# Military Procurement, Industry Structure and the Revolution in Military Affairs*

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**Abstract**

We examine a number of developments associated with the RMA, using a partial equilibrium model of military procurement with two-way international trade in a world where many of the recipients of this trade are non-producers engaged in regional arms races. We study the consequences for industry structure, procurement subsidies and the welfare of producers of: increased process innovation and the consequent rise in the fixed costs of R&D; increased trade between producers owing to the willingness to forgo the security benefits of domestic procurement; and a growing external market of non-producers. Our comparison of the non-cooperative and cooperative outcomes highlights two sources of inefficiencies from the position of the producers. Adopting a generalized Dixit-Stiglitz utility function to model military strength, the existence of a separate taste-for-variety parameter means governments acting independently choose to support too few firms and generate too little variety. The second source is from competition for the external market and here there are two effects. First export revenue per firm from exports rises as the total number of firms falls. This occurs because firms then compete less intensively and can spread their fixed costs over a larger market share. As the external market increases, governments then choose to support *less* firms. Under non-cooperation, however, this reduction in firm number is too little compared with the optimum because governments acting independently only care about competition between domestic firms. The second effect of an external market is to encourage too much investment in quality relative to the cooperative outcome. This is because the provision of quality has a beggar-thy-neighbour character. When governments raise quality unilaterally this increases market share. In equilibrium however the benefit to competitiveness disappears and countries are left with too much quality compared with that chosen cooperatively. A high investment into this quality raises fixed costs and reduces firm number further. The presence of an external market then tends to reduce firm number (i.e., raise concentration) and encourages excessive investment into quality. A high taste-for-variety has the opposite effect, raising firm numbers which, because there is a trade-off between quality and variety, reduces quality.

**JEL Classification**: F12, H56, L10

**Keywords**: Revolution in Military Affairs, military procurement, defence industry structure, arms trade

# 1 Introduction

In this paper, we construct a integrated model of military procurement, technological change, international trade and regional conflict. This work builds of previous work by the authors[1] and develops it in a number of directions. First, we draw upon an important recent literature[2] concerned with endogenous property rights, where agents allocate resources to attack or defence as well as to production and trade. The resulting distribution of property reflects agents ability to protect their resources from others or steal resources from others. In this literature the analogy to the production function is the conflict success function (CSF): the inputs are the fighting efforts of the two sides and the outputs are their relative degree of success: proportion of the stakes won, in the continuously divisible case or probability of winning, in an all or nothing case. Our use of CSFs to model the allocation of resources for for military goods provides rigorous micro-foundations for the concept of security, a feature that was lacking in earlier work.

The second development of work is aimed at addressing a number of important aspects of the ways in which new technologies and globalization are transforming fighting and the defence industry, the 'Revolution in Military Affairs'. Increasingly, the military is characterized by process innovation and fixed costs of R&D are escalating. Production is becoming concentrated in a smaller number of firms and a smaller number of producer countries. Trade in arms between producer countries is increasing with the greater willingness to forgo the security of domestic procurement. Governments are 'commercializing the military' by switching into COTS dual-use technology.

To address these issues associated with the RMA, we construct a partial equilibrium model of military procurement with two-way international trade in a world where many of the recipients of this trade are non-producers engaged in regional arms races. Governments in producer countries buy products from the domestic firms and import from the rest of the world; governments of non-producer countries do not have domestic production and therefore, cover their procurement needs through imports. Producer country governments have an interest in maintaining a domestic sector and therefore, pay domestic firms the

[1] Garcia-Alonso, 1999, 2000; Garcia-Alonso and Levine, 1997; Levine et al, 1994; Levine and Smith, 1995, 1997a, 1997b, 2000a, 2000b; Levine et al, 2000.

[2] See, for example, Anderton( 2000), Hirchleifer (2000), Skaperdas (19910, Garfinkel and Skerpadas (2000).

price that ensures their existence, i.e., the price that makes them meet the participation constraint. However, firms are price setters in the exports market.

A basic feature of our model is that governments choose the number of firms that compose the domestic procurement sector, whose existence they will guarantee. They also choose the 'quality' of level of technology. High fixed production costs associated with the latter in particular, imply that firms' existence depends on them being government providers and therefore, the government is actually choosing the number of domestic firms. In order to explain why governments want to keep several firms as domestic suppliers within the same sector, we use a 'taste for variety model'. This type of model has been traditionally used in the monopolistic competition literature starting with Dixit and Stiglitz (1977). Our paper however, uses a more general form of the Dixit-Stiglitz utility function, discussed in Benassy (1996), which allows for the taste for variety and the elasticity of substitution between the differentiated military goods to be independent.

The rest of the paper is organized as follows. Section 2 provides the basic set-up and the sequence of moves in the procurement game with governments and firms as players. There are three stages to the procurement and trade game. First, given their military expenditure (assumed to be exogenously fixed), producer governments choose the number of domestic suppliers to support, and the amount and quality of goods to procure from each. They also formulate a subgame perfect plan to import goods at the asking, possibly out-of-equilibrium prices, which they will implement at stage 3 of the game. At stage 2 given domestic procurement decisions, firms produce and take part in a Bertrand price-setting game in the export market. At stage 3 all governments make their import decisions given possibly out-of-equilibrium prices. In addition, importing government engaged in regional arms races, choose they levels of military expenditure. In a subgame perfect equilibrium of the entire game, sections 3, 4 and 5 solve for these stages starting at stage 3.

Section 5 characterizes the equilibrium which in general can accommodate various asymmetries between both producers and non-producers. Section 6 provides analytical results for the symmetric form of this equilibrium. Section 7 compares this equilibrium with the cooperative arrangement. Section 8 illustrates the results using numerical solutions and section 9 provides some concluding remarks.

## 2 The Set-up

### 2.1 The Model

We model an international market for a public service good, consisting of $\ell$ producing and importing countries and $r$ non-producers who only import. The total budget in each country available for this particular public service good is given. Producer country 1 produces differentiated goods $j = 1, 2, \cdots, n_1$, country 2 produces goods $j = n_1 + 1, n_1 + 2, \cdots, n_1 + n_2$ etc, so there are $\sum_{i=1}^{\ell} n_i = N$, say, goods in total. Governments procure from domestic firms (if they exist) and overseas firms who enter or exit the market freely.

As well as horizontal differentiation there is vertical differentiation: each good can also be produced in an unlimited number of vertically differentiated varieties or 'qualities'. If quality increases by a proportion $\lambda$, say, then one unit of the good provides $\lambda$ more services. The *maximum* quality of good $j$ in country $i = 1, 2, \cdots, \ell$ is $q_{ij}$ which is the quality of the domestically procured good. We assume that each firm can produce a lower quality good at the same cost and we allow for the possibility that there is an *arms export regime* in place that restricts the quality of the imported good by country i of variety j to $u_{ij} = \gamma_{ik} q_{kj}$, where $q_{kj}$ is the quality of the domestically procured good by country k of variety $j = n_{k-1} + 1, n_{k-1} + 2, \cdots, n_{k-1} + n_k$. The parameter $\gamma_{ik} \leq 1$ captures the extent of the arms export constraint by the exporting country $k$ on the importing country $i$. We take this regime to be exogenously imposed on the military authority making the procurement decisions and we do not go into details of how this regime can be sustained.[3]

It makes for a simpler presentation if we focus on decisions in producer country 1. Government 1 procures $d_{1j}, j = 1, 2, \cdots, n_1$ domestically produced goods with quality $q_{1j}$ and $m_{1j}, j = n_1 + 1, n_1 + 2, \cdots, N$ imported goods with quality $u_{1j}$. Military strength takes the form of a generalized Dixit-Stigliz CES utility function of the form

$$S_1 = [w_1 n_1 + (1 - w_1)(N - n_1)]^{\nu} \left[ w_1 \sum_{j=1}^{n_1} (q_{1j} d_{1j})^{\alpha} + (1 - w_1) \sum_{j=n_1+1}^{N} (u_{1j} m_{1j})^{\alpha} \right]^{\frac{1}{\alpha}} \; ; \; \alpha \in [0, 1), \, \nu > 0 \tag{1}$$

[3]At suitable stages of the game we impose symmetry between firms in the same country. Then each country produces the same quality of each variety and with $j = k$, the export control regime imposed by country k on country i is expressed by country i importing quality $u_{ik} = \gamma_{ik} q_{kk} = \gamma_{ik} q_k$, rewriting $q_{kk} = q_k$.

In (1) the weights $w_1$ and $1-w_1$, with $w_1 \in [\frac{1}{2}, 1]$, express possible preferences for domestic rather than imported procurement in country 1.[4] If we put $\nu = 0$ and $w = \frac{1}{2}$, (1) reduces to the familiar Dixit-Stiglitz utility function used in the new trade and endogenous growth literatures. But as Benassy (1996) points out, this form of utility is restricted in that it implies an on-to-one correspondence between the taste for variety and the elasticity of substitution.

To see the significance of this generalized form of the Dixit-Stiglitz utility function, suppose there are two producer countries. Define a function $v_1(n_1, n_2)$ to represent the proportional capability gain from spreading a certain amount of quality-adjusted output $y$ say between all $n_1 + n_2$ varieties rather than concentrating it on one variety in country 1; i.e.,

$$\begin{aligned} v_1(n_1, n_2) &= \frac{[w_1 n_1 + (1-w_1)n_2]^\nu \left[w_1 \sum_{j=1}^{n_1} (y)^\alpha + (1-w_1) \sum_{j=n_1+1}^{N} (y)^\alpha\right]^{\frac{1}{\alpha}}}{(n_1+n_2)y} \\ &= \frac{[w_1 n_1 + (1-w_1)n_2]^{\nu+\frac{1}{\alpha}}}{n_1+n_2} \end{aligned}$$

Suppose that the total number of varieties $N = n_1 + n_2$ increases keeping the proportion $\frac{n_1}{n_2}$ fixed. Then putting $n_1 = kN$ and $n_2 = (1-k)N$, $v_1 = v_1(N) = [w_1 k + (1-w_1)(1-k)]^{(\frac{1}{\alpha}+\nu)} N^{(\nu+\frac{1}{\alpha}-1)}$. We now define the *taste for variety* by the elasticity $\frac{N dv_1}{v_1 dN} = \tau$ say given by

$$\tau = \frac{N dv_1}{v_1 dN} = \nu + \frac{1}{\alpha} - 1$$

The significance of the extra term in (1) is now apparent. If $\nu = 0$, then the taste for variety $\tau = \frac{1}{\alpha} - 1 = \frac{1}{\sigma-1}$ which is determined solely by the elasticity of substitution $\sigma$. Thus this formulation establishes an arbitrary link between different characteristics: taste for variety and elasticity of substitution, the latter also determining the market power since the mark-up on marginal costs in the export market is $\frac{1}{\alpha}$. Introducing the extra term breaks this link and has important consequences for the subsequent analysis.

Governments in producer countries procure from domestic and foreign firms, possibly at different prices. Let $p_{1j}$ be the price of the procured domestic good and $P_j$ be the world

[4] Note that(1) can be given an 'iceberg' technology interpretation by writing it as $U_1 = [w_1 n_1 + (1-w_1)(N-n_1)]^\nu \left[\sum_{j=1}^{n_1} (q_{1j} d_{1j})^\alpha + \sum_{j=n_1+1}^{N} (T_1 u_{1j} m_{1j})^\alpha\right]^{\frac{1}{\alpha}}$, where $T_1 = \left(\frac{1-w_1}{w_1}\right)^{\frac{1}{\alpha}}$ is the fraction of the original good that actually arrives, the rest 'melting away' on route. However we do not distinguish between trade between close and distant countries, so we do not pursue this spatial interpretation.

market price of the traded good of variety j produced by firms in all producing countries $j = 1, 2, \cdot\cdot, N$. Then the budget constraint for government in producer country 1 is:

$$\sum_{j=1}^{n_1} p_{1j} d_{1j} + \sum_{j=n_1+1}^{N} P_j m_{1j} = G_1 \tag{2}$$

where $G_i$ is total procurement expenditure in country i.

For the non-producing country $i = \ell + 1, \ell + 2, \cdot\cdot\cdot, \ell + r$ military strength is given by

$$S_i = N^{\nu} \left[ \sum_{j=1}^{N} (u_{ij} m_{ij})^{\alpha} \right]^{\frac{1}{\alpha}} \tag{3}$$

where $u_{ij}$ is the quality allowed to importing non-producing country $i$. Their budget constraint is:

$$\sum_{j=1}^{N} P_j m_{ij} = G_i \tag{4}$$

The model is completed by specifying the following cost structure for the firm. Firm j produces $d_j$ units of variety j for its domestic government at a procurement price $p_j$ and exports $x_j$ units at a international market price $P_j$. The cost of producing total output $y_j = d_j + x_j$ for firm j in country i, at maximum quality $q_j$, is assumed to be

$$C(y_j, q_j) = F_i + f_i q_j^{\beta_i} + c_i y_j = H_i(q_j) + c_i y_i \tag{5}$$

say. The first term in (5) we associate with fixed capital costs and R&D, and the final term constitutes variable costs. It follows that the profit of this firm is

$$\pi_j = p_j d_j + P_j x_j - C_i(y_j, q_j) \tag{6}$$

and since there is free entry and exit, we must impose the participation constraint $\pi_j \geq 0$ on the procurement decision.

## 2.2 Sequencing of Events

We first consider the optimal decisions of a single government taking the decisions of other governments as given. The sequencing of events is as follows:

**1. Domestic Procurement by Producers.** Given military expenditure, the government in producer country 1 sets and procures domestic goods of quantity $d_{1j}$ and quality $q_{1j}$ at price $p_{1j}$, for $j = 1, 2, \cdot\cdot\cdot, n_1$. It also formulates a time-consistent plan to import goods $m_{1j}$ of quality $u_{1j}$, for $j = n_1 + 1, n_1 + 2, \cdots, N$ at the world market equilibrium price $P_j$. All decisions are subject to a budget constraint and a non-negative profit participation constraint for domestic firms. The procurement price may be greater or less than the international market price. Firms already participating in the international market will always accept domestic procurement as long as the procurement price exceeds the marginal cost. In general the world market price depends on procurement decisions at this stage.

**2. The Bertrand Equilibrium.** With a commitment to producing $d_{1j}$, in a Bertrand equilibrium of this stage of the game, firms in producer country 1 set world prices $P_j$ and export quantity $x_{1j}$ of quality $u_{ij}$ to countries $i = 2, \cdots, \ell + r$.

**3. Military Spending by Non-Producers and Demand for Imports by all Countries.** Given the world market price $P_j$ and quality $u_{ij}$, and military expenditure, governments in both producer and non-producer countries $i = 1, 2, \cdots, \ell + r$ procure imports of good, $m_{ij}$, $j = 1, 2, \cdots, N$ of quality $u_{ij}$, where $i \neq j$ for producer countries $i = 1, 2, \cdots, \ell$. Non-producers anticipating these decisions allocate resources between consumption and military expenditure.

To solve for the equilibrium[5] we proceed by backward induction starting at stage 3.

# 3 Military Spending by Non-Producers and Demand for Imports

## 3.1 Non-Producers

Consider non-producers in regions $i = \ell + 1, \ell + 2, \cdots, \ell + r$ as pairs of risk-neutral, countries, A and B say, involved in a regional conflict. They have an given, GDP $Y_{iA}, Y_{iB}$ which can be devoted to military expenditure $G_{iA}, G_{iB}$ or other forms of consumption expenditure $C_{iA}, C_{iB}$. Choose the price of the consumption good as the numeraire. The budget constraint is therefore

$$Y_{ik} = G_{ik} + C_{ik}\,; \;\; k = A, B \tag{7}$$

[5]Note that in the absence of procurement considerations the trade equilibrium corresponds exactly to the 'new-trade' model of Krugman (1979, 1980, 1981). Then stage 1 of our model is the free-entry process.

Consider now a war in region i leaving $\phi_i(C_{iA} + C_{iB})$ available for consumption. The parameter $\phi_i \in [0, 1]$ captures the destructive effect of a war in region i. Suppose that the prize for winning is some proportion of total consumption, $\theta_i\phi_i(C_{iA}+C_{iB})$. Then assuming the two countries maximize expected consumption the expected utility, following a war for country $k = A$ is given by

$$U_{iA}(S_{iA}, S_{iB}, G_{iA}, G_{iB}) = [p_{iA}(S_{iA}, S_{iB})\theta_i + (1 - p_{iA}(S_{iA}, S_{iB}))(1 - \theta_i)]\phi_i(C_{iA} + C_{iB}) \quad (8)$$

In (8), $p_i(S_{iA}, S_{iB})$ is a *Contest Success Function* (CSF) used extensively in the conflict literature. Dropping the regional subscript i in the rest of this subsection, a general form of the CSF , discussed in Skeperdas (1996), takes the form

$$p_A = \frac{f(S_A)}{f(S_A) + f(S_B)}\ ;\ f' > 0 \quad (9)$$

$$p_B = \frac{f(S_B)}{f(S_A) + f(S_B)} = 1 - p_A \quad (10)$$

Two forms of the CSF, discussed in Hirchleifer (2000) at some length, are the *ratio form* and the *difference form*. These take the forms respectively

$$p_A = \frac{(b_A S_A)^m}{(b_A S_A)^m + (b_B S_B)^m} \quad (11)$$

$$p_B = \frac{\exp(k b_A S_A)}{\exp(k b_A S_A) + \exp(k b_B S_B)} = \frac{1}{1 + \exp(k(b_B S_B - b_A S_A))} \quad (12)$$

In both these forms $b_i$, $i = A, B$ is a measure of the effectiveness of the same military capability in the hands of country i; m in (11) or k in (12) are *decisiveness parameters* scaling the degree to which a side's greater military strength translates into enhanced battle success. As we shall see the form of CSF is quite crucial in determining the effect of variety and quality on the choice of military expenditure.

At stage 3, given the price $P_j$, and the number of differentiated goods, in a regional Nash equilibrium, the importing government in non-producing producer country A chooses both total expenditure $G_A$ and a composition of imports $m_{Aj}, j = 1, 2, \cdots, N$ to maximize $U_A$ given by (8) subject to its budget constraint (7), given the corresponding decision $G_B$ of its rival . We decompose this optimization problem into two parts. First maximize the military strength $S_A, S_B$ that can be achieved with a given expenditures $G_A, G_B$. Let $S_A^*(G_A), S_B^*(G_B)$ be these maximized levels of military security. Then country A maximizes the utility $U_{iA}(S_{iA}^*(G_A), S_{iB}^*(G_B), G_{iA}, G_{iB})$ with respect to $G_A$ given its budget constraint and $G_B$, and country B acts similarly.

To carry out the first part of this optimization define a Lagrangian for non-producer country A

$$S_A - \lambda \left( \sum_{j=1}^{N} P_j m_{Aj} - G_A \right)$$

where $\lambda \geq 0$ is a Lagrange multiplier. Then the first-order conditions are:

$$\frac{1}{\alpha} \left[ \sum_{j=1}^{N} (u_{Aj} m_{Aj})^{\alpha} \right]^{\frac{1}{\alpha}-1} \alpha u_{Aj}^{\alpha} m_{Aj}^{\alpha-1} = \lambda P_j \; ; \; j = 1, 2, \cdots N \tag{13}$$

Returning to (13), dividing the $jth$ equation by the $kth$ equation we have

$$\left( \frac{u_{Aj} m_{Aj}}{u_{Ak} m_{Ak}} \right)^{\alpha-1} = \frac{u_{Ak} P_j}{u_{Aj} P_k}$$

Substituting back into the budget constraint (3) we get

$$\sum_{k=1}^{N} \left( \frac{P_k}{u_{Ak}} \right) u_{Aj} m_{Aj} \left( \frac{u_{Aj} P_k}{u_{Ak} P_j} \right)^{-\frac{1}{1-\alpha}} = \sum_{k=1}^{N} \left( \frac{P_k}{u_{Ak}} \right)^{1-\sigma} \left( \frac{P_j}{u_{Aj}} \right)^{\sigma} u_{Aj} m_{ij} = G_A$$

where $\sigma = \frac{1}{1-\alpha} > 1$. This results in the quality-adjusted demand by government A for good $j = 1, 2, \cdots, N$ given by

$$u_{Aj} m_{Aj} \quad = \quad \frac{G_A}{\left( \frac{P_j}{u_{Aj}} \right)^{\sigma} \sum_{k=1}^{N} \left( \frac{P_k}{u_{Ak}} \right)^{1-\sigma}} \tag{14}$$

To interpret and manipulate (14) it is convenient to define

$$\hat{P_A} = \left[ \sum_{k=1}^{N} \left( \frac{P_k}{u_{Ak}} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{15}$$

Then $\hat{P_A}$ is the quality-adjusted form of the familiar price index of imported goods, facing non-producer country A, used in the product differentiation literature (see, for example, Beath and Katsoulacas (1991), chapter 3). Now (14) and (15) can be written

$$u_{Aj} m_{Aj} = \frac{G_A}{\hat{P_A}^{1-\sigma}} \left( \frac{P_j}{u_{Aj}} \right)^{-\sigma} \tag{16}$$

The importance of (16) is that given $\hat{P}$, the elasticity of quality-adjusted demand for variety j on the world market with respect to the quality-adjusted price is constant with elasticity $-\sigma$. Substituting (16) into (3) and using $\sigma = \frac{1}{1-\alpha}$ we have

$$S_A^* = \frac{N^{\nu} G_A}{\hat{P}_A^{1-\sigma}} \left[ \sum_{j=1}^{N} \frac{P_j}{u_{Aj}}^{-\alpha\sigma} \right]^{\frac{1}{\alpha}} = \frac{N^{\nu} G_A}{\hat{P}_A^{1-\sigma}} \hat{P}_A^{\frac{1-\sigma}{\alpha}} = \frac{N^{\nu} G_A}{\hat{P}_A} \tag{17}$$

Hence in terms of optimal security, the budget constraint of country A takes the convenient form

$$C_A + G_A = C_A + N^{-\nu}\hat{P}_A S_A^* = Y_A \tag{18}$$

Country A now maximizes

$$U_A(S_A^*, S_B^*, G_A, G_B) = [p_A(S_A^*, S_B^*)\theta + (1 - p_A(S_A^*, S_B^*))(1-\theta)]\phi(C_A + C_B) \tag{19}$$

with respect to $G_A$, subject to (18) and a corresponding budget constraint for country B, given $G_B$.

The first order conditions defining the Nash equilibrium are

$$(2\theta - 1)\frac{\partial p_A}{\partial S_A^*} = \frac{[p_A(S_A^*, S_B^*)\theta + (1 - p_A(S_A^*, S_B^*))(1-\theta)]N^{-\nu}\hat{P}_A}{Y_A + Y_B - N^{-\nu}\hat{P}_A S_A^* - N^{-\nu}\hat{P}_B S_B^*} \tag{20}$$

$$(2\theta - 1)\frac{\partial p_B}{\partial S_B^*} = \frac{[p_B(S_A^*, S_B^*)\theta + (1 - p_B(S_A^*, S_B^*))(1-\theta)]N^{-\nu}\hat{P}_B}{Y_A + Y_B - N^{-\nu}\hat{P}_A S_A^* - N^{-\nu}\hat{P}_B S_B^*} \tag{21}$$

With $S_A^* = \frac{N^\nu G_A}{\hat{P}_A}$ and $S_B^* = \frac{N^\nu G_B}{\hat{P}_B}$, (20) and (21) define two *reaction functions* in $G_A$ and $G_B$. Their intersection is a Nash equilibrium; in general this is asymmetrical with asymmetries arising from differences in GDP $Y_A$ and $Y_B$ and differences in the prices facing each country. The latter can arise if export control regimes differ between the two countries. However for the most part we focus on symmetrical equilibria. Then putting $p_A = p_B = \frac{1}{2}$, $S_A = S_B = S$, say, $\hat{P}_A = \hat{P}_B = \hat{P}$, $Y_A = Y_B = Y^{np}$ and $G_A = G_B = G^{np}$, for all non-producers the, right-hand-side of the reaction functions become $\frac{N^{-\nu}\hat{P}}{4(Y^{np} - N^\nu \hat{P} S)}$. The left-hand-side however depends on the form of the CSF. For the ratio form in a symmetric equilibrium $\frac{\partial p_A}{\partial S_A} = \frac{m}{4S}$; for the difference form $\frac{\partial p_A}{\partial S_A} = \frac{kb}{4}$ where we have put $b_A = b_B = b$. Hence substituting into (20) or (21) we arrive at the symmetric Nash equilibria for the two cases

$$\text{Ratio form of CSF} \; : \; G^{np} = \frac{(2\theta - 1)m)Y^{np}}{(1 + (2\theta - 1)m} \tag{22}$$

$$\text{Difference form of CSF} \; : \; G^{np} = Y^{np} - \frac{N^{-\nu}\hat{P}}{(2\theta - 1)kb} \tag{23}$$

Thus an internal maximum $G > 0$ requires $\theta > \frac{1}{2}$ in both cases whilst for the difference form we must impose a further condition $\hat{P} < (2\theta - 1)bY^{np}$.

With the ratio form of the CSF we now see that military expenditure is independent of both the price index and the total number of varieties. This is a familiar property of the standard Dixit-Stigliz monopolistic competition model where the utility function is a Cobb-Douglas function of the numeraire good and the composite quantity index of differentiated goods (military strength in the context of our model). However with the difference form the optimal choice of government spending declines as the price index $\hat{P}$ increases, or the total number of varieties, $N$ decreases. Since in a market symmetric equilibrium $\hat{P} = \frac{PN^{-\frac{1}{(\sigma-1)}}}{\gamma q}$, and the price P is constant (as we shall see), it follows from (23) that

$$\frac{\partial G^{np}}{\partial N} = \left(\nu + \frac{1}{\sigma - 1}\right) \frac{N^{-\nu-1}\hat{P}}{(2\theta - 1)kb} > 0 \tag{24}$$

$$\frac{\partial G^{np}}{\partial q} = \frac{N^{-\nu}\hat{P}}{q(2\theta - 1)kb} > 0 \tag{25}$$

Since in the symmetric Nash equilibrium, *any* military expenditure is inefficient in this set-up, this means that the welfare of the non-producers actually *increases* if quality decreases and/or the arms control regime is strengthened (then $\gamma$ and therefore $\hat{P}$ falls) and/or $N$ decreases. This remarkable result is a consequence of the unique character of military goods–they bring security to the purchaser but insecurity to a threatened rivals. We summarize our result as:

**Proposition 1**

**With the ratio form of the CSF, the military expenditure of non-producers is independent of the number and quality of differentiated goods. However with the difference form, military expenditure falls and welfare increases as the number and quality decreases and/or the export regime is strengthened.**

### 3.2 Producers

As for non-producers we import demand for any good $j = 1, 2, \cdots, N$ can similarly be written as

$$\begin{aligned} u_{ij}m_{ij} &= \frac{[G_i - \sum_{j=n_{i-1}+1}^{n_{i-1}+n_i} p_{ij}d_{ij}]}{\left(\frac{P_j}{u_{ij}}\right)^{\sigma} \sum_{k \neq [N_{i-1}, N_i]}^{N} \left(\frac{P_k}{u_{ik}}\right)^{1-\sigma}}\,; j \neq N_{i-1}+1,\, N_{i-1}+2, \cdots, N_{i-1}+n_i \\ &= 0\,; \qquad j = N_{i-1}+1,\, N_{i-1}+2, \cdots, N_{i-1}+n_i \end{aligned} \tag{26}$$

where we have defined $N_i = n_1 + n_2 + \cdots + n_i$ for $i \geq 1$ (in which case $N_1 = n_1$ and $N_\ell = N$), country $i = 1, 2, \cdots, \ell$ produces varieties $j = N_{i-1}+1, N_{i-1}+2, \cdots, N_{i-1}+n_i = N_i$

and imports $m_{ij}$ units of variety $j = 1, 2, \cdots, N_{i-1}, N_i + 1, N_i + 2, \cdots, N$ (defining $N_0 = 0$). Again we can define an price index of imported goods for producer countries as

$$\hat{P}_i = \left[ \sum_{k \neq [N_{i-1}, N_i]}^{N} \left( \frac{P_k}{u_{ik}} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} ; \; i = 1, 2, \cdots, \ell \tag{27}$$

# 4 The Bertrand Equilibrium

Turning to stage 2 of the game, in producer country 1 firm $j = 1, 2, \cdots, n_1$ profit at stage 2 is given by

$$\pi_{1j} = (p_{1j} - c_1)d_{1j} + (P_j - c_1)x_{1j} - H_1(q_{1j}) \, ; \, j = 1, 2, \cdots, n_1 \tag{28}$$

where exports to producers and non-producers are given by

$$x_{1j} = \sum_{i=2}^{\ell+r} m_{ij} = \sum_{i=1}^{\ell} \frac{[G_i - \sum_{j=n_{i-1}+1}^{n_{i-1}+n_i} p_{ij} d_{ij}]}{P_j^{\sigma} \hat{P}_i^{1-\sigma}} + \sum_{i=\ell+1}^{\ell+r} \frac{G_i u_{i1}^{\sigma-1}}{P_j^{\sigma} \hat{P}^{1-\sigma}} \tag{29}$$

The first term in (29) consists of exports to other producing countries and depends on the procurement decisions already taken at stage 1 and on all prices set at stage 2 of the game. The second term consists of exports to non-producing countries and depend on the all prices set by firms at stage 2 of the game.

For producers let $\Gamma_i = G_i - \sum_{j=N_{i-1}+1}^{N_{i-1}+n_i} p_{ij} d_{ij}$ be the part of the government budget devoted to imports. Define $\Gamma_i = G_i$ and $\hat{P}_i = \hat{P}$ for non-producers. Then maximizing profits given by (28) with respect to $P_j$, gives the first-order conditions

$$(P_j - c_1) \frac{\partial x_{1j}}{\partial P_j} + x_{1j} = 0 \tag{30}$$

where from (29)

$$\frac{\partial x_{1j}}{\partial P_j} = -\frac{\sigma x_{1j}}{P_j} - \underbrace{P_j^{-\sigma} \sum_{i=2}^{\ell+r} \frac{\Gamma_i u_{i1}^{\sigma-1}}{\hat{P}_i^{2(1-\sigma)}} \frac{\partial(\hat{P}_i^{1-\sigma})}{\partial P_j}}_{\text{strategic interaction term}} \tag{31}$$

In working out the effect of a change in the price of variety firm j considers two effects: the first term takes the total price index of imports facing other countries $\hat{P}_i \, ; \, i = 2, 3, \cdots, \ell + r$ as given. The second *strategic* term considers the effect on each of these price indices of the firms export price. If $N$ is large, which we assume in this paper, then there are so many firms that we can ignore this strategic effect.[6] Then substituting (31) back into (30), the first order condition becomes

$$\left[-\frac{\sigma(P_j - c_1)}{P_j} + 1\right] x_{1j} = 0\,;\; j = 1, 2, \cdots, n_1 \tag{32}$$

Hence using (29) we obtain from (32) the *Lerner Index* for any variety $j \in [1, n_1]$ in country 1 as

$$L_1 = \frac{P_1 - c_1}{P_1} = \frac{1}{\sigma}$$

This is the familiar monopolistic competition result. The price of every good exported from country 1 is a constant mark-up on marginal cost: $P_1 = \frac{c_1}{\alpha}$. Similarly for country i, the price is given by $P_i = \frac{c_i}{\alpha}$

# 5 Domestic Procurement by Producers

We now complete the equilibrium by evaluating the optimal decisions of the government in country 1 at stage 1 of the game. Producer countries $i = 1, 2, \cdots, \ell$ face no one rival or enemy and devote resources to military expenditure. They prepare defensively and offensively to provide an 'insurance' in an uncertain world against a range of possible security needs. We do not attempt to model their expenditure decisions and take the military expenditure of producers as exogenous.

Each government in the producer countries then maximizes military strength for a given government spending. The government when choosing the procurement price, $p_1$, relaxes or tightens the firms' participation constraint and, in effect, chooses the number of domestic firms. Imposing symmetry between identical domestic firms, we have that $d_{1j} = d_1$ and $q_{1j} = q_1$ for all domestic varieties. Similarly, given the symmetry between all firms in countries $i = 2, 3, \cdots, \ell$ in the international market, government 1 will choose the same amount of imports of each variety from that country, $m_{1i}$ say, of quality $u_{1i}$.[7] We examine a complete information Nash equilibrium of stage 1 of the game, and a subgame

[6] However as the number becomes small this strategic term becomes significant - see Garia-Alonso and Levine (2003) for analysis of this case.

[7] Note that at stage 1 we have imposed symmetry between firms and have therefore defined $m_{ij}$ and $u_{ij}$ as the quantity and quality respectively imported by country i of **any good** from country j, $i, j = 1, 2, \cdots, \ell$. By contrast at stage 3 $j$ referred to the **variety** available on the world market, $j = 1, 2, \cdots, N$.

perfect equilibrium of the whole game, where for country 1, independent decision variables are $d_1$, $q_1$ and $n_1$.[8]

The optimization problem of the government in country 1 is to maximize military strength given by

$$S_1 = [w_1 n_1 + (1-w_1)(N-n_1)]^{\nu}\left[w_1 n_1 (q_1 d_1)^{\alpha} + (1-w_1)\sum_{i=2}^{\ell} n_i (u_{1i} m_{1i})^{\alpha}\right]^{\frac{1}{\alpha}} \tag{33}$$

with respect to the choice variables $d_1$, $q_1$, $\{m_{1i}\}$ and $n_1$, given the world prices $P_i = P = \frac{c_i}{\alpha}$; $i = 2, 3, \cdots, \ell$ of each variety from country $i$, the corresponding decisions of other countries, and two sets of constraints. These are the budget constraint $(BC_1)$ and the representative domestic firm's participation constraint $(PC_1)$ given by

$$BC_1 \;\; : \;\; p_1 n_1 d_1 + \sum_{i=2}^{\ell} P_i n_i m_{1i} = G_1$$

$$PC_1 \;\; : \;\; \pi_1 = (p_1 - c_1)d_1 + (P_1 - c_1)x_1 - F_1 - f_1 q_1^{\beta_1} \geq 0$$

and the corresponding constraints in the other countries. Clearly the PC constraint must bind so the procurement price is given by

$$p_1 = c_1 + \frac{F_1 + f_1 q_1^{\beta_1} - (P_1 - c_1)x_1}{d_1} = c_1 + \frac{H_1(q_1) - R_1(x_1)}{d_1} \tag{34}$$

where we have written export net revenue $(P_1 - c_1)x_1 = R_1(x_1)$ and we recall that total fixed production costs in country 1 are denoted by $H_1(q_1)$. It is useful to note from (35) that exports $x_1$ of each variety from country 1 can be written in terms of decision variables as

$$x_1 = \sum_{i=2}^{\ell+r} m_{i1} = \sum_{i=2}^{\ell} m_{i1} + \sum_{i=\ell+1}^{\ell+r} \frac{G_i u_{i1}^{\sigma-1}}{P_1^{\sigma}(n_1 P_1^{1-\sigma} u_{i1}^{\sigma-1} + n_2 P_2^{1-\sigma} u_{i2}^{\sigma-1} + \cdots + n_{\ell} P_{\ell}^{1-\sigma} u_{i\ell}^{\sigma-1})} \tag{35}$$

At stage 1, country 1 only commits to a *total* import budget $P_i n_i m_{1i}$ (determined by the $BC_1$ as a residual given other decisions by all countries ) and *not* to any procurement

[8]Any two from three possible decision variables, $d_1$, $p_1$ and $n_1$ can be assumed, but will lead to different Nash equilibria at stage 1. Our particular choice, $d_1$, and $n_1$ is made partly, for analytical convenience, but can be also justified by the need to **observe** decision variables in a more realistic incomplete information setting, where the process of dynamic adjustment towards the equilibrium, for example of a Cournot-type, needs to be addressed. It is plausible to assume that the domestic procurement decision, $d_i$, and the number of firms supported, $n_i$, $i = 1, 2, \cdots, \ell$ are more readily observed that the procurement price, $p_i$, $i = 1, 2, \cdots, \ell$, which involves a possibly hidden subsidy.

contract with any foreign firm. The import decision $m_{1j}, j = 1, 2, \cdots, \ell$ for any particular variety is decided at stage 3, given this total budget, and given out-of-equilibrium prices $P_j$ charged by firms.

Since we are assuming a Nash equilibrium in independent decision variables $d_1$, $q_1$ and $n_1$ for country 1, we can eliminate the the procurement price, $p_1$, using the $PC_1$ constraint. The $BC_1$ constraint now becomes

$$BC_1 \,:\, n_1(c_1 d_1 + H_1(q_1) - R_1(x_1)) + \sum_{i=2}^{\ell} P_i n_i m_{1i} = G_1 \tag{36}$$

and the government now maximizes $S_1$ given by (33) with respect to $d_1$, $q_1$, and $n_1$, given (36), and the corresponding budget constraints and independent decision variables of other governments. To carry out this constrained optimization, define a Lagrangian

$$\begin{aligned}\mathcal{L}_1 = S_1 \quad &- \quad \lambda_1[n_1(c_1 d_1 + H_1(q_1) - R_1(x_1)) + \sum_{i=2}^{\ell} P_i n_i m_{1i} - G_1] \\ &- \quad \sum_{i=2}^{\ell} \mu_i [n_i(c_i d_i + H_i(q_i) - R_i(x_i)) + \sum_{j=1,\, j\neq i}^{\ell} P_j n_j m_{ij} - G_i]\end{aligned}$$

where $\lambda_1 \geq 0$ is a Lagrange multiplier country 1 assigns to its own budget constraint, and $\mu_{1i} \geq 0, \;\; i = 2, 3, \cdots, \ell$ are Lagrange multipliers assigned to the other countries' budget constraints. Then country 1 maximizes $\mathcal{L}_1$ with respect to independent decision variables $d_1$, $q_1$, $n_1$, and with respect to endogenous variables $\{m_{ij},\, \lambda_i, \mu_i\}$, $i, j = 1, 2, \cdots, \ell$, $j \neq i$, given the independent decision variables of the other countries $\{d_i,\, q_i,\, n_i\}$, $i = 2, 3, \cdots, \ell$.

This optimization problem is greatly simplified as a result of the following Lemma:

**Lemma**

In a subgame perfect equilibrium (SPE) of the game $\mu_{1i} = 0$, $i = 2, 3, \cdots, \ell$.

**Proof**

The first-order condition with respect to $m_{1i}$ is given by

$$\frac{\partial S_1}{\partial m_{1i}} = S_1^{1-\alpha}[w_1 n_1 + (1-w_1)(N-n_1)]^{\alpha\nu}(1-w_1) n_i u_{1i}^{\alpha} m_{1i}^{\alpha-1} = \lambda_1 P_i n_i - \sum_{j=2}^{\ell} \mu_j n_j \frac{\partial R_j}{\partial m_{1j}}\,;\, i > 1 \tag{37}$$

From the counterpart of (35) for country i, we have $\frac{\partial R_i}{\partial m_{1i}} = (P_i - c_i)$. Hence, dividing the $i = r$ equation by the $i = s$ equation the relative demand by country for imported goods from countries $i = r, s$ is given by

$$\frac{m_{1r}}{m_{1s}} = \left(\frac{u_{1r}}{u_{1s}}\right)^{\sigma-1} \left[\frac{\lambda_1 P_s - \frac{1}{n_s}\sum_{j=2}^{\ell} \mu_{1j} n_j (P_j - c_j)}{\lambda_1 P_r - \frac{1}{n_r}\sum_{j=2}^{\ell} \mu_{1j} n_j (P_j - c_j)}\right]^{\sigma} \tag{38}$$

However from (26) at stage 3 of the game, the relative demand for two imported goods from countries $i = r, s$ is given by

$$\frac{m_{1r}}{m_{1s}} = \left(\frac{u_{1r}}{u_{is}}\right)^{\sigma-1} \left(\frac{P_s}{P_r}\right)^{\sigma} \tag{39}$$

In a SPE we must have agreement with the *anticipated* decision on imports given by (38) and the *actual* decision taken at stage 3 given by (39). This requires $\sum_{j=2}^{\ell} \mu_{1j} n_j (P_j - c_j) = 0$. Since $\mu_{1i} \geq 0$ it follows that $\mu_{1i} = 0$ for all $i = 2, 3, \cdots, \ell$. □

If at stage 1 the governments could commit to both domestic and overseas contracts, then imports of the later would satisfy the first-order condition (37) with $\mu_{1i} > 0$. According to (37), the marginal benefit (the left-hand-side) equals the marginal budgetary cost. The first term of the latter, on the right-hand-side, equals the shadow price of $BC_1$ multiplied by the procurement price. The second term equals the sum of the shadow price of $BC_i$ , $i > 1$ multiplied by the marginal revenue gain to each foreign country from exporting to country 1. Exports to country 1 relax these budget constraints and bring benefit to that country through allowing for more imports. Taking this into consideration lowers the effective cost of imports and there increases their volume.

Having made this commitment to importing more than it would in the absence of these strategic considerations, at stage 3 country 1 has a given import budget $G_1 - p_1 n_1 d_1$. If it were to re-optimise given world market prices, it would choose imports given by (26) and therefore set $\mu_{1i} = 0$ ; $i > 1$. The ex ante optimal contract at stage 1 is no longer optimal ex post at stage 3. The equilibrium is not subgame perfect in other words. The subgame perfection condition imposes $\mu_i = 0$, $i > 2$ and implies that at stage 1 country 1 ignores the budget constraints of other countries.

With $\mu_i = 0$, $i > 2$, the remaining first-order conditions for an internal solution (where

$n_1 \geq 0$ and $d_1 \geq 0$ and are not binding, but $BC_1$ does bind) are then

$$d_1 \quad : \quad \frac{\partial U_1}{\partial d_1} = S_1^{1-\alpha}[w_1 n_1 + (1-w_1)(N-n_1)]^{\alpha\nu} w_1 q_1^{\alpha} d_1^{\alpha-1} = \lambda c \tag{40}$$

$$\begin{aligned} n_1 \quad : \quad \frac{\partial S_1}{\partial n_1} &= \frac{S_1^{1-\alpha}}{\alpha} w_1 q_1^{\alpha} d_1^{\alpha}[w_1 n_1 + (1-w_1)(N-n_1)]^{\alpha\nu} + \nu w_1 S_1 [w_1 n_1 + (1-w_1)(N-n_1)]^{-1} \\ &= \lambda(c d_1 + H_1(q_1) - R(x_1) - n_1 \frac{\partial R_1}{\partial n_1}) \end{aligned} \tag{41}$$

$$q_1 \quad : \quad \frac{\partial S_1}{\partial q_1} = S_1^{1-\alpha}[w_1 n_1 + (1-w_1)(N-n_1)]^{\alpha\nu} w_1 q_1^{\alpha-1} d_1^{\alpha} = \lambda \left( \frac{\partial H_1}{\partial q_1} - \frac{\partial R_1}{\partial q_1} \right) \tag{42}$$

These 3 equations and (37) plus the constraint $BC_1$ solve for the decision variables $n_1$, $d_1, q_1$, and for endogenous variables $m_{1j}$ and $\lambda_1$.

To complete the equilibrium we need to evaluate the responses of net export revenue and quality, $\frac{\partial R_1}{\partial n_1}$ and $\frac{\partial R_1}{\partial q_1}$ respectively. First write total exports of each firm in country 1 given by (35) as the sum of exports to producers and non-producers, $x_1 = x_1^p + x_1^{np}$ where

$$x_1^{np} = \frac{1}{P_1} \sum_{i=\ell+1}^{\ell+r} \frac{G_i}{\sum_{k=1}^{\ell} n_k (\tilde{P}_{1k}^i)^{\sigma-1}} \tag{43}$$

and where we have defined the quality-adjusted price of good 1 relative to k, both exported to non-producer i.

$$\tilde{P}_{1k}^i = \frac{P_1 u_{ik}}{P_k u_{i1}} \tag{44}$$

According to (43), the value of exports to non-producers by each firm in country 1, $P_1 x_1^{np}$, depends positively on the 'competitiveness' of its good, $\frac{1}{\tilde{P}_{1k}^i}$ and expenditure by non-producers, and negatively on the numbers of competitors, $n_k$; $k = 1, 2, \cdots, \ell$. Differentiating (43) we then have

$$\begin{aligned} \frac{\partial R_1}{\partial n_1} &= (P_1 - c_1)\frac{\partial x_1^{np}}{\partial n_1} \\ &= -L_1 \sum_{i=\ell+1}^{\ell+r} \frac{G_i}{\left[\sum_{k=1}^{\ell} n_k (\tilde{P}_{1k}^i)^{\sigma-1}\right]^2} + L_1 \sum_{i=\ell+1}^{\ell+r} \frac{\frac{\partial G_i}{\partial n_1}}{\left[\sum_{k=1}^{\ell} n_k (\tilde{P}_{1k}^i)^{\sigma-1}\right]} \end{aligned} \tag{45}$$

$$\begin{aligned} \frac{\partial R_1}{\partial q_1} &= (P_1 - c_1)\frac{\partial x_1^{np}}{\partial q_1} \\ &= \frac{L_1(\sigma-1)}{q_1} \sum_{i=\ell+1}^{\ell+r} \frac{G_i \sum_{k=2}^{\ell} n_k (\tilde{P}_{1k}^i)^{\sigma-1}}{\left[\sum_{k=1}^{\ell} n_k (\tilde{P}_{1k}^i)^{\sigma-1}\right]^2} + L_1 \sum_{i=\ell+1}^{\ell+r} \frac{\frac{\partial G_i}{\partial q_1}}{\left[\sum_{k=1}^{\ell} n_k (\tilde{P}_{1k}^i)^{\sigma-1}\right]} \end{aligned} \tag{46}$$

where, in (46) we have used the expression for the export control regime on quality exported by country to country j given by $u_{j1} = \gamma_{j1} q_1$. Equations (45) and (46) are crucial for the results that follow in the paper. We have seen from proposition 1 that if we choose a difference form of the CSF then $\frac{\partial G_i}{\partial n_1} > 0$ and $\frac{\partial G_i}{\partial q_1} > 0$. Thus from (45) there are two opposing effects on the net revenue per firm in country 1 from increasing the number of firms, given $q_1$, $d_1$ and the corresponding decisions $n_i$, $q_i$ and $d_i$, $i > 1$ of other countries. For a given expenditure by non-producers an increase in $n_1$ spreads a given demand over more competing domestic firms. Since costs include a fixed component, average costs increase and revenue falls. This is the first negative term in (45). However with the difference form of the CSF, the second term is positive because an increase in $n_1$ increases the total variety available, $N$, and boosts demand from the external market. The sign of $\frac{\partial R_1}{\partial n_1}$ depends on which of these opposite forces dominates. By contrast, there is no ambiguity with respect to the effect on net revenue of a unilateral increase in quality. Given expenditure by non-producers, a unilateral increase in $q_1$ increases competitiveness and raises demand. It also raises expenditure and so both terms in (46) are positive.

We can now characterize a Nash equilibrium of stage 1 of the game. Using $H_1(q_1) = F_1 + f_1 q_1^{\beta}$ and dividing (41), and (37) by (40), in turn, we can eliminate the shadow price $\lambda$ to obtain

$$d_1 = \frac{\left[H(q_1) - R_1(x_1) - n_1 \frac{\partial R_1}{\partial n_1}\right]}{P_1\left[1 - \alpha + \frac{\alpha\nu}{[w_1 n_1 + (1-w_1)(N-n_1)]^{1+\alpha\nu}}\left(\frac{U_1}{d_1}\right)^{\alpha}\right]} \tag{47}$$

$$m_{1i} = d_1 \left(\frac{c_1(1-w_1)}{P_i w_1}\right)^{\sigma} \left(\frac{u_{1i}}{q_1}\right)^{\sigma-1} = \phi_1 d_1 \,;\ i > 1 \tag{48}$$

$$c_1 d_1 = \beta_1 f_1 q_1^{\beta_1} - q_1 \frac{\partial R_1}{\partial q_1} \tag{49}$$

The budget constraint $BC_1$, given by (36) and the expression for net revenue

$$R_1(x_1) = (P_1 - c_1)x_1 = (P_1 - c_1)[\sum_{i=2}^{\ell} m_{i1} + x_1^{np}] \tag{50}$$

with $x_1^{np}$ given by (43), completes the solution for the single economy given the decisions on $d_i$, $\{m_{ij}\}$, $q_i$ and $n_i$ by the other countries $i > 1$. Combining these equations with analogous ones for the remaining $\ell - 1$ producer countries completes the Nash equilibrium at stage 1 of the game and the Subgame Perfect Nash equilibrium of the whole game. In our set-up asymmetries between producer counries can arise from differences in costs $\{c_i, F_i, f_i, \beta_i\}$, expenditures $\{G_i\}$, the domestic bias parameter $\{w_i\}$ and the nature of the arms export control regime imposed by country $j$ on country $i$, $\{\gamma_{ij}\}$. For instance regarding the latter, if there are say 4 countries consisting of two alliances each with two countries, then a possible choice of $\gamma_{ij}$ is the matrix $\Gamma$ where

$$\Gamma = \begin{pmatrix} 1 & 1 & \gamma & \gamma \\ 1 & 1 & \gamma & \gamma \\ \gamma & \gamma & 1 & 1 \\ \gamma & \gamma & 1 & 1 \end{pmatrix}$$

where $0 \leq \gamma < 1$. Given the domination of one producer country in the world– the US – these asymmetries between producer countries are clearly of practical importance. However a non-symmetric non-cooperative equilibrium can only be solved by numerical solution. By contrast, a symmetric equilibrium is tractable and provides some valuable insights into the procurement and export arrangements in the EU which can be thought of as three approximately equally-sized size countries –the UK, Germany, and France – procuring and exporting arms. For the remainder of the paper we concentrate on the symmetric equilibrium.

## 6 A Symmetric Non-Cooperative Equilibrium

We solve for a symmetric non-cooperative equilibrium in which all producer countries and all non-producing countries are identical in every respect. Then $w_i = w$, $c_i = c$, $F_i = F$, $f_i = f$, $\beta_i = \beta$, $\gamma_{ij} = \gamma$, $G_i = G^p$ say, for producers $(i = 1, 2, \cdots, \ell)$ and $G_i = G^{np}$ for non-producers $(i = \ell + 1, \ell + 2, \cdots, \ell + r)$. Then $P_1 = P_2 = \cdots = P = \frac{c}{\alpha}$, $d_1 = d_2 = \cdots = d$, $n_1 = n_2 = \cdots = n$ etc, $N = \ell n$, $\phi_i = \phi = \left(\frac{\alpha(1-w)}{w}\right)^{\sigma} \gamma^{\sigma-1}$ and $\frac{U_i}{d_i} = \frac{U}{d} = n^{\nu+\frac{1}{\alpha}}[w + (1-w)(\ell-1)]^{\nu}[w + (1-w)(\ell-1)\phi^{\alpha}]^{\frac{1}{\alpha}}$ for producing countries. In addition from (45) and (46) we have

$$\frac{\partial R_1}{\partial n_1} = \frac{\partial R_2}{\partial n_2} = \cdots \quad = \quad -\frac{Lr}{N}\left[\frac{G^{np}}{N} - \frac{\partial G^{np}}{\partial N}\right] \tag{51}$$

$$\frac{\partial R_1}{\partial q_1} = \frac{\partial R_2}{\partial q_2} = \cdots \quad = \quad \frac{Lr}{N}\left[\frac{(\sigma-1)(\ell-1)G^{np}}{q\ell} + \frac{\partial G^{np}}{\partial q}\right] \tag{52}$$

The first-order condition (47) now becomes

$$d = \frac{(H(q) - R(x) + \Theta_1)}{P(1 - \alpha + \Theta_2)} \tag{53}$$

where we have defined

$$\Theta_1 = \frac{Lr}{\ell}\left[\frac{G^{np}}{N} - \frac{\partial G^{np}}{\partial N}\right]$$
$$\Theta_2 = \frac{\alpha\nu[w+(1-w)(\ell-1)\phi^{\alpha}]}{[w+(1-w)(\ell-1)]}$$

and we have used (24). Substituting for $H(q)-R(x)$ from (53) into (34) we arrive at the procurement price in the non-cooperative symmetric equilibrium

$$p = P(1+\Theta_2) - \frac{\Theta_1}{d} \tag{54}$$

Hence for a 'traditional' Dixit-Stigliz utility function where $\nu = \Theta_2 = 0$ and in the limit as the external market becomes small (but still of sufficient size to determine the world market price), $\Theta_1 \to 0$ and we have that $p = P$; i.e., the procurement price equals the market price. Generally however either $p > P$, in which case the procurement process involves a subsidy, or $p < P$ implying that the government taxes away part of the monopolistic profits. A high taste for variety $\nu$ encourages the former and whilst the effect of large external market depends on the sign of $\Theta_1$. It is clear from (54) that $\Theta_1 > 0$ iff the elasticity $\frac{N\partial G^{np}}{G^{np}\partial G^{np}} < 1$. If the CSFs are of the ratio form, this elasticity is zero and $\Theta_1 > 0$ unambiguously. For the difference form of the CSFs $\Theta_1$ can be negative if the elasticity is sufficiently high. In our calibration this does not happen so that a larger external sector encourages the taxation of monopoly profits. The intuition behind this external effect is that increasing the number of differentiated goods, each produced by a single firm, reduces the net export revenue to the external market per firm and tightens the participation constraint. In a non-cooperative equilibrium each government takes into account only their own contribution to the world supply of differentiated goods and, through reducing the procurement price, lowers its optimal number of domestic firms as the external market becomes more important. We summarize this result as:

**Proposition 2: The Procurement Price**

**In a symmetric, non-cooperative equilibrium without strategic pricing by firms, the procurement price may be above or below the world market price. A high taste for variety encourages the former and provided the elasticity $\frac{N\partial G^{np}}{G^{np}\partial G^{np}} < 1$, a large external market encourages the latter.**

In general the full solution to the symmetric non-cooperative equilibrium requires numerical solutions which are provided later in the paper. However we can derive explicit expressions for the total number of firms in the case where CSFs are of ratio form and $\frac{\partial G^{np}}{\partial G^{np}} = 0$. To do this, first put $R(x) = (P-c)x = (P-c)((\ell-1)\phi d + \frac{rG^{np}}{NP})$. Then (53) becomes

$$d = \frac{\left[H(q) - \frac{rG^{np}}{\sigma \ell n}\left(1 - \frac{1}{\ell}\right)\right]}{P\left[(1-\alpha)(1+(\ell-1)\phi) + \Theta_2\right]} \tag{55}$$

Writing the symmetric budget constraint as

$$nd(p + P(\ell-1)\phi) = G^p \tag{56}$$

and using (54) some algebra leads to

$$d = \frac{G^p + \frac{rG^{np}}{\sigma \ell^2}}{nP\left(1+(\ell-1)\phi + \Theta_2\right)} \tag{57}$$

Equation (55) says that given quality, the producer countries respond to a increases in external demand per variety. $\frac{rG^{np}}{\ell n}$ by reducing the size of the firm. Thus the right-hand-side of (55) is upward-sloping in n. Equation (57) reflects the trade-off between size and number arising from the budget constraint and the right-hand-side of is downward-sloping in n. Hence given quality, we can solve for the equilibrium number of differentiated goods (equals the number of firms), $n$, and hence the total world number $N = \ell n$.

To complete the non-cooperative equilibrium we need to determine quality, using (49). Using (52), (49) becomes

$$cd = \beta f q^{\beta} - (\sigma - 1)\frac{\Theta_1(\ell-1)}{q} \tag{58}$$

Hence combining (57) and (58) and substituting $\Theta_1 = \frac{rG^{np}}{\sigma \ell N}$, quality can be expressed in terms of the firm number per country, n, as:

$$\beta f q^{\beta} = \frac{1}{n}\left[(\sigma-1)(\ell-1)\frac{rG^{np}}{\sigma \ell^2} + \frac{c\left(G^p + \frac{rG^{np}}{\sigma \ell^2}\right)}{P\left(1+(\ell-1)\phi+\Theta_2\right)}\right] \tag{59}$$

Thus there is a *trade-off* between firm number and quality. According to (60), for a given quality, firm number is decreasing with that quality. According to (59), for a given number, quality is decreasing with that firm number. The equilibrium levels of $n$ and $q$ are determined by the intersection of these two downward-sloping curves, given government expenditures and the other parameters of the model.

We express the following result for $N$ in terms of the total world expenditure $G = \ell G^p + rG^{np}$ and the relative size of the external market of non-producers $\frac{rG^{np}}{G}$:

$$N = \frac{G}{\beta F}\left[\theta - \frac{rG^{np}}{G}\left(\theta\left(1 - \frac{1}{\sigma \ell}\right) - \frac{1}{\sigma \ell}(\ell-1)(\beta - \sigma + 1)\right)\right] \tag{60}$$

where we have defined

$$\theta = \frac{\beta(1-\alpha)(1+(\ell-1)\phi) + \beta\Theta_2 - \alpha}{1+(\ell-1)\phi+\Theta_2} \in ((1-\alpha), 1)$$

Again we can examine the special case of a 'traditional' Dixit-Stigliz utility function where $\nu = \Theta_2 = 0$, there is no investment in quality ($\beta \to \infty$) and the limit as the external market becomes small. Then $\theta = \beta(1-\alpha)$ and we have that $N = \frac{G(1-\alpha)}{F}$, a familiar result for a closed economy monopolistic competition model.

From (60) and the definition of $\Theta_1$ given after (53) we can now examine the effect on the world number of firms of changes in the taste for variety parameter $\nu$, the preference for domestic supply parameter $w \in [\frac{1}{2}, 1]$ and the relative size of the external market $\frac{rG^{np}}{G}$.[9] First note that $\theta \in [1-\alpha, 1]$ as $\nu$ increases from 0 to $\infty$. Furthermore, from (60), $N$ is increasing in $\theta$ if $1 > \frac{rG^{np}}{G}\left(1 - \frac{1}{\sigma\ell}\right)$. Since $\frac{rG^{np}}{G} < 1$, $\sigma > 1$ and $\ell \geq 1$ this condition is satisfied. Hence it follows that $N$ is an increasing function of $\nu$ and we arrive at the expected result that an increase in the taste for variety in producer countries increases the number of differentiated goods.

Next consider an increase in the domestic procurement bias parameter, $w$. In the range $w \in [\frac{1}{2}, 1]$, $\phi$ falls from $\alpha^{\sigma}$ to 0 and $\Theta_2$ goes from $\frac{\alpha\nu[1+(\ell-1)\alpha^{\sigma\alpha}]}{\ell}$ to $\alpha\nu$. Since $\alpha^{\alpha\sigma} < 1$, $\frac{1+(\ell-1)\alpha^{\sigma\alpha}}{\ell} < 1$ and therefore this represents an increase in $\Theta_2$ and therefore $\theta$. A strengthening of the arms control regime, modelled as a decrease in $\gamma \in [0,1]$ has exactly the same effect as an increase in $w$. We have already shown that $N$ is an increasing function of $\theta$. It follows that as producer countries become less concerned with domestic supply and/or relax the arms control regime, $\Theta_2$ falls and therefore the equilibrium number of firms, $N$, falls.

Finally from (60), $N$ decreases with the relative size of the external market, $\frac{rG^{np}}{G}$, if the following condition is satisfied:

$$\theta > \frac{1}{\sigma\ell}(\theta + \ell - 1) \tag{61}$$

Since $\theta < 1$, the right-hand side of (61) is an increasing function of $\ell$ and at $\ell = \infty$ equals $\frac{1}{\sigma}$. But $\theta > 1 - \alpha$. Hence (61) holds.

[9] In the analysis up to the end of this section we have let $\beta \to \infty$, hence suppressing investment into R&D. However the results can be shown to go through without this assumption..

A willingness to procure from abroad, export more arms and the growing relative size of the international market are three features one may plausibly associate with *military globalization.* In that sense we may conclude that globalization is associated with a *decrease* in the number of firms in the world market. Summarizing our results:

**Proposition 3: The Number of Firms**

**In the case of the ratio form of the CSFs, in a symmetric, non-cooperative equilibrium, the number of firms increases as the taste for variety by producer countries increases. Military 'globalization' in the form of a reduction in preferences of producer countries for domestic supply, a relaxation of arms controls and an increase in the relative size of the external market results in a decrease in the number of firms.**

# 7 Cooperation Between Producers

Staying with the case of the ratio form of the CSFs, we now examine a symmetric cooperative agreement at stage 1 where there are no export controls between producers and a common export control regime with respect to non-producers. Then $u_{ij} = q_j$ for producers $i = 1, 2, \cdots, \ell$, and $u_{ij} = \gamma q_j$ for non-producers $i = \ell+1, \ell+2, \cdots, \ell+r$. $\ell$ identical producers would then choose $d_1 = d_2 = \cdots = d_\ell = d$, $n_1 = n_2 = \cdots = n_\ell = n$, $q_1 = q_2 = \cdots = q_\ell = n$ and commit, at stage 1, to $m_1 = m_2 = \cdots = m_\ell = m$ to maximize $U_1 = U_2 = \cdots = U_\ell = U$ where

$$U = [w + (1-w)(\ell-1)]n^{\nu+\frac{1}{\alpha}} q[wd^\alpha + (1-w)(\ell-1)m^\alpha]^{\frac{1}{\alpha}} \tag{62}$$

subject to budget constraints $BC_1 = BC_2 = \cdots = BC$ and participation constraints $PC_1 = PC_2 = \cdots = PC$ where

$$\begin{aligned} BC &: n[pd + P(\ell-1)m] = G^p \\ PC &: \pi = (p-c)d + R(x) - H(q) = 0 \end{aligned}$$

In PC the net revenue is given by

$$R(x) = (P-c)x = (P-c)(x^p + x^{np}) = (P-c)\left[(\ell-1)m + \frac{rG^{np}}{\ell nP}\right] \tag{63}$$

Using (63) we can consolidate the BC and PC constraints as

$$n[c(d + (\ell-1)m) + H(q)] = L\frac{rG^{np}}{\ell} + G^p \tag{64}$$

Hence the optimal procurement decision for the producers together is found by maximizing $n^{\nu+\frac{1}{\alpha}}q[wd^\alpha + (1-w)(\ell-1)m^\alpha]^{\frac{1}{\alpha}}$ with respect to $n$, $d$ and $m$ subject to the consolidated constraint (64).

To carry out this optimization define a Langrangian

$$n^{\nu+\frac{1}{\alpha}}[wd^\alpha + (1-w)(\ell-1)m^\alpha]^{\frac{1}{\alpha}}q - \lambda\left[n[c(d+(\ell-1)m)+H(q)] - L\frac{rG^{np}}{\ell} - G^p\right]$$

where $\lambda \geq 0$ is a Lagrangian multiplier. The first-order conditions are:

$$\begin{aligned}
n &: (\nu+\frac{1}{\alpha})n^{(\nu+\frac{1}{\alpha}-1)}[wd^\alpha + (1-w)(\ell-1)m^\alpha]^{\frac{1}{\alpha}}q = \lambda\left[c(d+(\ell-1)m)+H(q) - Lr\frac{\partial G^{np}}{\partial N}\right] \\
d &: n^{(\nu+\frac{1}{\alpha})}[wd^\alpha + (1-w)(\ell-1)m^\alpha]^{\frac{1}{\alpha}-1}wd^{\alpha-1}q = \lambda nc \\
m &: n^{(\nu+\frac{1}{\alpha})}[wd^\alpha + (1-w)(\ell-1)m^\alpha]^{\frac{1}{\alpha}-1}(1-w)(\ell-1)m^{\alpha-1}q = \lambda nc(\ell-1) \\
q &: n^{(\nu+\frac{1}{\alpha})}[wd^\alpha + (1-w)(\ell-1)m^\alpha]^{\frac{1}{\alpha}} = \lambda\left[nH'(q) - \frac{Lr}{\ell}\frac{\partial G^{np}}{\partial q}\right]
\end{aligned}$$

Dividing the first, the third and the fourth first-order condition by the second we arrive at:

$$m = \left(\frac{1-w}{w}\right)^\sigma d \quad = \quad \bar{\phi}d \text{ say,} \tag{65}$$

$$cd\left[\left(\nu+\frac{1}{\alpha}\right)(w+(1-w)(\ell-1)\bar{\phi}^\alpha - w(1+(\ell-1)\bar{\phi})\right] \quad = \quad w\left[H(q) - \frac{Lr}{\ell}\frac{\partial G^{np}}{\partial N}\right] \tag{66}$$

$$cd = q\left[\beta f q^{\beta-1} - \frac{Lr}{N}\frac{\partial G^{np}}{\partial q}\right]\frac{wd^\alpha}{wd^\alpha + (1-w)(\ell-1)m^\alpha} \tag{67}$$

Equations (65), (66), (66) together with the constraint (64) characterize the optimal co-operative procurement agreement. Equations (66) and (67) can be simplified somewhat by noting that $[w+(1-w)(\ell-1)\bar{\phi}^\alpha] = w\left[1+\left(\frac{1-w}{w}\right)(\ell-1)\bar{\phi}^\alpha\right] = w\left[1+\left(\frac{1-w}{w}\right)^{\frac{1}{1-\alpha}}(\ell-1)\right] = w[1+\bar{\phi}(\ell-1)]$. Then putting $\frac{\partial G^{np}}{\partial N} = 0$ for the case of the ratio form of the CSFs and substituting into (66), a little algebra results in

$$N = \frac{(1-\alpha+\alpha\nu)G\left[1-\frac{rG^{np}}{G}\left(1-\frac{1}{\sigma\ell}\right)\right]}{H(q)(1+\alpha\nu)} \tag{68}$$

whilst (65) and (67) now give

$$cd = \frac{\beta f q^\beta}{(1+\bar{\phi}(\ell-1))} \tag{69}$$

We can compare this result with the corresponding non-cooperative equilibrium given by (60). Putting $\ell = 1$ in the latter expression we find, as expected, that the non-cooperative equilibrium and the cooperative arrangement are the same if there is only one country. A more interesting result follows from (68). The right-hand-side is *independent of the domestic production bias parameter, $w$*. Since imports $m = \bar{\phi} d$ where $\bar{\phi} = \left(\frac{1-w}{w}\right)^{\sigma}$, an increase in $w$ has no effect on the total number of firms (varieties) in the cooperative arrangement and only affects the trade between producers.[10] Note that this contrasts with the non-cooperative arrangement where an decrease in $w$ leads to a decrease in the total number of firms (see proposition 2). As with the non-cooperative equilibrium, however, since $L < 1$, from (68) we can see that an increase in the relative size of the external market leads to a lower total number of firms under cooperation, and comparing (68) with (60), cooperation enhances this 'external effect' on the total firm number. To summarize:

**Proposition 4: Optimal Cooperative Procurement**

**In the case of the ratio form of the CSFs, in the optimal cooperative procurement arrangement, the total number of firms is independent of the preferences of producer countries for domestic supply. As with the non-cooperative equilibrium, given quality an increase in the relative size of the external market leads to a lower total number of firms under cooperation, which enhances this 'external effect' on the total firm number.**

# 8 Numerical Results

We now turn to numerical solutions of the non-cooperative and cooperative outcomes. We confine ourselves to the case where CSFs are of ratio form and therefore the expenditure of non-producers as a proportion of their GDP is fixed. The more interesting case where CSFs are of difference form and the expenditure of non-producers is endogenously determined by equilibrium levels of firm number, quality and by the nature of the arms export regime is more difficult to calibrate. This is discussed in an appendix.

## 8.1 Parameters

By choice of units we can put $c = f = 1$. We exclude arms export regimes for now and put $\gamma = 1$. We examine a symmetric equilibrium of three countries (say, the UK, Germany and

[10] Compare the trade equation in the non-cooperative equilibrium, where $m = \phi d$ and $\phi = \left(\frac{c(1-w)}{Pw}\right)^{\sigma}$. With cooperation, trade is valued not at the world market price, but at the marginal cost, resulting in *more* trade.

France in a EU setting) so $\ell = 3$. We can calibrate the parameter $\alpha$ as follows. From the binding participation constraint we have that revenue equals total costs, $P(d + x) = Py$ where we recall that $d$ =domestic procurement, $x$ =exports and $y = d + x$=output, all per firm. In equilibrium the procurement price equals the international market price $P = \frac{c}{\alpha}$ where $c$ =marginal cost (equals average production cost given our assumption of constant returns to scale). Thus we have

$$Py = \frac{c}{\alpha} y = \text{Total Costs(TC)} = F + fq^{\beta} + cy \tag{70}$$

where $q$ is is quality. In (70) let us associate the second quality component of total costs with R&D, the third with variable cost leaving F as fixed capital cost. Denote the shares of fixed, R&D and variable cost in total cost as $\gamma_F$, $\gamma_R$ and $\gamma_V$. Thus

$$\frac{cy}{\frac{cy}{\alpha}} = \frac{\text{variable costs}}{\text{total cost}} = \alpha = \gamma_V \tag{71}$$

From Dunne et al (2002), a reasonable value for $\gamma_V = 0.5$ for Europe which is therefore our chosen value for $\alpha$.

We calibrate the parameter $F$ as follows. We use a baseline non-cooperative equilibrium where utility is given by the traditional Dixit-Stigliz CES function with $\nu = 0$ and there is no external market of non-producers. Then $\Theta_1 = \Theta_2 = 0$ and the procurement price $p = P$. Using the results of section 6 the equilibrium then becomes

$$d = \frac{\alpha F \beta}{c[\beta(1-\alpha)(1+(\ell-1)\phi) - \alpha]} \tag{72}$$

$$n = \frac{G^p}{Pd(1+(\ell-1)\phi)} = \frac{G^p[\beta(1-\alpha)(1+(\ell-1)\phi) - \alpha]}{F\beta(1+(\ell-1)\phi)} \tag{73}$$

$$m = \phi d \equiv \left[\frac{\alpha(1-w)}{w}\right]^{\sigma} d \tag{74}$$

$$p = P = \frac{c}{\alpha} \tag{75}$$

$$q = \left[\frac{cd}{\beta f}\right]^{\frac{1}{\beta}} \tag{76}$$

We normalize $G = \ell G^p = 1$ and put $N = \hat{N} = 100$ in the baseline. Then $F$ is calibrated as

$$F = \frac{G^p[\beta(1-\alpha)(1+(\ell-1)\phi) - \alpha]}{\hat{N}\beta(1+(\ell-1)\phi)} \tag{77}$$

We now examine the non-cooperative and cooperative outcomes as three parameters change: $\beta$ the R&D investment parameter, $W$ the domestic bias parameter and $\Phi$, the proportion of world demand for military goods coming from the external market of non-producers. Each parameter is allowed to vary in turn with central values: $\beta = 1.5$, $w = \Phi = 0.5$.

### 8.2 Process Innovation

In our first experiment we examine the effect of more process innovation and escalating fixed costs associated with R&D investment. In our model this effect is captured by a fall the the parameter $\beta$. In figures 1 and 3 we see that with a high value $\beta$, R&D as a proportion to total output is low, and comparing the non-cooperative equilibrium with the optimal cooperative arrangement the number of firms is too high. As $\beta$ falls, $R\&D$ investments rises until at $\beta = 1.5$ we can reproduce data on R&D as a proportion of output which suggests figures of 20-25% (see Dunne et al (2002)). The firm number falls substantially seeing a total of around 160 at the low R&D end to around 100 at $\beta = 1.5$, at the latter high-investment end an absence of cooperation sees an insufficient number of firms and an excessive production of quality. The beggar-thy-neighbour aspect of quality in the external market drives this result. When countries order high-tech, high quality specifications for domestic procurement, acting independently they improve the competitiveness of their exports to the external market. In a Nash equilibrium however these gains are wiped out: R&D expenditure is high but there is no improvement in competitiveness. A subsidy (seen in figure 2 where the procurement price exceeds the world price is then required and figure 4 shows that the gains from cooperation between producer countries (to those countries) rises substantially with more process innovation.[11]

### 8.3 Changes in Domestic Procurement Bias

In our second experiment we set $\beta = 1.5$ and allow the domestic procurement bias parameter, $w$ to increase from $w = 0.5$ to $w = 1$ at which point producing countries are self-sufficient, and only exporting to non-producers.

In figure 5 the number of firms per country in the non-cooperative equilibrium first falls and then rises rises with w. As countries become more self-sufficient they internalize

[11]Let $U^C$ and $U^{NC}$ be the utilities under cooperation and non-cooperation, respectively. Utility loss is then defined as $\frac{U^C - U^{NC}}{U^C} \times 100$.

the benefits of variety arising from $\nu > 0$ and choose to support more domestic firms. On the other hand from figure 9, they also internalize the benefits of investment into quality, raising fixed costs and tending to reduce the number of firms. The net effect is the $\bigcup$-curve. Under cooperation firm number is independent of $w$ as predicted by proposition 4. In Figure 5, for higher values of $w$, both firm number and quality increases in the noncooperative equilibrium and consequently, from figure 6 the subsidy (procurement price minus the world price) also increases. From proposition there are two affects at work here: taste for variety $\nu > 0$ tends to encourage subsidy whilst the external market effect encourages the opposite (a tax on export profits). With our parameter values in the non-cooperative equilibrium the former effect dominates for higher values of $w$.

Figure 7 shows total output per firm in the non-cooperative equilibrium broken down into exports to non-producers and producers and domestic procurement. As $w$ increases exports to producers fall and initially output is diverted to domestic procurement. With the increase in the number of firms, the total size of each firm falls and all three components eventually fall for higher values of $w$. The utility loss to producers from failing to cooperative are shown in figure 8. Considering the welfare of producers only, in the absence of cooperation are 'too few' firms and they produce 'too much quality'. The latter is shown in figure 9 which plots R&D as a proportion of the total output of the firm.

### 8.4 Changes in the Composition of World Demand

We now fix the preference parameter at $w = 0.5$, so there is no domestic bias in the procurement decision, and we allow the proportion of world demand from non-producers, $\frac{rG^n}{\ell G^p} = \Phi$, say, increase from $\Phi = 0.5$ towards unity. Figures 10 to 14 show the numerical results. Figure 10 shows that the the subsequent fall in the firm number under both non-cooperation and cooperation as $\Phi$ rises as as before there are too few firms and too much quality in the absence of cooperation. From figure 11 these changes in industry structure are brought about by initially a subsidy under non-cooperative giving way to a tax at higher values of $\Phi$. The optimal (cooperative) procurement price for the producers, by contrast, involves a substantial tax throughout the full range of $\Phi$. All these results are consistent with the results of propositions 2 to 4.

A falling number of firms as $\Phi$ rises is associated with a rise in the size of each firm.

Figure 12 shows this happening and a switch of output from domestic procurement and internal trade to the external market. Figure 13 shows that the gains to cooperation between producers rise substantially as the external market becomes more important. This is largely the result of excessive investment into quality as figure 14 shows, but a close examination of figure 10 reveals that the difference in firm number between the cooperative and non-cooperative outcome also rises contributing to this welfare deficiency.

# 9 Conclusions

In order to examine a number of developments associated with the RMA, this paper has constructed a partial equilibrium model of military procurement with two-way international trade in a world where many of the recipients of this trade are non-producers engaged in regional arms races. We have studied the consequences for industry structure, procurement subsidies and the welfare of producers of: increased process innovation and the consequent rise in the fixed costs of R&D; increased trade between producers owing to the willingness to forgo the security benefits of domestic procurement; and a growing external market of non-producers.

Our comparison of the non-cooperative and cooperative outcomes has highlighted two sources of inefficiencies from the position of the producers. Adopting a generalized Dixit-Stiglitz utility function to model military strength, the existence of a separate taste-for-variety parameter ($\nu$) means governments acting independently choose to support too few firms and generate too little variety. Competition for the external market generates two effects. First export revenue from exports rises as the total number of firms falls. This occurs because firms then compete less intensively and can spread their fixed costs over a larger market share. As the external market increases, governments then choose to support *less* firms. Under non-cooperation, however, this reduction in firm number is too little compared with the optimum because governments acting independently only care about competition between domestic firms.

The second effect of an external market is to encourage too much investment in quality relative to the cooperative outcome. This is because the provision of quality has a beggar-thy-neighbour character. When governments raise quality unilaterally this increases market share. In equilibrium however the benefit to competitiveness disappears and countries are left with too much quality compared with that chosen cooperatively. A high investment into this quality raises fixed costs and reduces firm number further. The presence of an external market then tends to reduce firm number (i.e., raise concentration) and encourages excessive investment into quality. A high taste-for-variety has the opposite effect, raising firm numbers which, because there is a trade-off between quality and variety, reduces quality.

A number of important issues have not been addressed, the most important being the effect of these changes in industry structure and process innovation on the military expenditure and welfare of non-producers. In the ratio form of the CSF used in our simulations there are no such affects, but with the possibly more plausible difference form of CSFs, the military expenditure of non-producers rises as the number and quality of military goods increase. Since cooperation reduces the level of quality (but can raise the total number of goods, unless the taste-for-variety parameter is small) the possibility emerges that a common defence policy in the EU can *be mutually beneficial to both producers and recipients of arms.* This will be investigated in future work.

# A The Difference Form of the CSFs

With the difference form of the CSFs, military expenditure of non-producers is now endogenous and responds to industry structure, the quality of arms and the nature of the arms export regime. Letting $Y^p$ be the GDP per producer, we choose a different normalization: $\ell Y^p = 100$. Let $g^p$ be the proportion of GDP devoted to military expenditure in producer countries. Then $G^p = g^p Y^p$. Finally we put $N = \hat{N} = 100$ in the baseline. Choosing observed data $g^p = \hat{g}^p$, $F$ is calibrated as

$$F = \frac{100\hat{g}^p[\beta(1-\alpha)(1+(\ell-1)\phi)-\alpha]}{\hat{N}\beta(1+(\ell-1)\phi)} \tag{A.1}$$

Turning to the non-producers let the relative economic size of this market be denoted by $\Phi = \frac{rY^{np}}{\ell Y^p}$ so fixing $\Phi$ and $r$, $Y^{np}$ is given by $Y^{np} = \frac{100\Phi}{r}$. For the ratio form of the CSFs there are no further parameters to calibrate. For the difference form from (23) we require the composite parameter $(2\theta-1)kb = \Psi$, say. We calibrate $\Psi$ by assuming in a second baseline non-cooperative equilibrium with external producers, $\Phi$ is fixed, we have traditional Dixit-Stigliz utility function ($\nu = 0$) and the difference form of the CSFs. We let military expenditure in non-producers as a proportion of their GDP $g^{np}$ corresponds to data $\hat{g}^{np}$. Then from (23) this imposes the condition

$$\Psi = \frac{\hat{P}}{Y^{np}(1-\hat{g}^{np})} \tag{A.2}$$

where $\hat{P} = \frac{P}{\gamma q} N^{\frac{1}{1-\sigma}}$. Then

$$\begin{aligned} \frac{\partial G^{np}}{\partial N} &= \left(\nu + \frac{1}{\sigma - 1}\right) \frac{N^{-(\nu+1)}}{\Psi} && \text{(A.3)} \\ \frac{\partial G^{np}}{\partial q} &= \frac{N^{-\nu}}{q\Psi} && \text{(A.4)} \end{aligned}$$

Then in the non-cooperative equilibrium the parameter to be calibrated $\Psi$ is an endogenous variable that can be solved with the rest of the solution.

This procedure can be generalized to calibrate a number of 'difficult' parameters. Suppose we take these to be $[F,\ \Psi,\ \nu, \beta] = \Xi$, say . Then we can compute the non-cooperative equilibrium, as a function of $\Xi$. Suppose that we have data for four outputs: firm number per country $n = \hat{n}$, military expenditure in the non-producer countries as a proportion of GDP, $\hat{g}^{np}$, R&D expenditure by firms as a proportion of output , $\hat{RD}$ and the subsidy as a proportion of the world price $s = \frac{p-P}{P} = \hat{s}$. From the non-cooperative equilibrium we have a solution $n = n(\Xi)$, $g^{np} = g^{np}(\Xi)$, $RD = RD(\Xi)$ and $s = s(\Xi)$. Then $\Xi$ can be calibrated as the solution to:

$$\begin{aligned} \hat{n} &= n(\Xi) \\ \hat{g^{np}} &= g^{np}(\Xi) \\ \hat{RD} &= RD(\Xi) \\ \hat{s} &= s(\Xi) \end{aligned}$$

The result of this exercise is a model calibrated in a non-cooperative equilibrium to be consistent with stylized facts regarding firm number, military expenditure by non-producers, R&D expenditure and the level of subsidies given to the defence industry. Clearly this procedure can be extended to other parameters such as $\alpha$ if we had more stylized facts.

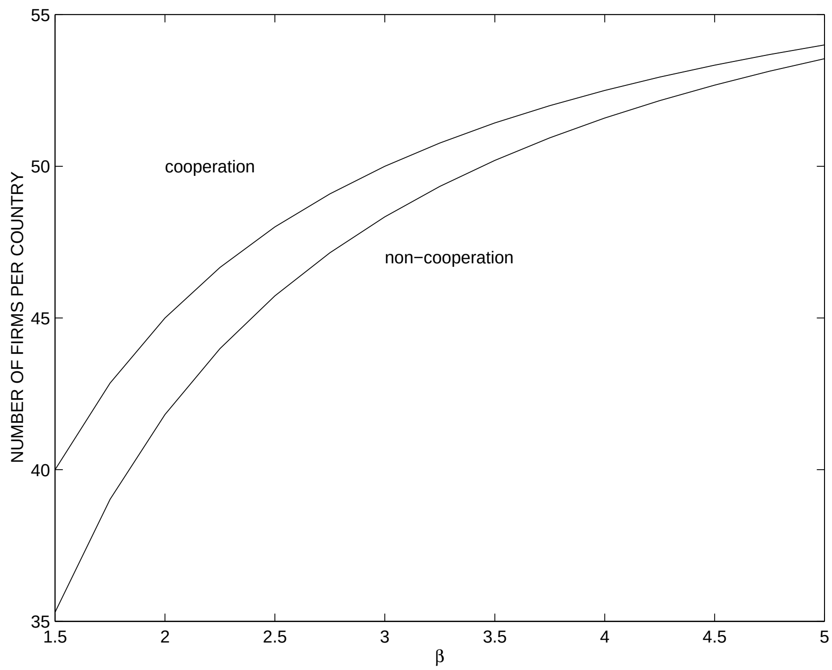


Figure 1: **Number of Firms per Country as $\beta$ increases: Non-Cooperation compared with Cooperation.**

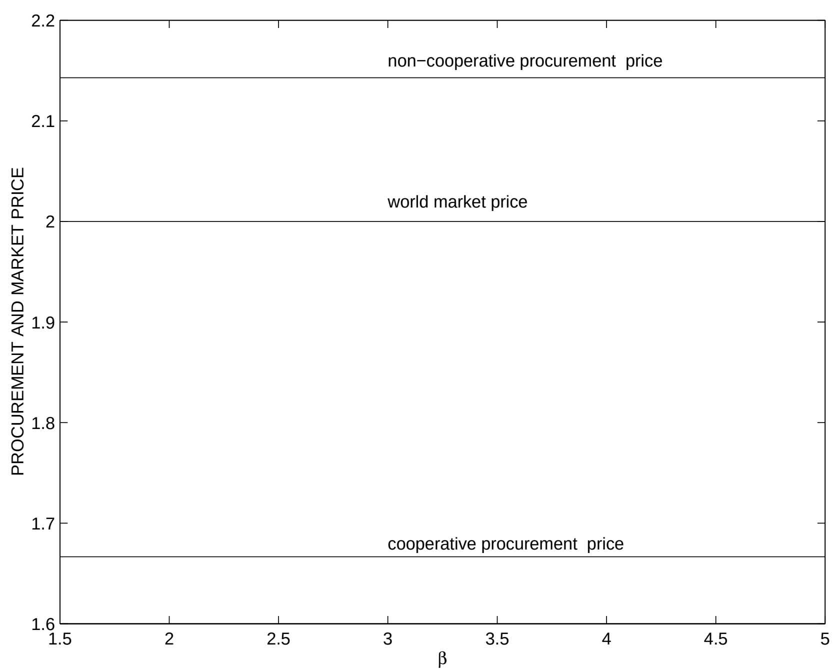


Figure 2: **Non-Cooperative and Cooperative Procurement and World Market Prices as $\beta$ increases.**

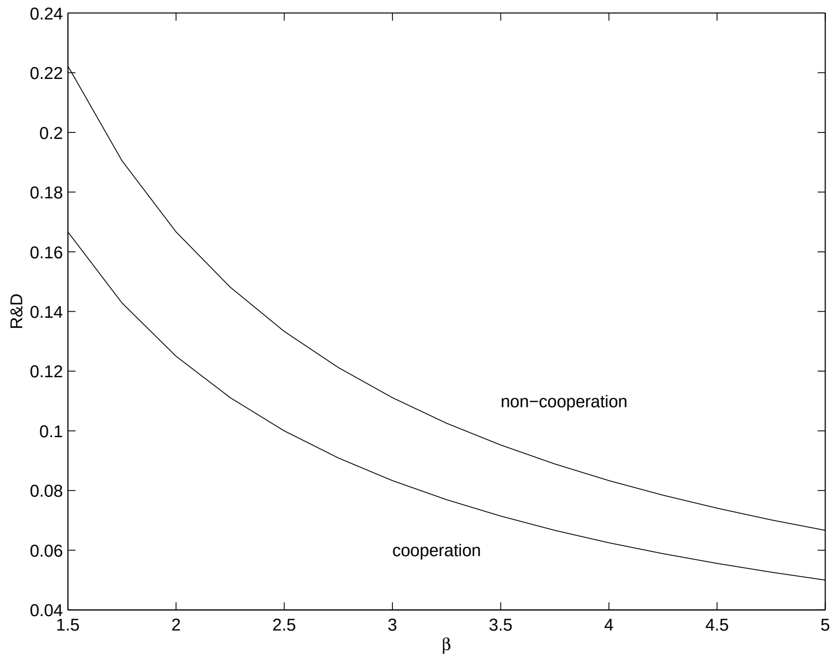


Figure 3: **R&D Expenditure as a Proportion of Output as $\beta$ increases.**

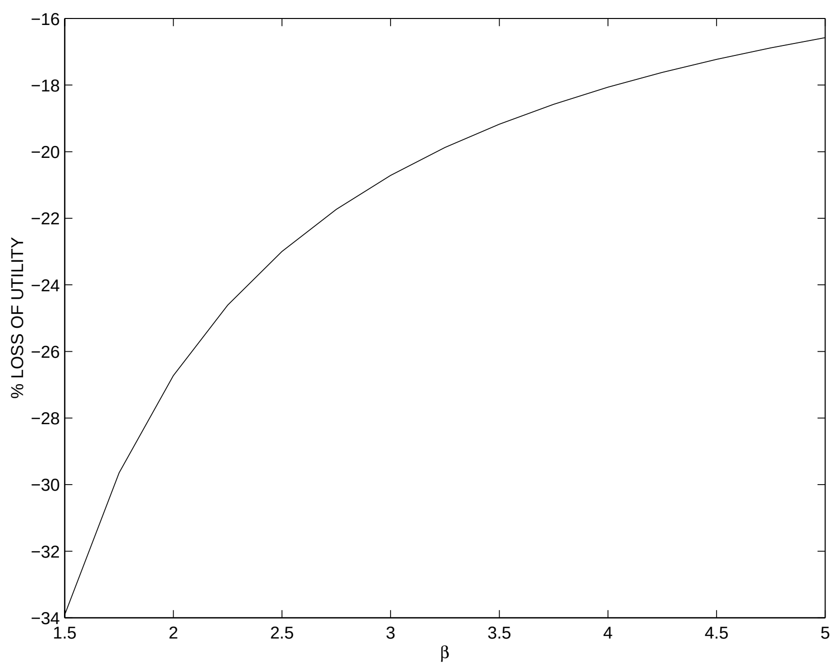


Figure 4: **Loss of Utility: Cooperation compared with Non-Cooperation as $\beta$ increases.**

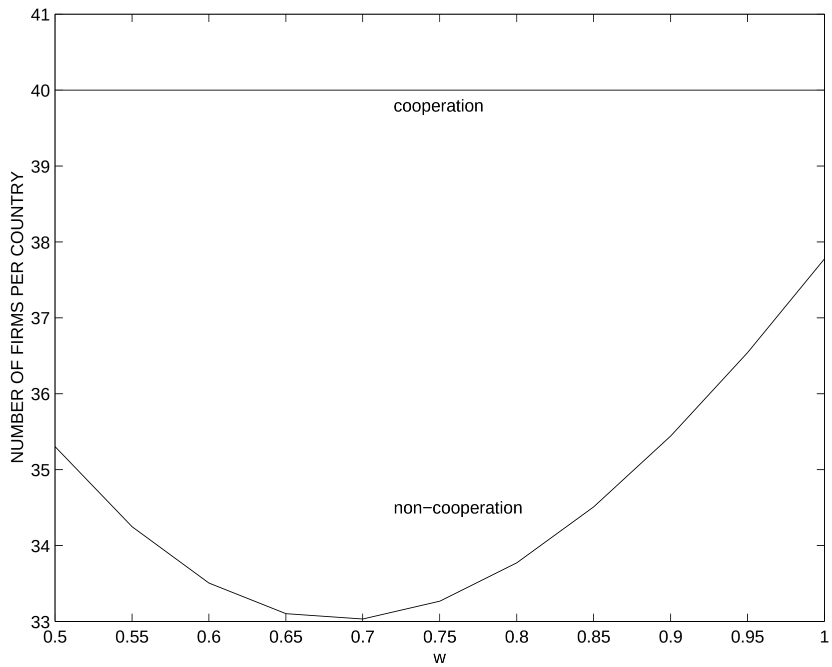


Figure 5: **Number of Firms per Country as $w$ increases: Non-Cooperation compared with Cooperation.**

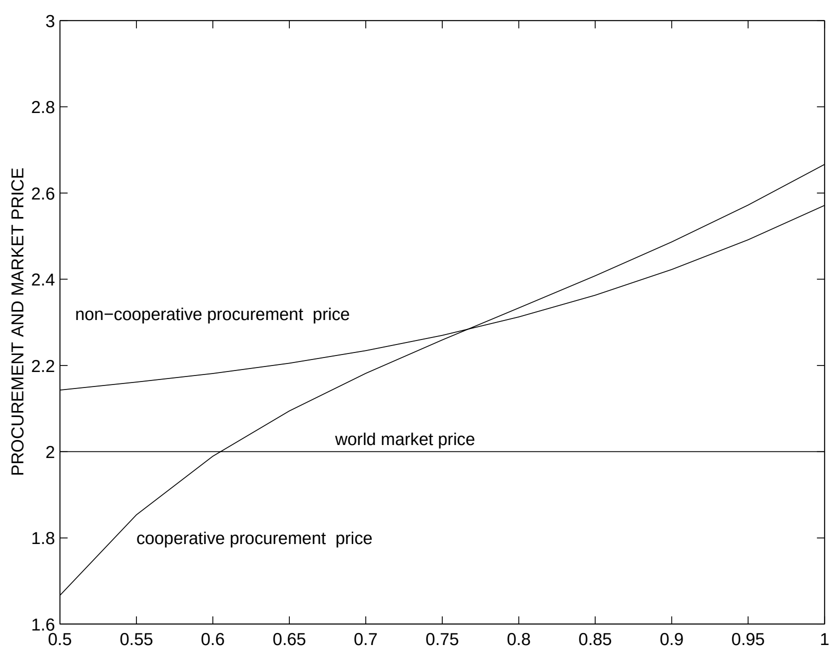


Figure 6: **Non-Cooperative and Cooperative Procurement and World Market Prices as $w$ increases.**

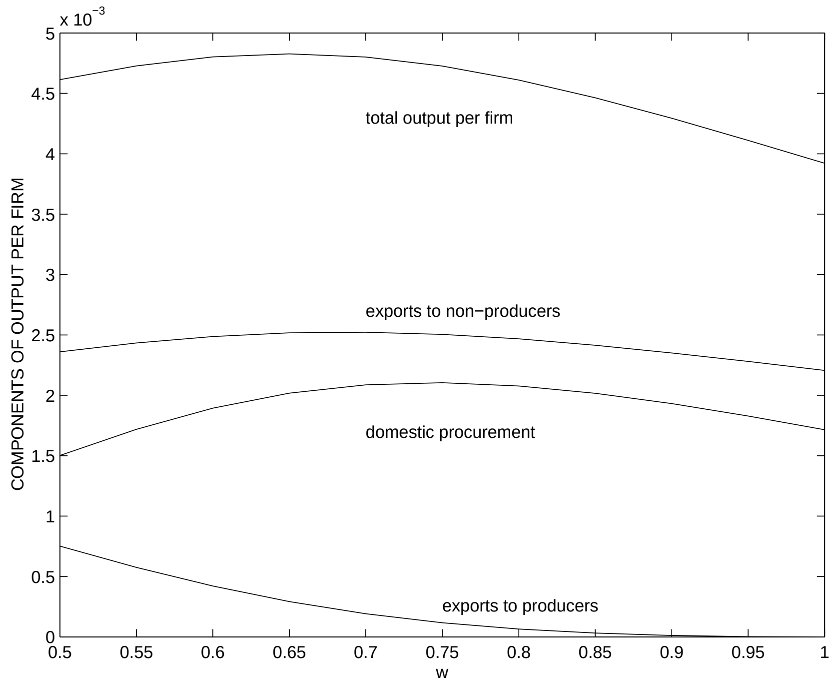


Figure 7: **Components of Output per Firm in the Non-Cooperative Equilibrium as $w$ increases.**

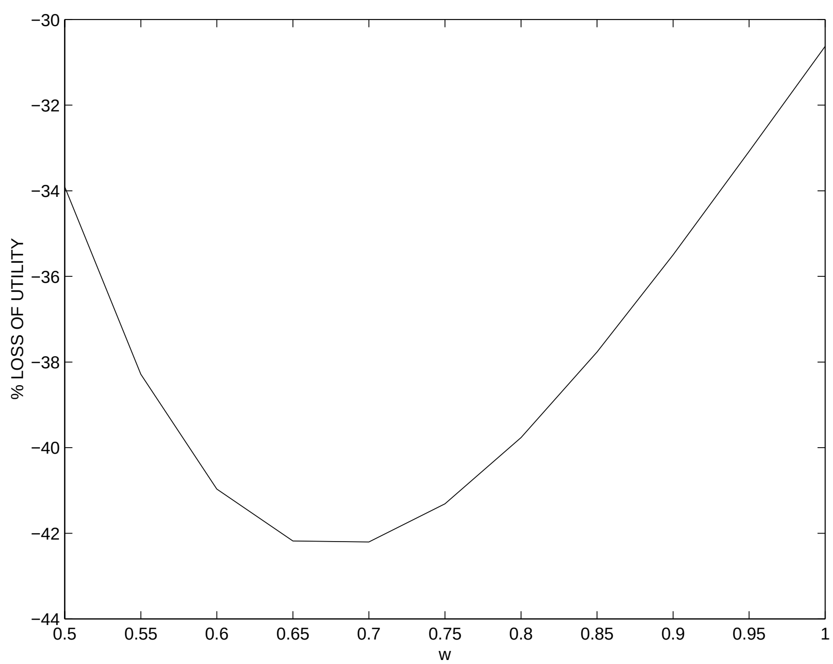


Figure 8: **Loss of Utility: Cooperation compared with Non-Cooperation as $w$ increases.**

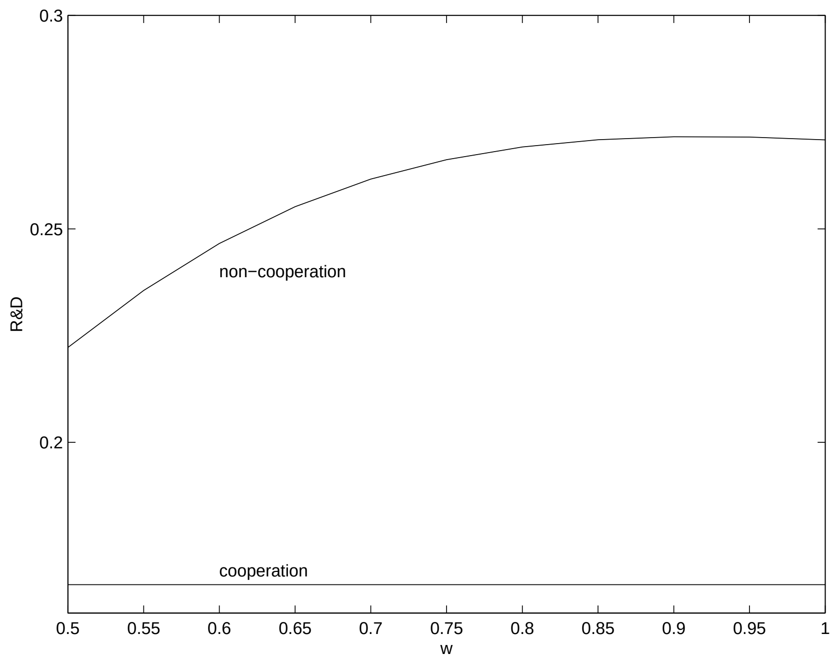


Figure 9: **R&D Expenditure as a Proportion of Output as $w$ increases:**

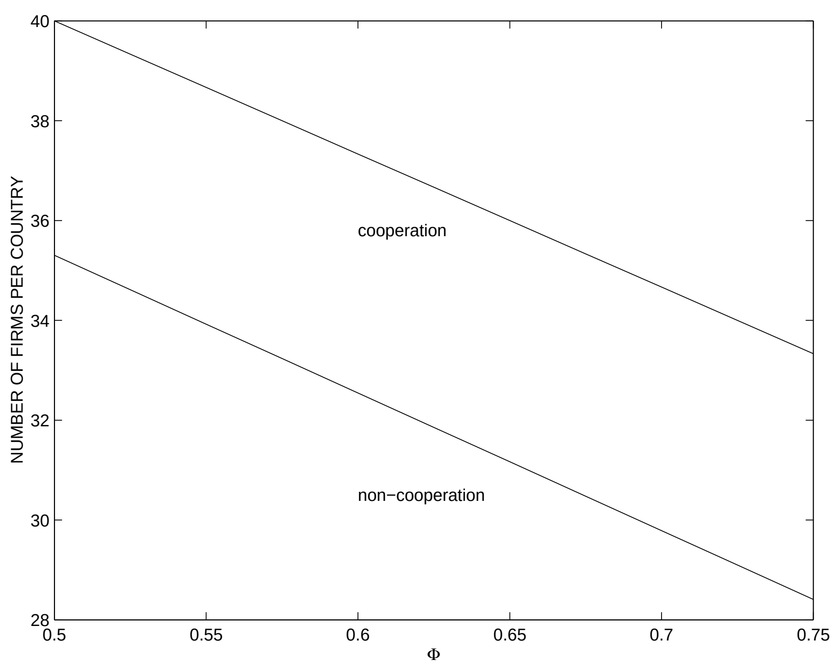


Figure 10: **Number of Firms per Country as $\Phi$ increases: Non-Cooperation compared with Cooperation.**

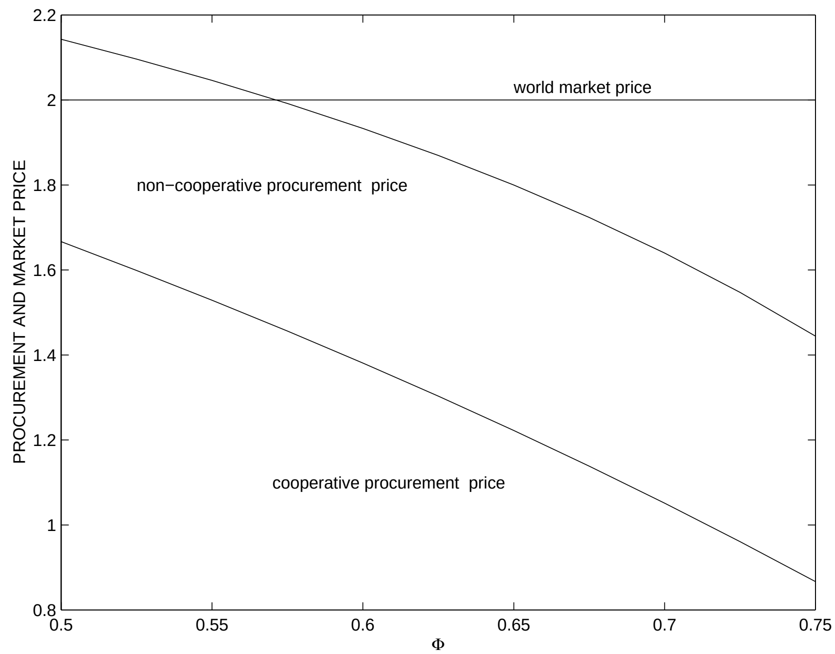


Figure 11: **Non-Cooperative and Cooperative Procurement and World Market Prices as $\Phi$ increases.**

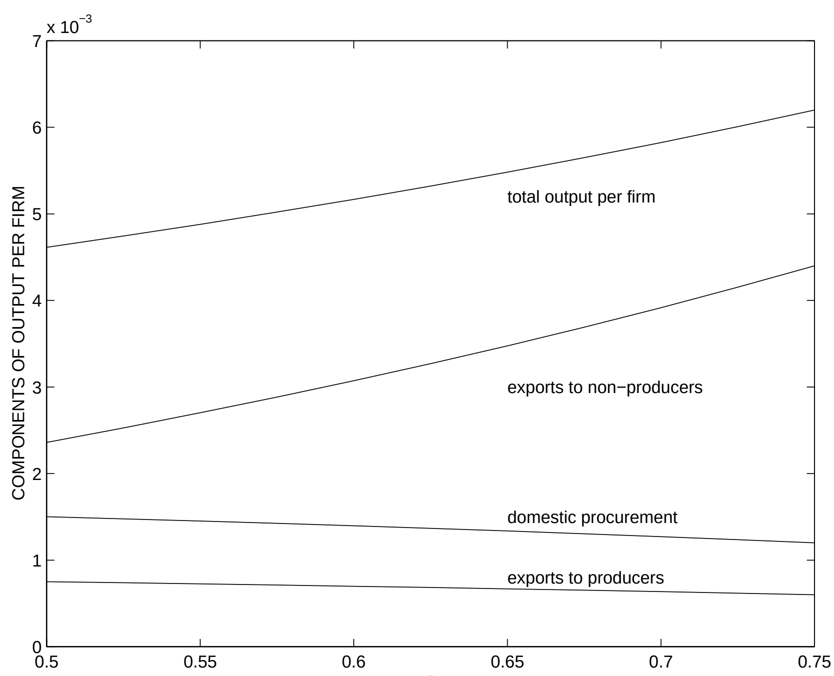


Figure 12: **Components of Output per Firm in the Non-Cooperative Equilibrium as $\Phi$ increases.**

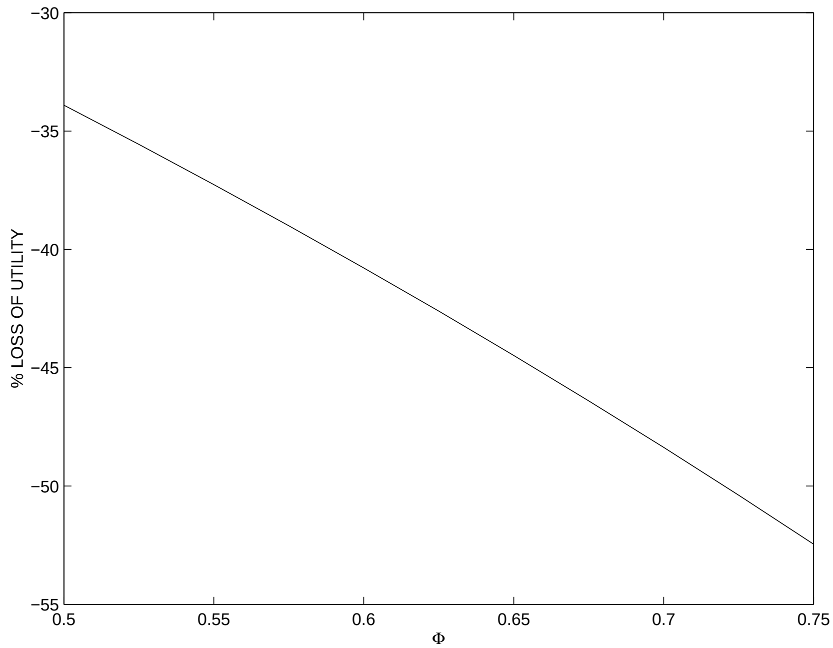


Figure 13: **Loss of Utility: Cooperation compared with Non-Cooperation as $\Phi$ increases.**

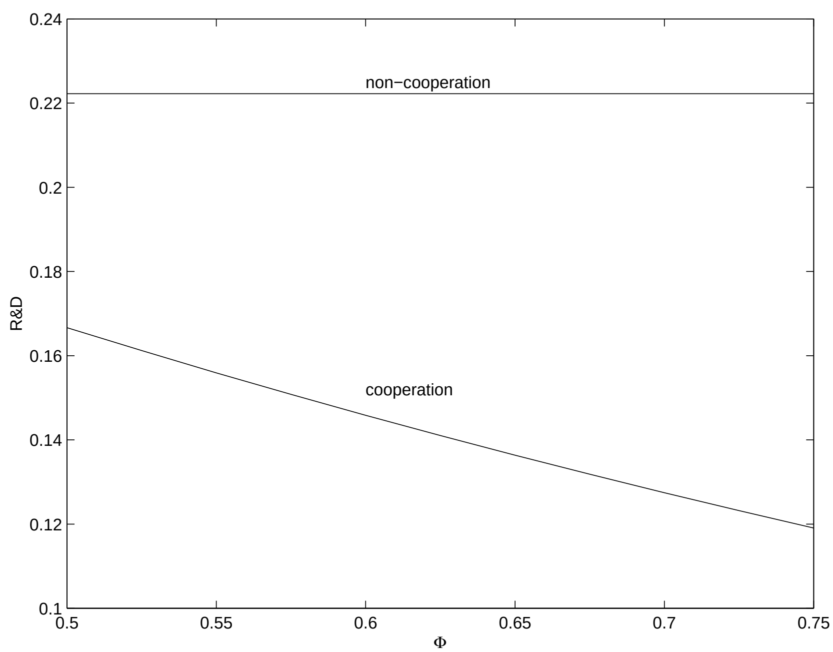


Figure 14: **R&D Expenditure as a Proportion of Output as $\Phi$ increases:**