M. H. and Pesaran, B. "MICROFIT 4.0, Windows version", 1997, Oxford University Press, ISBN 0-19-268531-7. This is the manual for the MICROFIT package that you will be using and contains useful examples and exercises. It is denoted as MICROFIT MANUAL (MM) hereafter.

More advanced texts

Davidson, R. and MacKinnon, J. G. (1993), ‘Estimation and Inference in Econometrics’, Oxford University Press.
Very comprehensive

Greene, W. H. (1997) ‘Econometric Analysis’, Macmillan.
Comprehensive

Hendry, D. F. (1995), ‘Dynamic Econometrics’, Oxford University Press.

Intriligator M, R Bodkin, C Hsaio (1996) “Econometric Models, Techniques and Applications”, Prentice Hall.

Judge, G. G., Griffiths, W. E., Hill, R. C., Lutkepohl, H., and Lee, T. C. (1985) ‘The Theory and Practice of Econometrics’, Second Edition, Wiley.

Maddala, G. S. (1977) ‘Econometrics’, McGraw Hill.
A popular reference text for researchers

Maddala, G S and Kim (1999) “Unit Roots, Cointegration and Structural Change”, Cambridge University Press.

Mills T (1999) “The Econometric Modelling of Financial Time Series”, Cambridge University Press.

Patterson, K (2000) “An Introduction to Applied Econometrics”, Palgrave.

If you already have a textbook that is not on this list and you are happy with it then carry on and use it to follow the first part of the course. All texts follow roughly the same material.

7. Applied Exercises:

Introduction

These exercises introduce you to Microfit 4.1, a very powerful interactive econometrics package that is also relatively easy to use. It has a clear structure of screen editors and windows facilities for data processing, file management, graphic display, estimation, hypothesis testing and forecasting windows. It is driven by icons and written commands.

It is important that you follow the steps exactly the first time you use the package. You will soon get used to moving through the screens to find the task you want. When typing in commands always use the "return" / "enter" key at the end.

Documentation: There is a Microfit manual available in the library, but you will not need to reference it at this stage, as the applied exercises are fairly straightforward and there is a very good help facility in the programme. MFIT is menu driven you do not need a manual to run the programme.

Important: Becoming familiar with Microfit is an essential part of this course. It will require considerable amounts of practice outside of the timetabled sessions.

EXERCISE 1

1. Getting Started

Logon to the network and access Microfit

2. Data Input:

- Click on File and choose the New option
- Choose Annual as the data frequency
- Give 1977 as the Start year and 1987 as the End year
- Give 2 as the number of variables
- Click on OK

In the next window overwrite X1 and X2 with RC and RPDI respectively and insert descriptions of the variables, RC is real consumption, RPDI is real personal disposable income.

Click on Go

Now input the following data in the relevant cells

	RC	RPDI
1977	176.0	195.6
1978	186.0	209.9
1979	193.8	221.7
1980	193.8	224.9
1981	193.8	222.2
1982	195.6	221.7
1983	204.3	227.9
1984	207.9	232.4
1985	215.3	237.8
1986	226.8	244.8
1987	238.5	252.2

When you have finished you can check and edit if necessary and when you are ready click on Go.

There are a number of icons above the data window with Variables, Data, Process etc on them:

- Click on Variables to check the information you have input, then click on Close
- Click on Data to check the inputted data (same window)
- Click on Process to get to the data processing screen.

3. Data Processing

You are now in the main screen and can undertake various tasks. For now:

a. Check the data:

- Type LIST RC RPDI
- Click on Close

b. Compute variables:

- Compute the savings ratio type: S=(RPDI-RC)/RPDI
  Then click Go
- Compute a constant type: C=1
  Then click Go
- Take logs of variables type: LRPDI=LOG(RPDI);
  LRC=LOG(RC);
  LS=LOG(S);
  Then click Go.

The ; allows you to run more than one command at a time.

You can also compute other transformations in the same way. For example first differences for DC would require DRC=RC-RC(-1).

c. Plot variables over time: Type:
PLOT RPDI
PLOT RC
PLOT S
PLOT RPDI RC
Click on Go after each of these and on Close in the graphics Window

d. Plot variables against each other: Type
XPLOT RPDI RC
Click on Go
Click on Close

e. Get simple descriptive statistics and correlations: Type
COR RPDI RC
Click on Go
(You should get correlation coefficient of 0.9676)
Click on Close

After you have done all of these type click on the Single icon, which will give you the Linear regression - Ordinary Least Squares screen.

4. Running a Regression

Click on the empty window and type in the dependent variable a constant and the independent variable as follows:
RC C RPDI
and then click on the Start icon.

This will give you regression results and various statistics. They should be equivalent to:

$$RC = \underset{(-2.4)}{-52.21} + \underset{(11.5)}{1.13}\ RPDI$$

$$R^2 = 0.94 \qquad DW = 0.596$$

You can print this out or save to

Further possible statistics are offered when you click on the Close icon. For the moment choose no. 3 (List/plot/save residuals and fitted values) and then click on OK.

5. Analysing the Residuals

Choose the option no. 3 Plot residuals and click on Ok
You will now see a plot of the residuals

It should be clear that there are problems with this regression!

You can print this graph out or save to a file using icons at the top left.

- Click on Close to return to the menu
- Accept option 0, Return to the Post Regression Menu
- Choose option 0, Return to Backtracking Menu
- Choose option 1, Return to Single Equation Estimation Window

6. Saving Data and Variables

- Click on File menu
- Choose Save As option
- Give the filename MAEG1.FIT
- Specify a directory and drive or floppy disk
- Accept the first and last observations offered by clicking on OK

- Exit from Microfit:

Either go to the File menu and choose Exit.
Or click on the X in the top right hand corner. Click on OK when you receive the warning about data being lost as you have already saved it.

7. Using saved FIT files

- Start Microfit again.
- Click on the File menu and choose Open
- Give the filename MAEG1.FIT

Note that you may have to specify a drive or directory, depending on where you saved the file.

You are now back with your data and all of the variables you created above are still there.

Check that this is the case by using the icons and then rerun the regression

You can now either exit or move on to exercise three, by clicking on the File menu and choosing the option Open

EXERCISE 2

The steps below only provide instructions. You will need to go through the same procedures as in Exercise 1 to complete the tasks.

1. Enter Microfit

2. Click on the File menu and choose the Open option. Give the filename as MAEG2.FIT with the directory/disk details. This file contains data for 1959-87 on consumption and income

3. Using OLS, estimate the linear model: RC = a + b RPDI

4. Estimate a log-linear model: LRC = a + b LRPDI
where LRC = LOG(RC) and LRPDI = LOG(RPDI)

5. Estimate an extended model introducing inflation: LRC = a + b LRPDI + c DLPC
Where the inflation term DLPC is constructed using:
LPC = LOG(PC)
DLPC = LPC - LPC(-1)

6. Consider the results for each of these regressions and why each one improves on the previous one.

7. To consider how to interpret multiple regression.
-Make a note of the results of the extended model (parameter estimates/ t ratios/ DW/ Rsq)
-Estimate LRC = a_1 + b_1 DLPC
-Save the residuals. Give them the name RES1
-Estimate LRPDI = a_2 + b_2 DLPC
-Save the residuals as RES2
-Estimate RES1 = a_3 + b_3 RES2

You will see that b_3 is the same as b (the estimate in the extended model). Consider what this tells you about the interpretation of parameter estimates in multiple regression analysis.

EXERCISE 3: Capital Asset Pricing Model

This exercise is based upon those at the end of Chapter 2 in Berndt. The data is the same except that it has been collected in a Microfit file.

1. Start Microfit
2. Read in the file CHAP2.FIT, which contains monthly share return data.
3. Have a look at the descriptions of the variables.
4. Plot MARKET from Jan 78 to Dec 87 and then IBM over the same period.
5. What do the plots tell you?
6. Construct the risk premium measures $r_p - r_f$ and $r_m - r_f$, using:

RIBM = IBM - RKFREE
RMARKET = MARKET - RKFREE
Plot these and look at what happened October 1987.

7. Get the sample means using:

COR IBM MARKET RKFREE RIBM RMARKET
Work out the values you would expect for β if you estimated
$(r_j - r_f) = \alpha_j + \beta_j (r_m - r_f) + e_j$

8. Estimate β for IBM using RIBM = $\alpha_j + \beta_j$ RMARKET + e_j
9. Are the results what you would expect?. Look at the residuals and consider any outliers.
10. Choose two companies, a highly risky one and a relatively safe one and estimate their values of β. Are they what you would expect?
11. Look at the plots over time and the plots of the residuals. Are there any outliers? How would you interpret them?
12. Test the null hypothesis that $\alpha = 0$ and the hypothesis that $\beta = 1$. What are the implications of these results?
13. If you wish follow the rest of the Exercises in Berndt Chapter 2.

EXERCISE 4

This exercise will extend the previous two, both in terms of the data coverage and the techniques used.

Part 1

A Microfit file named CONS95.FIT contains the data.

Run Microfit and access this file.

The data consist of:
CE: Consumers Expenditure in current prices
RCE: Consumers Expenditure in 1995 prices
RDE: Expenditure on Durables in 1995 prices
PDI: Personal Disposable Income in current prices
RPDI: Real Personal Disposable Income in 1995 prices
C: A variable with the value one for each observation

When you have read in the data, check the data definitions using TITLE.
Then:
Explain what RPDI measures and how it differs from personal income.
Plot RCE and RPDI and comment on the main features

Generate:

```
S=log((PDI-CE)/PDI)
RS=log((RPDI-RCE)/RPDI)
```

Plot these two series and comment on their meaning and the difference between them.

Generate:

```
LC=log(RCE-RDE)
GC=LC-LC(-1)
LY=log(RPDI)
GY=LY-LY(-1)
LP=log(CE/RCE)
GP=LP-LP(-1)
Z=LC-LY
```

What do these series measure?

Use the following commands and explain the output you get:

```
PLOT LC LY
PLOT GC GY
COR LC
COR LC LY GC GY
LIST C LC LY GC GY
XPLOT LC LY
```

Run a regression with RCE as the dependent variable and C and RPDI as the independent variables, **using sample 1950 to 1980.** Note and interpret the regression results.

Save your dataset in a special Microfit file with a different name to the original (eg consnew.fit). Use this file from now on.

Part 2

Run the following regressions using OLS, on **sample 1950 1980**, the first variable is the dependent variable, the rest the independent ones. In each case interpret and comment on the main features of the regression results, diagnostic tests A to D, the plot of actual and predicted values and the plot of the residuals.

1. LC C LY

Test whether the coefficient of LY is significantly different from zero and then from one, at the 5% level.

2. GC C GY

After estimating the equation go to the hypothesis testing menu and conduct a variable addition test to see whether LC(-1) and LY(-1) are jointly significant using the F statistic and individually significant using the t statistics

3. GC C GY GP

Repeat as for 2 and comment on the significance of the lagged values.

4. LC C LY LY(-1) LC(-1)

5. GC C GY LY(-1) LC(-1)

Compare the results for 4 and 5 in terms of coefficients, standard errors, log-likelihoods, and the sum of squared residuals. What is the relationship between them.

6. GC C GY LY LY(-1) LC(-1)

Explain what happens when LY is added to 5

7. GC C GY LY(-1) GP LP(-1) LC(-1)

Calculate the long run elasticities of consumption with respect to the price level and income. What does economic theory predict about the coefficient of LP(-1). Test this prediction.

8. GC C GY GP Z(-1)

Test 8 against 7. Is this a well specified equation. Explain the economic interpretation of 8. What is the long run elasticity of consumption with respect to prices and incomes in this model.

9. GC C

Interpret this model and carry out a variable addition test for the significance of LY(-1) and GY(-1). Interpret the result.

Part 3

Run equation 8 for the samples 1950 to 1965 and 1966 to 1980. Test whether the variances are equal in the two periods and whether the coefficients are equal.

Use the equation to forecast 1980 to 1985. Interpret your results.
Estimate equation 8 for 1980 to 1985. Note the results.

Construct dummy variables for 1986, 1987, 1988 and 1989 using the following commands:

```
SAMPLE 1948 1989
D86=0
D87=0
D88=0
D89=0
SAMPLE 1986 1986
D86=1
SAMPLE 1987 1987
D87=1
SAMPLE 1988 1988
D88=1
SAMPLE 1989 1989
```

D89=1
SAMPLE 1948 1989

Run the following regression for 1950 to 1989:
GC C GY GP Z(-1) D86 D87 D88 D89

Compare the results from this equation with the ones obtained when equation 8 was run over 1950 to 1985. Interpret the coefficients and standard errors of the dummy variables.

Part 4

Reestimate equation 1 assuming AR(1) disturbances, test this against 3, using a likelihood ratio test.

What problems arise in testing 4 against 8?
Which of the models you have estimated are restricted versions of 7?
What are the restrictions in each case?

Construct a tree showing the relationship between this family of models and the test statistics.

Re-estimate equation 8 using the instrumental variable estimator, using as instruments:
C LY(-1) LC(-1) LP(-1) LY(-2) LC(-2) LP(-2)

Part 5

Using the whole data set 1948 1998:

Use the general to specific methodology to find the best parsimonious consumption function.

Using DF and ADF tests investigate the time series properties of the series and whether they are cointegrated. If you find they are estimate the error correction model.

Use the Johansen procedure to investigate the consumption function relationship and the dynamics.

Compare the results from the different methods and discuss any differences.

Part 6

Compare the results for each of the samples used above and discuss the differences.

Undertake tests for structural stability across the subsamples.

Investigate the evolution of the coefficients using CUSUM squared procedure

Discuss the results, their implications for the study of consumption and possible ways to move forward.

Revision Exercise

Indicate whether the following are True, False of Uncertain and give a brief, at most one page, explanation or proof of your assertion.

Do not simply state your answer, explain!

1. The least squares estimator is the best linear unbiased estimator (BLUE) as long as the error terms are normally distributed.

2. We cannot use OLS methods with a lagged dependent variable as they will be neither best nor unbiased.

3. The OLS method can only be used to estimate regressions that are linear in the variables.

4. In multiple regression a high correlation in the sample among the regressor implies that the least squares estimators of the parameters are biased. The solution is to drop one of the regressors.

5. Heteroscedasticity in the errors leads to biased estimators of the regression coefficients and their standard errors.

6. Estimating a time series regression in first differences is preferable to estimating it in levels.

7. The Durbin Watson statistic tests whether the errors are autocorrelated. If it is significant we should estimate the parameters using the Cochrane-Orcutt method.

8. In a simultaneous equation system the more exogenous variables there are the better.

9. If two variables are not cointegrated then there is no long run relation between them.

10. The linear probability model is an adequate way to deal with the problem of a limited dependent variable.

Table E-4 Cumulative student's t distribution*

$$F(t) = \int_{-\infty}^{t} \frac{\Gamma\left(\frac{n+1}{2}\right)}{\Gamma(n/2)\sqrt{\pi n}\left(1+\frac{x^2}{n}\right)^{(n+1)/2}}\,dx$$

F n	.75	.90	.95	.975	.99	.995	.9995
1	1.000	3.078	6.314	12.706	31.821	63.657	636.619
2	.816	1.886	2.920	4.303	6.965	9.925	31.598
3	.765	1.638	2.353	3.182	4.541	5.841	12.941
4	.741	1.533	2.132	2.776	3.747	4.604	8.610
5	.727	1.476	2.015	2.571	3.365	4.032	6.859
6	.718	1.440	1.943	2.447	3.143	3.707	5.959
7	.711	1.415	1.895	2.365	2.998	3.499	5.405
8	.706	1.397	1.860	2.306	2.896	3.355	5.041
9	.703	1.383	1.833	2.262	2.821	3.250	4.781
10	.700	1.372	1.812	2.228	2.764	3.169	4.587
11	.697	1.363	1.796	2.201	2.718	3.106	4.437
12	.695	1.356	1.782	2.179	2.681	3.055	4.318
13	.694	1.350	1.771	2.160	2.650	3.012	4.221
14	.692	1.345	1.761	2.145	2.624	2.977	4.140
15	.691	1.341	1.753	2.131	2.602	2.947	4.073
16	.690	1.337	1.746	2.120	2.583	2.921	4.015
17	.689	1.333	1.740	2.110	2.567	2.898	3.965
18	.688	1.330	1.734	2.101	2.552	.2878	3.922
19	.688	1.328	1.729	2.093	2.539	2.861	3.883
20	.687	1.325	1.725	2.086	2.528	2.845	3.850
21	.686	1.323	1.721	2.080	2.518	2.831	3.819
22	.686	1.321	1.717	2.074	2.508	2.819	3.792
23	.685	1.319	1.714	2.069	2.500	2.807	3.767
24	.685	1.318	1.711	2.064	2.492	2.797	3.745
25	.684	1.316	1.708	2.060	2.485	2.787	3.725
26	.684	1.315	1.706	2.056	2.479	2.779	3.707
27	.684	1.314	1.703	2.052	2.473	2.771	3.690
28	.683	1.313	1.701	2.048	2.467	2.763	3.674
29	.683	1.311	1.699	2.045	2.462	2.756	3.659
30	.683	1.310	1.697	2.042	2.457	2.750	3.646
40	.681	1.303	1.684	2.021	2.423	2.704	3.551
60	.679	1.296	1.671	2.000	2.390	2.660	3.460
120	.677	1.289	1.658	1.980	2.358	2.617	3.373
∞	.674	1.282	1.645	1.960	2.326	2.576	3.291

* This table is abridged from the "Statistical Tables" of R. A. Fisher and Frank Yates published by Oliver & Boyd, Ltd., Edinburgh and London, 1938. It is here published with the kind permission of the authors and their publishers.

Table E-5[1] Durbin-Watson statistic (d). Significance points of d_L and d_U: 5%

n	$k' = 1$		$k' = 2$		$k' = 3$		$k' = 4$		$k' = 5$	
	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U
15	1.08	1.36	0.95	1.54	0.82	1.75	0.69	1.97	0.56	2.21
16	1.10	1.37	0.98	1.54	0.86	1.73	0.74	1.93	0.62	2.15
17	1.13	1.38	1.02	1.54	0.90	1.71	0.78	1.90	0.67	2.10
18	1.16	1.39	1.05	1.53	0.93	1.69	0.82	1.87	0.71	2.06
19	1.18	1.40	1.08	1.53	0.97	1.68	0.86	1.85	0.75	2.02
20	1.20	1.41	1.10	1.54	1.00	1.68	0.90	1.83	0.79	1.99
21	1.22	1.42	1.13	1.54	1.03	1.67	0.93	1.81	0.83	1.96
22	1.24	1.43	1.15	1.54	1.05	1.66	0.96	1.80	0.86	1.94
23	1.26	1.44	1.17	1.54	1.08	1.66	0.99	1.79	0.90	1.92
24	1.27	1.45	1.19	1.55	1.10	1.66	1.01	1.78	0.93	1.90
25	1.29	1.45	1.21	1.55	1.12	1.66	1.04	1.77	0.95	1.89
26	1.30	1.46	1.22	1.55	1.14	1.65	1.06	1.76	0.98	1.88
27	1.32	1.47	1.24	1.56	1.16	1.65	1.08	1.76	1.01	1.86
28	1.33	1.48	1.26	1.56	1.18	1.65	1.10	1.75	1.03	1.85
29	1.34	1.48	1.27	1.56	1.20	1.65	1.12	1.74	1.05	1.84
30	1.35	1.49	1.28	1.57	1.21	1.65	1.14	1.74	1.07	1.83
31	1.36	1.50	1.30	1.57	1.23	1.65	1.16	1.74	1.09	1.83
32	1.37	1.50	1.31	1.57	1.24	1.65	1.18	1.73	1.11	1.82
33	1.38	1.51	1.32	1.58	1.26	1.65	1.19	1.73	1.13	1.81
34	1.39	1.51	1.33	1.58	1.27	1.65	1.21	1.73	1.15	1.81
35	1.40	1.52	1.34	1.58	1.28	1.65	1.22	1.73	1.16	1.80
36	1.41	1.52	1.35	1.59	1.29	1.65	1.24	1.73	1.18	1.80
37	1.42	1.53	1.36	1.59	1.31	1.66	1.25	1.72	1.19	1.80
38	1.43	1.54	1.37	1.59	1.32	1.66	1.26	1.72	1.21	1.79
39	1.43	1.54	1.38	1.60	1.33	1.66	1.27	1.72	1.22	1.79
40	1.44	1.54	1.39	1.60	1.34	1.66	1.29	1.72	1.23	1.79
45	1.48	1.57	1.43	1.62	1.38	1.67	1.34	1.72	1.29	1.78
50	1.50	1.59	1.46	1.63	1.42	1.67	1.38	1.72	1.34	1.77
55	1.53	1.60	1.49	1.64	1.45	1.68	1.41	1.72	1.38	1.77
60	1.55	1.62	1.51	1.65	1.48	1.69	1.44	1.73	1.41	1.77
65	1.57	1.63	1.54	1.66	1.50	1.70	1.47	1.73	1.44	1.77
70	1.58	1.64	1.55	1.67	1.52	1.70	1.49	1.74	1.46	1.77
75	1.60	1.65	1.57	1.68	1.54	1.71	1.51	1.74	1.49	1.77
80	1.61	1.66	1.59	1.69	1.56	1.72	1.53	1.74	1.51	1.77
85	1.62	1.67	1.60	1.70	1.57	1.72	1.55	1.75	1.52	1.77
90	1.63	1.68	1.61	1.70	1.59	1.73	1.57	1.75	1.54	1.78
95	1.64	1.69	1.62	1.71	1.60	1.73	1.58	1.75	1.56	1.78
100	1.65	1.69	1.63	1.72	1.61	1.74	1.59	1.76	1.57	1.78

n = number of observations.
k' = number of explanatory variables.

[1] This Table is reproduced from *Biometrika*, vol. 41, p. 173, 1951, with the permission of the Trustees.

Table E-3 Cumulative chi-square distribution*

$$F(u) = \int_0^u \frac{x^{(n-2)/2} e^{-x/2}\,dx}{2^{n/2}\Gamma(n/2)}$$

n \ F	.005	.010	.025	.050	.100	.250	.500	.750	.900	.950	.975	.990	.995
1	$.0^4393$	$.0^3157$	$.0^3982$	$.0^2393$	.0158	.102	.455	1.32	2.71	3.84	5.02	6.63	7.88
2	.0100	.0201	.0506	.103	.211	.575	1.39	2.77	4.61	5.99	7.38	9.21	10.6
3	.0717	.115	.216	.352	.584	1.21	2.37	4.11	6.25	7.81	9.35	11.3	12.8
4	.207	.297	.484	.711	1.06	1.92	3.36	5.39	7.78	9.49	11.1	13.3	14.9
5	.412	.554	.831	1.15	1.61	2.67	4.35	6.63	9.24	11.1	12.8	15.1	16.7
6	.676	.872	1.24	1.64	2.20	3.45	5.35	7.84	10.6	12.6	14.4	16.8	18.5
7	.989	1.24	1.69	2.17	2.83	4.25	6.35	9.04	12.0	14.1	16.0	18.5	20.3
8	1.34	1.65	2.18	2.73	3.49	5.07	7.34	10.2	13.4	15.5	17.5	20.1	22.0
9	1.73	2.09	2.70	3.33	4.17	5.90	8.34	11.4	14.7	16.9	19.0	21.7	23.6
10	2.16	2.56	3.25	3.94	4.87	6.74	9.34	12.5	16.0	18.3	20.5	23.2	25.2
11	2.60	3.05	3.82	4.57	5.58	7.58	10.3	13.7	17.3	19.7	21.9	24.7	26.8
12	3.07	3.57	4.40	5.23	6.30	8.44	11.3	14.8	18.5	21.0	23.3	26.2	28.3
13	3.57	4.11	5.01	5.89	7.04	9.30	12.3	16.0	19.8	22.4	24.7	27.7	29.8
14	4.07	4.66	5.63	6.57	7.79	10.2	13.3	17.1	21.1	23.7	26.1	29.1	31.3
15	4.60	5.23	6.26	7.26	8.55	11.0	14.3	18.2	22.3	25.0	27.5	30.6	32.8
16	5.14	5.81	6.91	7.96	9.31	11.9	15.3	19.4	23.5	26.3	28.8	32.0	34.3
17	5.70	6.41	7.56	8.67	10.1	12.8	16.3	20.5	24.8	27.6	30.2	33.4	35.7
18	6.26	7.01	8.23	9.39	10.9	13.7	17.3	21.6	26.0	28.9	31.5	34.8	37.2
19	6.84	7.63	8.91	10.1	11.7	14.6	18.3	22.7	27.2	30.1	32.9	36.2	38.6
20	7.43	8.26	9.59	10.9	12.4	15.5	19.3	23.8	28.4	31.4	34.2	37.6	40.0
21	8.03	8.90	10.3	11.6	13.2	16.3	20.3	24.9	29.6	32.7	35.5	38.9	41.4
22	8.64	9.54	11.0	12.3	14.0	17.2	21.3	26.0	30.8	33.9	36.8	40.3	42.8
23	9.26	10.2	11.7	13.1	14.8	18.1	22.3	27.1	32.0	35.2	38.1	41.6	44.2
24	9.89	10.9	12.4	13.8	15.7	19.0	23.3	28.2	33.2	36.4	39.4	43.0	45.6
25	10.5	11.5	13.1	14.6	16.5	19.9	24.3	29.3	34.4	37.7	40.6	44.3	46.9
26	11.2	12.2	13.8	15.4	17.3	20.8	25.3	30.4	35.6	38.9	41.9	45.6	48.3
27	11.8	12.9	14.6	16.2	18.1	21.7	26.3	31.5	36.7	40.1	43.2	47.0	49.6
28	12.5	13.6	15.3	16.9	18.9	22.7	27.3	32.6	37.9	41.3	44.5	48.3	51.0
29	13.1	14.3	16.0	17.7	19.8	23.6	28.3	33.7	39.1	42.6	45.7	49.6	52.3
30	13.8	15.0	16.8	18.5	20.6	24.5	29.3	34.8	40.3	43.8	47.0	50.9	53.7

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* This table is abridged from "Tables of percentage points of the incomplete beta function and of the chi-square distribution," *Biometrika*, Vol. 32 (1941). It is here published with the kind permission of its author, Catherine M. Thompson, and the editor of *Biometrika*.

Table 14.1 Critical Values for Unit Root Tests

Sample Size	*K-Test*		*t-Test*		*F-Test*[a]	
	1%	*5%*	*1%*	*5%*	*1%*	*5%*
AR (1)						
25	−11.9	−7.3	−2.66	−1.95		
50	−12.9	−7.7	−2.62	−1.95		
100	−13.3	−7.9	−2.60	−1.95		
250	−13.6	−8.0	−2.58	−1.95		
500	−13.7	−8.0	−2.58	−1.95		
∞	−13.8	−8.1	−2.58	−1.95		
AR (1) with constant						
25	−17.2	−12.5	−3.75	−3.00		
50	−18.9	−13.3	−3.58	−2.93		
100	−19.8	−13.7	−3.51	−2.89		
250	−20.3	−14.0	−3.46	−2.88		
500	−20.5	−14.0	−3.44	−2.87		
∞	−20.7	−14.1	−3.43	−2.86		
AR (1) with constant and trend						
25	−22.5	−17.9	−4.38	−3.60	7.24	10.61
50	−25.7	−19.8	−4.15	−3.50	6.73	9.31
100	−27.4	−20.7	−4.04	−3.45	6.49	8.73
250	−28.4	−21.3	−3.99	−3.43	6.34	8.43
500	−28.9	−21.5	−3.98	−3.42	6.30	8.34
∞	−29.5	−21.8	−3.96	−3.41	6.25	8.27

[a] $K = T(\hat{\rho} - 1)$, $t = (\hat{\rho} - 1)/\mathrm{SE}(\hat{\rho})$ and F-test is for $\gamma = 0$ and $\rho = 1$ in $y_t = \alpha + \gamma t + \rho y_{t-1} + u_t$.

Source: W. A. Fuller, *Introduction to Statistical Time Series* (New York: Wiley, 1976), p. 371 for the *K*-test and p. 373 for the *t*-test; D. A. Dickey and W. A. Fuller, "Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root," *Econometrica*, Vol. 49, No. 4, 1981, p. 1063 for the *F*-test.

Table 14.2 Critical Values (5%) for the Cointegration Tests

n	*T*	*CRDW*	*DF*	*ADF*[a]
2	50	0.78	−3.67	−3.29
	100	0.39	−3.37	−3.17
	200	0.20	−3.37	−3.25
3	50	0.99	−4.11	−3.75
	100	0.55	−3.93	−3.62
	200	0.39	−3.78	−3.78
4	50	1.10	−4.35	−3.98
	100	0.65	−4.22	−4.02
	200	0.48	−4.18	−4.13
5	50	1.28	−4.76	−4.15
	100	0.76	−4.58	−4.36
	200	0.57	−4.48	−4.43

[a] $CRDW = \sum (\hat{u}_t - \hat{u}_{t-1})^2 / \sum \hat{u}_t^2$, CRDW means "cointegrating regression Durbin–Watson" statistic; DF = t-test for $\alpha = 0$ in $\Delta \hat{u}_t = \alpha \hat{u}_{t-1} + \eta_t$; ADF = t-test for $\alpha = 0$ in $\Delta \hat{u}_t = \alpha \hat{u}_{t-1} + \sum_{1}^{p} \phi_i \Delta \hat{u}_{t-1} + \eta_t$. In all these tests $\hat{u}_t$ is the residual from the cointegrating regression.

Source: R. F. Engle and S. Yoo, "Forecasting and Testing in Cointegrated Systems," *Journal of Econometrics*, Vol. 35, 1987.

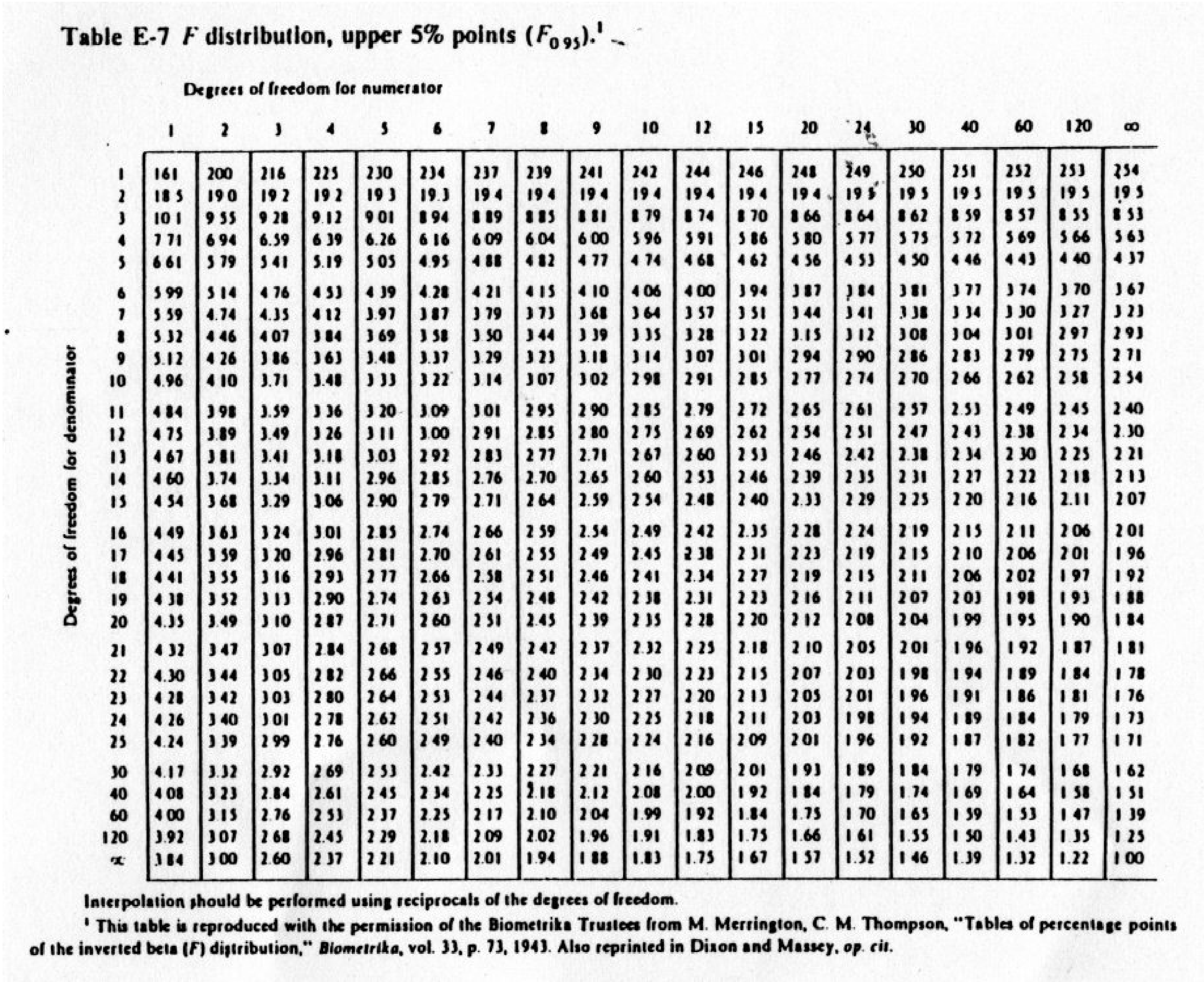

Table E-7 *F* distribution, upper 5% points ($F_{0.95}$).[1]

Degrees of freedom for numerator

Degrees of freedom for denominator	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161	200	216	225	230	234	237	239	241	242	244	246	248	249	250	251	252	253	254
2	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.5	19.5	19.5	19.5	19.5	19.5
3	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.37
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

Interpolation should be performed using reciprocals of the degrees of freedom.

[1] This table is reproduced with the permission of the Biometrika Trustees from M. Merrington, C. M. Thompson, "Tables of percentage points of the inverted beta (*F*) distribution," *Biometrika*, vol. 33, p. 73, 1943. Also reprinted in Dixon and Massey, *op. cit.*