THE STRUCTURE OF THE DEFENSE INDUSTRY AND THE SECURITY NEEDS OF THE COUNTRY: A DIFFERENTIATED PRODUCTS MODEL

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7/4/2003

Abstract

This paper models the interactions between security levels of two arms-producing countries (the USA and Western Europe, say) and the defense industry market structure. The defense firms in each arms-producing country produce heterogeneous defense goods and sell them to their own governments and to the "rest of the world". There is no production of defense goods in the rest of the world. The security level of each of the two arms-producing countries depends on its purchase of the defense good **relative** to the amounts of the defense goods that are purchased by the rest of the world. Target security levels in the arms-producing countries are exogenous. Decisions in the model are taken in two stages. First, the governments of the two arms-producing countries commit themselves, simultaneously and non-cooperatively, to the amounts of the defense goods that they will purchase, at the world price, from their own defense firms. Second, the defense firms in the two producer countries determine the prices of their defense goods in order to maximize their profits. The results show, among other things, that (i) the net defense costs of the producer countries are lower when the number of defense firms in each country is small, (ii) an increase in the target security levels in the two arms-producing countries results in larger net defense costs in these countries, higher prices and lower exports of the defense goods.

Key words: Defense industry, security levels, net defense cost, industry profits

1. Introduction

Many countries spend substantial resources on procurement, maintenance and training of their armies in order to deter a war and, if necessary, be prepared for a possible major attack against them. Major wars are rare, their timing is difficult to predict, their scale exhibits very large variability over time and geographical area and the opposing armies may use technologies and equipment that are different, sometimes vastly so. Thus, it is extremely difficult to measure the benefits of the expenditure on national defense of a particular country. This difficulty is the reason for the heated debate on the overall expenditure on defense, as well as the structure of this expenditure. On the one hand, governments try to reduce defense expenditure and direct resources to other pressing social and economic needs. That is, defense expenditure is difficult to justify in times of peace. On the other hand, an adequate response to a major enemy is impossible without investing substantial resources over a long period of time.

Since the end of the Cold War, military expenditures have fallen sharply, but production capability has decreased only slightly (Markusen, 2000). As a result, export markets for military goods have become very competitive. Several processes have characterized the evolution of the defense industry since the end of the Cold War and the disintegration of the USSR. These are: (i) the emergence of a small number of giant defense firms in the USA and Western Europe (the consolidation process), (ii) the "buy local" policy in the USA and Western Europe, and (iii) the emergence of the USA and Western Europe as the only dominant players in the production and exports of advanced weapon systems.

The consolidation process in the USA accelerated when the American secretary of defense recognized in 1993 that the defense market was too small for the existing players at that time, and a reduction in the number of firms in order to lower the US procurement costs was necessary (Markusen, 2000). The consequent emergence of four giant American defense firms was the catalyst for a similar consolidation process across the Atlantic (Piggot, 1999; Becker, 2000; Nativi and Bonsignore, 2000; James, 2001). Size, so it seems, had become a significant factor in the defense industry (The Economist, 1997), though it did not ensure either efficiency or survival and growth in the market (Dunne, 1995).

Despite the privatization and consolidation processes that took place in the defense industry during the 1990s, economic constraints and pressure from local firms forced governments to intervene in the markets for weapon systems and subsidize development costs in order to preserve their local defense industries. In the USA and in Western Europe this strengthening of the local defense industry is expressed in a “buy local” policy (Adams, 2001; Cornu, 2001; Cobble, 2000; Flamm, 2000; James 2000; Lovering, 2000; Markusen, 2000; Serfati, 2000).

The literature on defense economics includes specific models aimed at studying and explaining specific phenomena such as the influence of defense expenditures on economic growth, the defense structure of specific countries, and studies of local, regional and global arms races, as well as global models, covering industry structure, international relations, interactions between security and economic conditions, local, regional and global arms races, arms imports and exports, etc.

Global arms trade models covering industry structure, interactions between security and economic conditions, arms imports and exports and arms races must address and specify a large variety of model characteristics. Among them are: (i) the definition of security, (ii) the definition and scope of the "enemies" in the model, (iii) the relations between the governments and the defense firms in the arms-producing countries, (iv) the scope of the arms races among countries other than the arms-producing countries, (v) the procurement rules (methods of pricing the defense goods) in the arms-producing countries, (vi) the type and scope of the game (Cournot, Bertrand, etc.) that describes the defense industry oligopoly, (vii) the variety of defense goods, and (viii) the decision rule that the governments follow in determining their security level.

Anderton (1995), Sandler and Hartley (1995), Levine and Smith (1997b) and Bolks and Stoll (2000) provide surveys and assessments of models of international arms trade. Detailed theoretical models of arms trade can be found, among others, in Levine, Sen and Smith (1994), Levine and Smith (1995, 1997a, 1997b, 2000), Garcia-Alonso (1999, 2000), Bolks and Stoll (2000), Garcia-Alonso and Hartley (2000), Blume and Tishler (2001) and Golde and Tishler (2002).

This study contributes to the literature in three important ways. First, most other studies define security as the **difference** between the country's stock of weapon systems and its rivals' stock of weapon systems (see Levine et al. (1994), Levine and Smith (1995, 1997b, 2000), Garcia-Alonso (1999, 2000)). This paper follows Blume and Tishler (2001) and Golde and Tishler (2002) in defining the country's security level as its stock of weapon systems **relative** to that of its potential enemies[1]. Second, we assume that the potential enemies of each producing bloc (country) are not confined to a particular region in the world. Rather, **all** countries in the rest of the

[1] Bolks and Stoll (2000) choose the ratio rather than the difference of arms stocks in analyzing naval arms races. Smith (1980), who developed an econometric model for Great Britain's defense expenditure, points out that the ratio, rather than the difference, between US defense expenditures and those of the USSR is statistically significant in the model. Hirshleifer (2000) discusses the characteristics of the measures of security and points out when each should be used.

world are potential rivals of the two producing blocs. Third, most global models assume that the governments choose their security levels in order to maximize a welfare function (see Levine et al. (1994), Levine and Smith (1995, 1997b, 2000), Garcia-Alonso (1999, 2000) and Golde and Tishler (2002)). This paper assumes that the target security level of the country is exogenously determined by its generals and politicians[2]. Finally, this paper does not consider arms races in the developing world. Rather, it emphasizes the interactions between the major developed countries' defense needs and the defense industry market structure[3].

This paper presents a model of the interactions between the choice of defense levels and market structure. It extends the model in Blume and Tishler (2001) and is based on similar assumptions, with two major exceptions. Whereas the model of Blume and Tishler (2001) is based on ***Cournot's conjecture*** in a market of one ***homogeneous*** good, this study utilizes a ***Bertrand*** differentiated products model with ***heterogeneous*** goods. The model views two developed countries (corresponding to the USA and Western Europe) that produce heterogeneous defense goods. The "rest of the world" includes all the other countries in the world. There is no production of defense goods in the rest of the world. The two producer countries are allies but have potential enemies in the rest of the world. The rest of the world features downward-sloping demand functions for the various defense goods and it purchases the defense

[2] Clearly, this decision rule may be viewed as a proxy for welfare maximization subject to the availability of the country's resources. However, this decision rule emphasizes the strong connection between the country's defense expenditure and that of its potential enemies, and the seemingly small correlation between defense expenditure and other government expenditures (on education, welfare, health, etc.). A-priori it is not clear which decision rule is a better approximation of reality.

[3] See Levine et al. (1994), Levine and Smith (1995, 1997a, 1997b, 2000), and Garcia-Alonso (1999, 2000) for models of arms races among the countries in the "rest of the world".

goods produced by the two developed countries at the equilibrium world prices[4]. The security level of each of the two developed countries depends on their purchase of defense goods (from their own defense industry) **relative** to the amounts of the defense goods that are purchased by the rest of the world. Target security levels are exogenous in this model. They are assumed to be determined by the country's culture, social fabric, political structure, religions, beliefs, etc.

The model includes two stages. First, the governments of the two producer countries commit themselves, simultaneously and non-cooperatively, to the amounts of the defense good that they will purchase, at the world price, from their defense industry[5]. Second, the defense firms in the two producer countries play a Bertrand game to determine the prices of the goods that they produce, sell to their own government and export to the rest of the world.

Some of the results of this paper are intuitive and straightforward while others depend on strategic interaction between market structure and security. First, a rise in

[4] We shall also comment on the solution of the model when the defense firms sell the defense good to their own government at a price that is equal to their marginal cost plus a markup (see Blume and Tishler (2001) on this issue). Following Levine et al. (1994), Garcia-Alonso (1999), Levine and Smith (1995, 1997b, 2000) and Blume and Tishler (2001), we assume that suppliers in the two producing countries know the aggregate demand functions of the recipients (countries in the rest of the world), can discriminate between recipients through export licenses and the like, and know how the military capability of the recipients will affect their own security (see Garcia-Alonso (2000) and Levine and Smith (2000) on the optimal policy of security restrictions on exports by the producing countries).

[5] We assume, as is the case in reality, that the governments of the producing countries do not altogether prohibit, or tax, exports. The interplay of market structure and government trade policy whereby governments simultaneously and non-cooperatively choose whether or not to tax or provide subsidies for their firms is found in Balboa, Daughety and Reinganum (2001).

the rest of the world's demands for the defense goods (due, say, to increased regional conflicts or an increase in the GDP of the rest of the world) increases exports to the rest of the world as well as the equilibrium world prices. These changes, in turn, increase the defense industry's profits, but normally also increase the arms-producing countries' net defense cost (their expenditure on defense minus the profits of their defense industry). Second, a rise in the producer-countries' target security levels increases these countries' purchase of the defense good and the world prices. As a result, we observe an increase in the producer-countries' defense expenditures, an increase in the defense industry's profit, and, generally, a rise in the producer-countries' net defense cost. Third, the net defense costs of the producer-countries are minimal when the number of defense firms in the world is small. Fourth, current prices of modern weapon systems are very high (compared to the reservation prices derived from the demand functions of the rest of the world). Exports of weapon systems may vanish in response to a further increase in these prices. Fifth, an increase in the target security level in the USA, following the events of September 11 and the war in Ieaq, will likely cause an increase in the USA's spending on military procurement and, hence, an increase in the world prices of the defense goods and a reduction in the rest of the world's imports of the defense goods. Finally, an increase in production cost and a continuing consolidation of the defense industry are in favor of the USA and Western Europe (these effects will reduce these countries' net defense costs). Therefore, mergers and acquisitions across the Atlantic are likely to develop, and the massive investments made by the USA and Western Europe in military R&D are expected to continue, aiming, in part, at raising world prices to a level that is likely to crowd out the rest of the world from the market for modern weapon systems.

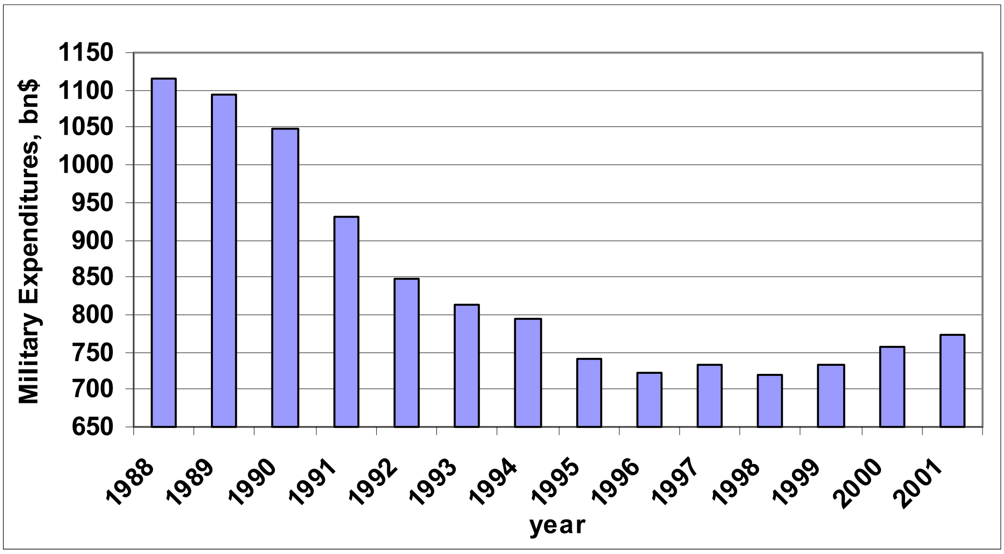
The structure of this paper is as follows: Section 2 provides a background and data on the defense industry. Section 3 presents the concept of security and describes the model. The model, with one defense firm in each arms-producing country, is solved in Section 4 and the optimal solution is analyzed in Section 5. Section 6 extends the model to include several, possibly many, firms in each of the arms-producing countries. Section 7 presents simulations of the model, based on real data. Section 8 discusses policy issues and concludes. Mathematical details of the development of the model and the subsequent results are found in the Appendix.

2. The International Arms Trade – Background

Reliable data on defense expenditures, production and exports are scarce. Quality data on prices and production costs of defense goods are anecdotal. Thus, although the general trends in the development of the markets for defense goods are fairly clear, the interpretation of the data that we present here should be cautious. The main sources for aggregate defense data are SIPRI (Stockholm International Peace Research Institute) and WMEAT (World Military Expenditures and Arms Transfers). Other important data sources are the International Institute for Strategic Studies (IISS), the UN Arms Transfer Register, and the International Monetary Fund.

The end of the Cold War and the disintegration of the USSR caused a sharp decline in the world's defense expenditures, procurement and exports of weapon systems. Defense budgets in the world declined by over 30% during the period that these events were taking place, most notably in the developed countries, where the share of defense spending in GDP declined by about 50% from 1985 to 1998 (Mantin, 2001). The reversal of this decline towards the end of the 1990s (see Figure 1) was caused by the eruption of local disputes in the Middle East, Kosovo, East Timor and other countries, and the trend is expected to continue for the near future.

Figure 1: Defense expenditures in the world during 1988-2001 (1998 prices)

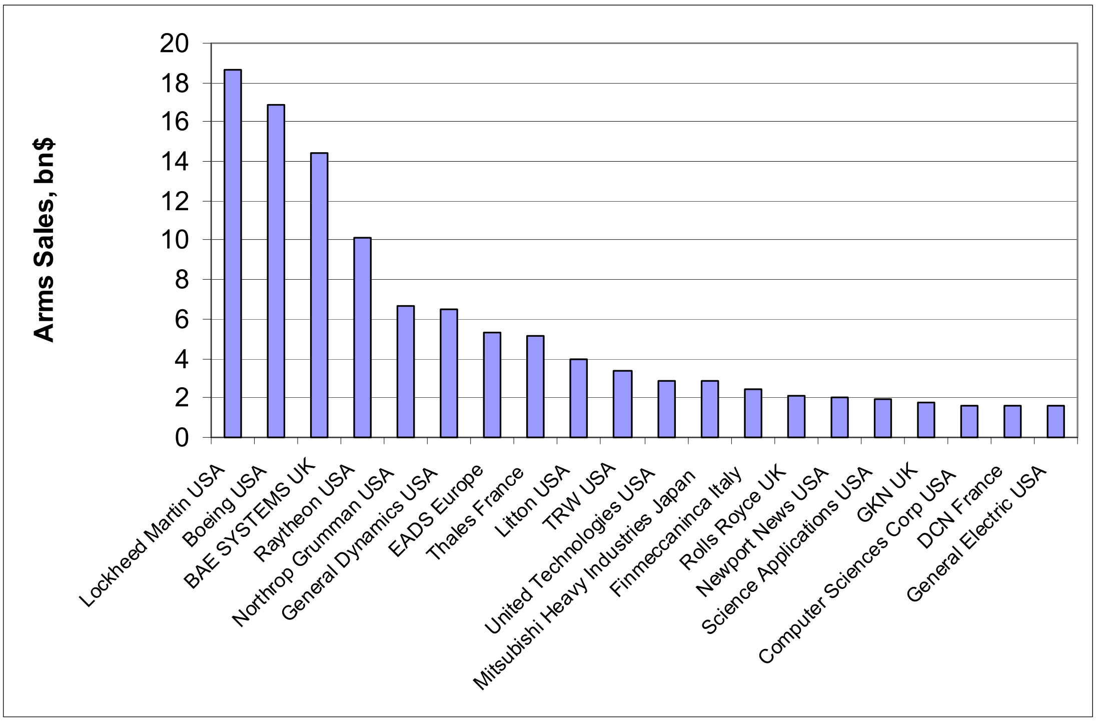


Source: SIPRI 1998, 2001, 2002

European countries have been purchasing locally made, or at the very least European made, defense systems (Cobble, 2000, James, 2000, Lovering, 2000, Serfati, 2000), while the USA enforces a "buy American" policy (Flamm, 2000, Markusen, 2000). Thus, the "buy local" policy of the arms-producing countries has meant that almost all of the procurements in the USA and Western Europe have been from the local industry. It also means that global arms exports are much smaller than global procurements.

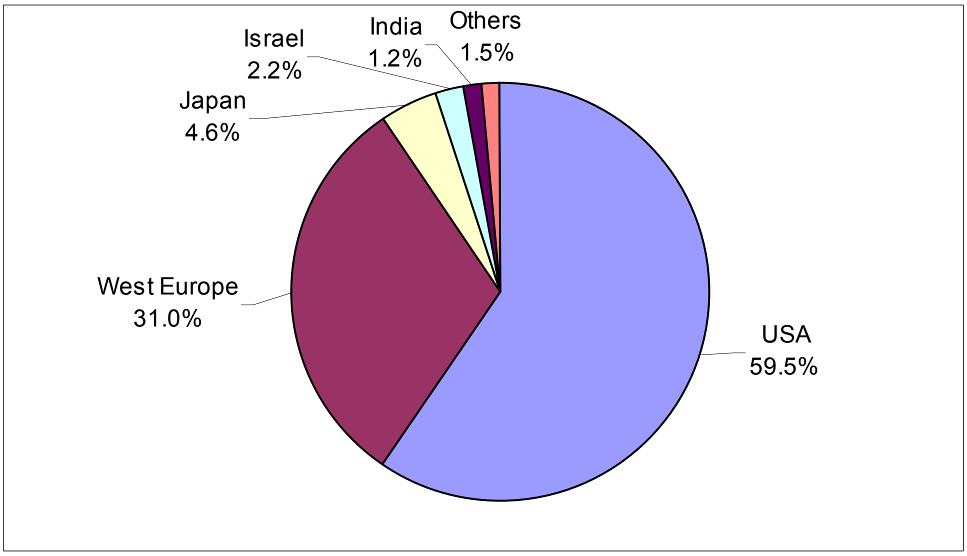
The superior technology and the economic might of the USA and Western Europe are evident from Figures 2 and 3; in 2000, 19 out of the 20 largest defense firms in the world belong to these two blocs. Clearly, the USA and the countries of Western Europe are the only ones that possess the technological and economic capabilities to produce and procure highly sophisticated and expensive weapon systems.

Figure 2: Sales of weapon systems by the 20 largest defense firms in the world (2000)



Source: SIPRI 2002

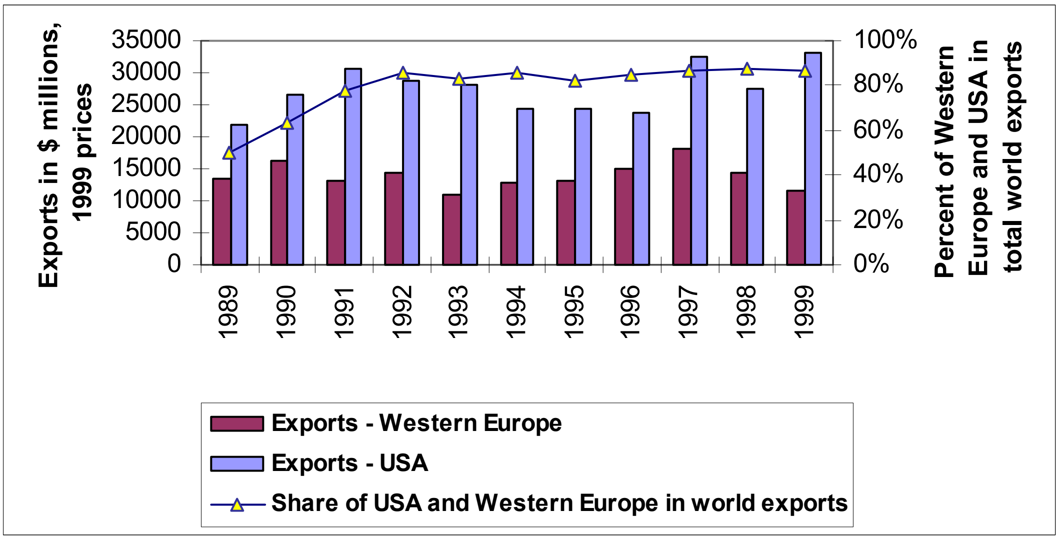
Figure 3: The share of the largest 100 defense firms in the world in 2000, by national origin of the defense firm



Source: SIPRI 2002

The combined share of the USA and Western Europe (mainly – UK, France, Germany, Italy, Sweden, Switzerland and Spain) in the world's exports of weapon systems has been about 85% since 1992 (see Figure 4).

Figure 4: Exports of weapon systems by the USA and Western Europe and the share of these exports in the world's total (1989-1999)



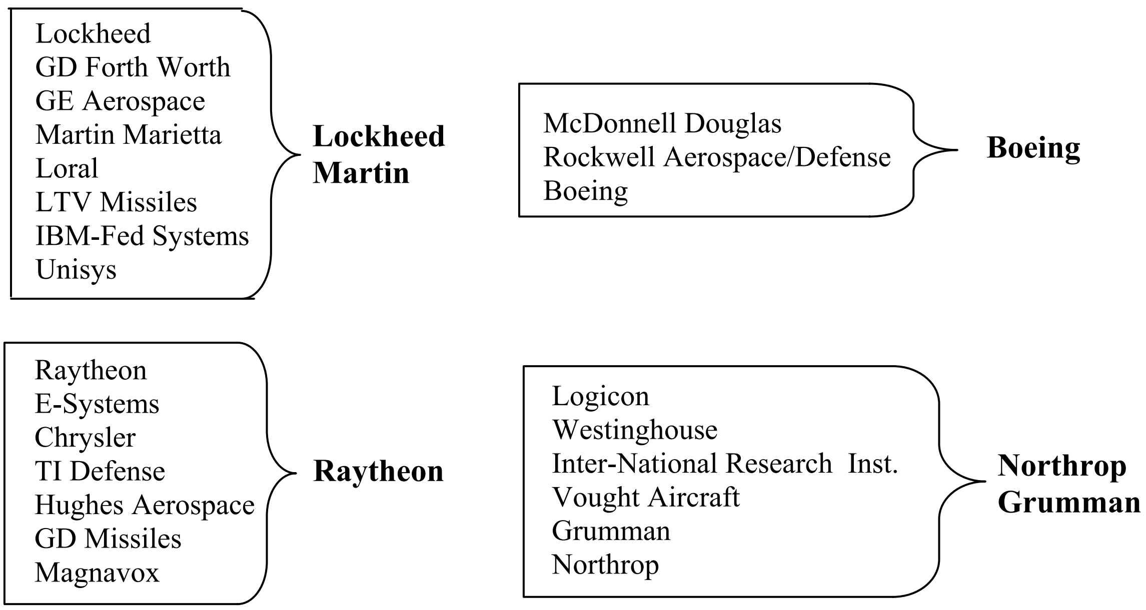
Source: WMEAT 2000

The decline in defense expenditure in the world prompted the US government to adopt a new policy with regard to the structure of its defense industry. The objection of earlier administrations to mergers and acquisitions in the defense industry was reversed by the Clinton administration in the 1990s, resulting in the formation of four huge defense firms in the USA between 1990 and 1998 (Lockheed Martin, Boeing, Northrop Grumman and Raytheon – see Figure 5).

A similar process took place in Western Europe, though the pace was somewhat slower. It started in the 1980s with the emergence of national champions, and reached its peak in 1999 with two mega-mergers: BAE Systems and EADS (European Aeronautic, Defence and Space Company), and the acquisition of the

British firm Racal Electronics by the French electronics firm Thales (formerly Thomson-CSF) (see Cobble, 2000, The Economist, 1997, 2002).

Figure 5: Mergers and acquisitions in the US defense industry between 1990 and 1998



Source: Mantin 2001, The Economist 2002

As a result of the consolidation process, the concentration in the world's defense industry increased dramatically. On average, the sales of each of the largest 100 defense firms declined from \$2.2 billion to \$1.5 billion (1995 prices) between 1990 and 1998, while each of the five largest defense firms increased their defense sales from \$9.5 billion to \$11.7 billion (see SIPRI 2000, Dunne and Smith (2001)).

3. The Model

The model of this paper describes the optimal behavior of the defense industries in two developed countries, country ***A*** (representing the USA) and country ***B*** (representing Western Europe)[6]. Following Garcia-Alonso (1999) we first assume

[6] Extending the model to several countries would complicate the presentation but would not change the nature of the results.

that there is one profit maximizing defense firm in each developed country[7]. The products of the defense firms are substitutes (these products may be similar, but they are not identical). The product, the defense good, of each firm is an aggregate of modern platforms (fighter planes, missiles, integrated weapon and intelligence systems, etc.) and their peripherals, as well as sophisticated munitions and other high-tech equipment. We denote the output of the defense industry in country *i*, *i=A,* ***B*** ,by x_i. The "rest of the world", denoted *W*, includes all the other countries in the world. There is no production of defense goods in the rest of the world. Countries ***A*** and ***B*** are allies, but they may have enemies or potential enemies in the rest of the world. The rest of the world features a downward-sloping demand function for the defense good (the quantity demanded is inversely related to the world price).

The defense industry in country *i* (*i=**A**, **B***) must satisfy local demand by its government (x_i^i) before it is allowed to export the defense good to the "rest of the world" (x_i^W). The governments of ***A*** and ***B*** purchase the defense goods only from the local defense industry. Clearly, $x_i^i + x_i^W = x_i$, *i=A, B*.

Security in each of the arms-producing countries is a function of the size of the country's stock of weapon systems relative to the stock of weapons of its potential enemies. Thus, the security, S_i, of country *i* is dependent on the amount of the defense good x_i^i in country *i*, and the amount of the defense good in the "rest of the world" (sold by the defense industries of countries ***A*** and ***B*** to the "rest of the world"), $x_A^W + x_B^W$. That is, we define the level of security of each country as follows:

[7] In Section 6 we extend the model to include N and K defense firms in countries *A* and *B*, respectively.

$$S_A = \frac{x_A^A}{x_A^W + x_B^W}, \tag{1a}$$

$$S_B = \frac{x_B^B}{x_A^W + x_B^W} \ . \tag{1b}$$

The *target* security level of country i ($S_i^0 \geq 0$) is determined by military and political decision makers who assess the country's potential enemies. The required security level is exogenously given in this model[8].

The "rest of the world" features the following demand functions for the defense goods that are produced by the defense industries of countries ***A*** and ***B*** (p and q are the unit prices of the defense goods of countries ***A*** and ***B***):

$$x_A^W = a_0 + a_1 p + \alpha_1 q \,, \tag{2a}$$

$$x_B^W = b_0 + \beta_1 p + b_1 q \,, \tag{2b}$$

[8]In the model we assume that the defense goods exhibit a similar quality. The definition of security is somewhat different when the defense goods produced in country ***A*** are different in quality from those produced in country ***B***. This extension can be handled as follows. Denote by λ the quality of the defense good produced in ***B*** relative to the one produced in ***A*** (that is, ***A***'s quality equals 1). Expressions (1a) and (1b) become $S_A = x_A^A / (x_A^W + \lambda x_B^W)$ and $S_B = \lambda x_B^B / (x_A^W + \lambda x_B^W)$, respectively. It is straightforward to further extend the model to allow each country to exhibit a different perception of the quality of its defense good relative to the defense good produced by the other country. The results of the model without differentiated qualities are similar in nature (though somewhat less complicated) to those of the model with differentiated qualities. To simplify the analysis and presentation, we proceed to analyze differentiated defense products with similar qualities.

where $a_0 > 0$, $b_0 > 0$, $a_1 < 0$, $b_1 < 0$, $\alpha_1 > 0$, $\beta_1 > 0$ are constant parameters. We further assume that the absolute values of the own-price effects are larger than the cross-price effects, that is, $|a_1| > \alpha_1$ and $|b_1| > \beta_1$.

The technology in country *i* (*i*=***A***, ***B***) is represented by the following cost function:

$$C(x_i) = C_i + c_i x_i \tag{3}$$

Where x_i is total production in country *i*. $C_i > 0$ and $c_i > 0$ are constant parameters.

The decisions in this model are taken in the following order:

Stage 1 – conditional on the values of the S_i^0s, the governments of the arms-producing countries commit themselves, simultaneously and non-cooperatively, to the amounts of the defense goods that they will purchase from their defense industries. That is, each country seeks to make its actual security level, S_i, match its target security level S_i^0 (which may be interpreted as a minimization of the quadratic loss function $(S_i - S_i^0)^2$).

Stage 2 – given the commitments of the arms-producing governments to purchase the defense good from their own defense industries, and the rest of the world's demand functions for the defense goods, the defense firms in countries ***A*** and ***B*** determine the prices of their defense goods in order to maximize their profits. Specifically, the firms in countries ***A*** and ***B*** play a Bertrand differentiated products game to decide on the price of their defense goods and, as a result, how much to sell to the rest of the world.

4. The Solution of the Model

The model is solved in two steps. First we solve the differentiated products game among the defense firms. Then, given the reaction functions of the firms, the governments of ***A*** and ***B*** determine their optimal purchase of the defense good (their commitment levels) such that the initial security measures, S_i^0, are simultaneously attained in both countries.

In the following analysis, we assume that the governments of ***A*** and ***B*** are paying the world price for their purchase of the defense goods from their own industries. However, since most of the defense industry's R&D cost in each country is borne by the government (Goolsbee, 1998), and because the government purchases its defense goods only from its own industry (James, 2000; Markusen, 2000; Serfati, 2000), we shall also comment on the solution of the model when governments pay their defense industries a price which is equal to the marginal production cost of that country's defense industry plus a markup factor. Presumably, this procurement rule will result in a price that is lower than the world price[9].

[9] Blume and Tishler (2001) assume that government ***A*** and government ***B*** each independently chooses one of two possible procurement rules: they may purchase the defense good from their defense industry either at the world price or at a price that is equal to the marginal production cost of that country's defense industry plus a markup factor. The solution of their model is similar under these two procurement rules. Generally, Blume and Tishler (2001) show that both governments are better off paying the world price to their defense industries. Another plausible assumption is that the governments pay a markup over average costs. Our model cannot be solved analytically under this assumption. Thus, we do not use this assumption in this paper. Several authors (see, for example, Levine and Smith (1997b) and Garcia-Alonso (1999)) assume that the government pays its defense industry a unit price such that the overall profits of the industry (from selling to its government and from exports) are zero. That is, the industry's fixed costs are very high and cannot be recovered from

Suppose that government ***A*** purchases the defense good at the world price. Then, the profit function of the defense firm monopoly of country ***A*** is given by:

$$\pi_A = (p - c_A)(x_A^A + x_A^W) - C_A = (p - c_A)(x_A^A + a_0 + a_1 p + \alpha_1 q) - C_A \tag{4a}$$

If government ***A*** purchases the defense good from its defense industry monopoly at marginal cost plus a markup, that is, at a price equal to $(1+\mu)c_A$, the profit function of the monopoly of country ***A*** is given by:

$$\pi_A = (p - c_A)x_A^W + \mu c_A x_A^A - C_A = (p - c_A)(a_0 + a_1 p + \alpha_1 q) + \mu x_A^A - C_A \tag{4b}$$

where $\mu \geq 0$ is the markup factor. The expressions for the profits of the defense firm of country ***B*** are similar.

Solution of stage 2

The solution of the second stage of the model is as follows (mathematical details are in the Appendix). Solving $\partial\pi_A/\partial p = 0$ (conditional on the price of the defense firm in ***B*** and the demand functions of the rest of the world) and $\partial\pi_B/\partial q = 0$ (conditional on the price of the defense firm in ***A*** and the demand functions of the rest of the world) yields the following reaction functions of the defense firms of ***A*** and ***B*** (the optimal prices of the defense goods produced by the firms in countries ***A*** and ***B*** as functions of the commitments of the governments of ***A*** and ***B***):

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marginal cost pricing. Hence, the government is paying a price (which may be **higher** than the world price obtained from the rest of the world) to keep its defense industry in business. This assumption may be a good approximation for the UK defense industry but less so for the US defense industry which is, generally, profitable. In this paper we assume that the governments pay, in addition to the world price, a pre-specified part of the R&D and, possibly, other fixed costs.

$$p = A_1 + A_2 x_A^A + A_3 x_B^B, \quad (5a)$$

$$q = B_1 + B_2 x_A^A + B_3 x_B^B, \quad (5b)$$

where $A_1 \geq 0$, $A_2 \geq 0$, $A_3 \geq 0$, $B_1 \geq 0$, $B_2 \geq 0$, and $B_3 \geq 0$ are simple functions of the basic parameters of the demand functions (2), the cost functions (3), and the functional form of the profit function (4a or 4b). It is straightforward to show that $A_2 = A_3 = B_2 = B_3 = 0$ (the optimal price of the defense firm is not dependent on the commitments of the government of ***A*** and ***B***) when the prices that the governments of ***A*** and ***B*** pay their local producers equal the producers' marginal cost plus a markup.

By substituting *p* and *q* in (5) into the demand functions (2) we obtain the optimal exports of countries ***A*** and ***B*** as functions of these governments' commitments:

$$x_A^W = \widetilde{A}_1 + \widetilde{A}_2 x_A^A + \widetilde{A}_3 x_B^B, \quad (6a)$$

$$x_B^W = \widetilde{B}_1 + \widetilde{B}_2 x_A^A + \widetilde{B}_3 x_B^B, \quad (6b)$$

where $\widetilde{A}_1$, $\widetilde{A}_2 \leq 0$, $\widetilde{A}_3 \geq 0$, $\widetilde{B}_1$, $\widetilde{B}_2 \geq 0$, and $\widetilde{B}_3 \leq 0$ are simple functions of the basic parameters of the demand functions (2), the cost functions (3), and the functional form of the profit function (4a or 4b). Requiring $x_A^W > 0$ and $x_B^W > 0$ (to ensure that the rest of the world purchases a positive quantity of defense goods) implies that $\widetilde{A}_1 > 0$ and $\widetilde{B}_1 > 0$. Again, it is straightforward to show that $\widetilde{A}_2 = \widetilde{A}_3 = \widetilde{B}_2 = \widetilde{B}_3 = 0$ (the optimal exports of the defense firms are not dependent on the commitments of the government of ***A*** and ***B***) when the prices that the governments of ***A*** and ***B*** pay to their local producers equal the producers' marginal cost plus a markup.

Expressions (5) and (6) suggest that if the government of one of the arms-producing countries (say, ***A***) increases its commitment, the world prices of the defense goods will rise (and the exports of ***A*** will decrease while the exports of ***B*** will increase) when the governments of ***A*** and ***B*** pay the world price to their local producers (since $A_2 > 0, B_2 > 0$, see (5)), and will be unaffected (implying no change in ***A***'s and ***B***'s exports) when they pay according to the marginal cost plus a markup (since $A_2 = B_2 = 0$ in (5)). Moreover, an increase in the demand for any one of the defense goods (an increase in a_0 or b_0 due, for example, to an increase in the rest of the world's GDP, or an increase in local conflicts in the rest of the world) leads to higher export levels and world prices.

<u>Solution of stage 1</u>

Substituting the solutions of the optimal exports (expression (6)) in stage 2 into the security functions (1) and solving for x_A^A and x_B^B yields:

$$x_A^A = \frac{D_1 S_A^0}{1 - D_2 S_A^0 - D_3 S_B^0}, \qquad (7a)$$

$$x_B^B = \frac{D_1 S_B^0}{1 - D_2 S_A^0 - D_3 S_B^0}, \qquad (7b)$$

where $D_1 \geq 0$, $D_2 \leq 0$, and $D_3 \leq 0$ are functions of the basic parameters of the demand functions (2), the cost functions (3), and the functional form of the profit function (4a or 4b) (see Appendix). Clearly, $x_A^A \geq 0$ and $x_B^B \geq 0$ since $S_A^0 \geq 0$ and $S_B^0 \geq 0$[10]. It is straightforward to show that $D_2 = 0$ and $D_3 = 0$ when each of the

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[10] If the products are quality differentiated, where λ is the quality of the defense good of country ***B*** relative to the quality of the defense good of country ***A*** (that is, ***A***'s quality equals 1), equation (7)

arms-producing governments pays a price that is equal to marginal cost plus a markup for the defense good that it purchases from its local defense industry[11].

The optimal solution of the model as given in expression (7) implies the following conclusions, when the governments of the arms-producing countries pay the world price to their defense industries:

1. An increase in the target security level of say ***A*** increases its purchase of the defense good, x_A^A, (which, in turn, increases the prices of both defense goods), reduces ***B***'s purchase of the good, x_B^B, and reduces the exports to the rest of the world (due to the increase in prices).
2. An increase in the demand for any one of the defense goods (an increase in a_0 or b_0 due, for example, to an increase in the rest of the world's GDP, or an increase in local conflicts in the rest of the world) causes an increase in the commitments of both arms-producing countries.

5. Analysis of the Model: Analytical Results under Symmetry

The large number of basic parameters in the set up of the model makes its analysis complicated and cumbersome, with results that are often dependent on the values of the parameters. Therefore, in this section we present analytical results for restricted versions of the model. First, adopting a procedure that is common in the

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becomes: $x_A^A = \lambda D_1 S_A^0 / \left(\lambda - \lambda D_2 S_A^0 - D_3 S_B^0\right)$, $x_B^B = D_1 S_B^0 / \left(\lambda - \lambda D_2 S_A^0 - D_3 S_B^0\right)$, where $D_1 \equiv \widetilde{A}_1 + \lambda \widetilde{B}_1 > 0$, $D_2 \equiv \widetilde{A}_2 + \lambda \widetilde{B}_2 < 0$, and $D_3 \equiv \widetilde{A}_3 + \lambda \widetilde{B}_3 > 0$.

[11] If the defense goods are quality differentiated, we obtain: $x_A^A = D_1 S_A^0$, $x_B^B = D_1 S_B^0 / \lambda$, where $D_1 \equiv \widetilde{A}_1 + \lambda \widetilde{B}_1 > 0$.

literature (see, among others, Garcia-Alonso, 1999; Levine and Smith 1997b), we impose symmetry, as follows: (i) the rest of the world's demand functions for the defense goods are identical, and (ii) the cost functions of all firms are identical. Now, straightforward analysis of expressions (6) and (7) yields the effects of changes in the target security levels on the production in and exports of the arms-producing countries, as shown in ***Result 1***.

Result 1:

An increase in the target security level in one arms-producing country leads to higher production, but lower exports, in that country, and lower production, but higher exports, in the other country.

In the rest of this section we assume symmetry and identical target security levels in ***A*** and ***B*** ($S_A^0 = S_B^0 \equiv S^0$). The effects of changes in the target security levels, in the levels of demand from the rest of the world, and in marginal production cost on the arms-producing countries' net defense cost, government expenditure, industry profits, prices and exports are summarized in ***Results 2*** and ***3*** below (the proofs are in the Appendix).

Result 2a:

An increase in the target security levels in ***A*** and ***B*** implies:

- An increase in the profits of the defense firms in ***A*** and ***B***.
- An increase in government expenditure on defense in ***A*** and ***B***.
- An increase in net defense costs in ***A*** and ***B*** (provided $a_0 \le [c_A\, a_1(4a_1 + 3\alpha_1)]/\alpha_1$).

The increase in the target security levels in ***A*** and ***B*** induces the arms-producing countries to increase their purchases of the defense goods from their local industries which, in turn, cause an increase in the world price, a subsequent decline (due to the increase in price) in the quantities demanded by the rest of the world (a decline in exports) and an increase in the defense firms' profits. The increase in government expenditure is, mostly, larger than the increase in the profits of the defense firms, yielding an increase in the net defense costs of both arms-producing countries.

Next, we analyze the effect of an increase in the level of the rest of the world's demand for the defense goods (an increase in a_0 and b_0). This increase may be the result of an increase in regional conflicts, or, for example, due to an increase in the GDP of countries in the rest of the world.

Result 2b:

An increase in the level of demand for the defense goods implies:

- An increase in the profits of the defense firms in ***A*** and ***B***.
- An increase in government expenditure on defense in ***A*** and ***B***.
- An increase in the arms-producing countries' net defense cost when the demand is low, and a decrease in the arms-producing countries' net defense cost when the demand is high (net defense cost is a quadratic function of a_0 and b_0).

The increase in the defense industry's profits is a straightforward result of the increase in demand. The increase in the government expenditures is due to the increase in the demand of the rest of the world and the requirement to maintain the target security levels. The third part of ***Result 2b*** is less obvious. That is, when the demand is low, an increase in the rest of the world's demand for the defense goods is harmful to the arms-producing countries (it increases their net defense costs). The increase in the rest of the world's demand for the defense goods increases the profits of the firms that produce them but, when the demand is relatively low, it increases these countries' expenditure on the defense goods at a faster rate. This effect is reversed when the demand of the rest of the world is "high". Golde and Tishler (2002) produced similar results, showing a "low" demand from the rest of the world in 2000, that is, in 2000 the increase in the rest of the world's demand was harmful to the USA and Western Europe.

An increase in the defense good's quality, in its complexity or in the prices of its components, may lead to an increase in its marginal production cost, which affects the optimal solution of the model in the following way.

Result 2c:

An increase in the marginal production cost of the defense goods causes:

- A decrease in the profits of the defense firms in ***A*** and ***B***.
- An increase in the arms-producing governments' defense expenditures when $S \geq -\alpha_1 /[2(a_1 + \alpha_1)] > 0$. When $0 < S < -\alpha_1 /[2(a_1 + \alpha_1)] > 0$ the arms-producing governments' defense expenditures increase when marginal production cost is "low", and decrease when marginal production cost is "high".
- The net defense cost of the arms-producing countries increases when marginal production cost is low, and decreases when it is high (net defense cost is a quadratic function of the marginal production cost). When target security levels are zero, net defense cost is an increasing function of marginal production cost.

The decline in the defense industry's profit is straightforward. Government expenditure may increase when marginal cost increases (when, following an increase in the world price, the rest of the world reduces its demand proportionally less than the increase in the world price), and then decline (when, following an increase in the world price, the rest of the world reduces its demand proportionally more than the increase in the world price). Net defense cost rises when marginal production cost is very low and declines when it is high (net defense cost is a quadratic function of c, since profits and government expenditures are quadratic functions of c). Hence, the governments in the arms-producing countries may have an interest to increase or decrease their firms' marginal production cost, depending on the actual values of the model parameters, particularly the level of the marginal production costs themselves. Marginal production cost increases may arise from supporting or subsidizing R&D expenditures, or requiring high quality weapon systems, for example.

Finally, it is straightforward to compare the model results when the governments of the arms-producing countries pay their local defense industry the world price (denoted ***WP***) or a price equal to marginal production cost plus a markup (denoted ***MU***). The following conclusions hold.

Result 3:

- The world price under ***WP*** is higher than or equal to the world price under ***MU*** (since under ***MU*** the commitments of the governments of the arms-producing countries do not affect the optimal price, see expression (5)).
- Exports of the defense goods under ***MU*** are higher than or equal to the exports under ***WP*** (since the world price is lower under ***MU***).
- The expenditure of the governments of ***A*** and ***B*** on the procurement of the defense good is lower under ***MU*** than under ***WP***. Clearly, world price is lower under ***MU***. Thus, under ***MU***, the governments of ***A*** and ***B*** pay a lower price (than they would have under ***WP***) to their defense firms, but have to increase the volume of their purchases (commitments) in order to maintain their target security levels, while

the rest of the world purchases a higher volume of defense goods under ***MU*** relative to ***WP*** (since the world price is lower under ***MU***). The first effect (lower price) is stronger than the second (higher volume of commitment).

- Generally, net defense costs under ***WP*** are lower than those under ***MU***. That is, the lower expenditure on procurement under ***MU*** is smaller, in absolute value, than the larger profit of the defense firms under ***WP***.

6. The Model with Several Firms in each of the Arms-Producing Countries

Governments may have a role in determining the number of firms in their defense industries. For example, the US government initiated and then supported the consolidation process in the defense industry during the 1990s. However, when it sensed that the large defense firms (integrators) had become too few in number, the US government blocked the merger of Lockheed Martin with Northrop Grumman in 1997, and the acquisition of Newport News Shipbuilding by General Dynamics in 1999[12].

To assess the effect of the number of defense firms in each country we proceed to extend the model to include K and N firms in countries ***A*** and ***B***, respectively. Denote:

x_{ij}^{i} - procurement of government *i* from its j-th defense firm.

x_{ij}^{W} - exports of the *j*-th firm in country *i*.

$x_{ij} \equiv x_{ij}^{i} + x_{ij}^{W}$ - production of firm *j* in country *i*.

[12] Eventually, in April 2001, General Dynamics acquired Newport News Shipbuilding.

$x_i \equiv \sum_j x_{ij}$ - total production of the defense industry in country *i*.

Finally, let $x_i^i \equiv \sum_j x_{ij}^i$ and $x_i^W \equiv \sum_j x_{ij}^W$. The definition of the security level of country *i*, S_i, is given by expression (1).

The rest of the world's demand functions for the defense good produced by the *i*-th firm in country ***A*** and the *k*-th firm in country ***B***, respectively, are given by:

$$x_{Ai}^W = a_{i0} + \sum_{j=1}^{K} a_{ij} p_j + \sum_{j=1}^{N} \alpha_{ij} q_j, \qquad i = 1,..., K. \tag{8a}$$

$$x_{Bk}^W = b_{k0} + \sum_{j=1}^{K} \beta_{kj} p_j + \sum_{j=1}^{N} b_{kj} q_j, \qquad k = 1,..., N, \tag{8b}$$

where $a_{i0} > 0$, $b_{k0} > 0$, $a_{ii} < 0$, $b_{kk} < 0$, $a_{ij} > 0 \;\; \forall j \neq i$, $b_{kj} > 0 \;\; \forall j \neq k$, $\alpha_{ij} > 0$, $\beta_{kj} > 0$ are constant parameters. We further assume that the value of the own-price effect in each demand function is larger, in absolute value, than the sum of all cross-price effects. That is, $|a_{ii}| > \sum_{j=1, j\neq i}^{K} a_{ij} + \sum_{j=1}^{N} \alpha_{ij}$, $|b_{kk}| > \sum_{j=1}^{K} \beta_{kj} + \sum_{j=1, j\neq k}^{N} b_{kj}$.

The cost function of firm *j* in country *m*, *m*=***A***, ***B***, is given by:

$$C(x_{mj}) = C_{mj} + c_{mj} x_{mj}, \tag{9}$$

where C_{mj} and c_{mj} are known parameters.

The (operating) profit function of firm *i* in country ***A***, under ***WP***, is given by:

$$\pi_{Ai} = (p_i - c_{Ai})(x_{Ai}^A + x_{Ai}^W) = (p_i - c_{Ai})(x_{Ai}^A + a_{i0} + \sum_{j=1}^{K} a_{ij} p_j + \sum_{j=1}^{N} \alpha_{ij} q_j), \tag{10a}$$

and the (operating) profit function of firm *i* in country ***A***, under ***MU***, is given by:

$$\pi_{Ai} = (p_i - c_{Ai})x_{Ai}^W + \mu c_{Ai} x_{Ai}^A = (p_i - c_{Ai})(a_{i0} + \sum_{j=1}^{K} a_{ij} p_j + \sum_{j=1}^{N} \alpha_{ij} q_j) + \mu c_{Ai} x_{Ai}^A . \quad (10b)$$

Finally, the decisions in the model are taken in two stages, as in Section 3.

The solution of the model, under symmetry[13], is similar to that in Section 4 (the exact expressions for the optimal prices, exports and production quantities are in the Appendix). As the large number of parameters in the model prevents a meaningful analytical evaluation of its solution, we proceed to characterize the model by using simulations. To further simplify the analysis we assume: (i) N=K (***A*** and ***B*** have the same number of firms), (ii) all the defense firms in ***A*** (or ***B***) sell the same quantity of the defense good to the government, and (iii) all the cross-price coefficients in each demand function are identical, that is, $a_{ij} = a_{ik}$, $b_{ij} = b_{ik}$, $\alpha_{ij} = \alpha_{ik}$ and $\beta_{ij} = \beta_{ik}$.

Solving the model by using the same steps as in Section 4 yields:

$$x_A^A = \frac{D_1 S_A^0}{1 - \frac{D_2 S_A^0}{K} - \frac{D_3 S_B^0}{N}} \quad (11a)$$

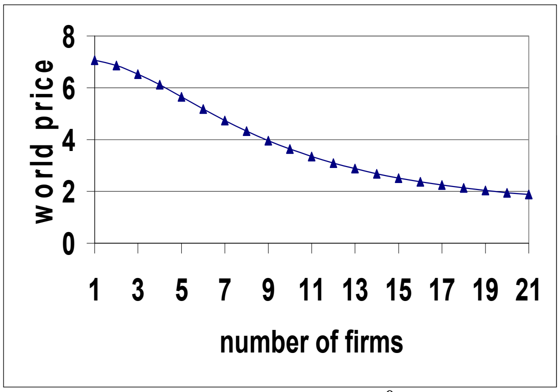
$$x_B^B = \frac{D_1 S_B^0}{1 - \frac{D_2 S_A^0}{K} - \frac{D_3 S_B^0}{N}} \quad (11b)$$

[13] The symmetry assumption here is identical to that in Section 5. That is: (i) the relevant parameters of the demand functions of the rest of the world for the defense goods in ***A*** and ***B*** are identical (in other words, $a_{i0} = b_{i0} \equiv a_0^N > 0 \ \forall i$, $a_{ij} = b_{ij} \equiv \alpha_1^N > 0 \ \forall i \neq j$, $a_{ii} = b_{ii} \equiv a_1^N < 0 \ \forall i$). (ii) the cost functions of all firms in the two arms-producing countries are equal ($C_{Ai} = C_{Bi} \equiv C$, $c_{Ai} = c_{Bi} \equiv c > 0 \ \forall i$), and (iii) target security levels in the two countries are equal ($S_A^0 = S_B^0 \equiv S^0$).

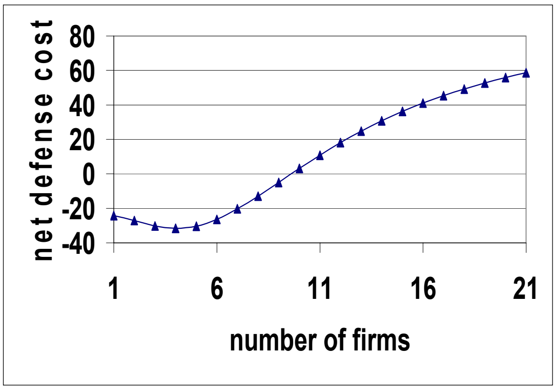
where $D_1 \geq 0$, $D_2 \leq 0$, and $D_3 \leq 0$ are simple functions of the basic parameters of the demand functions (8), the cost functions (9), and the functional form of the profit function (10a or 10b). Clearly, $x_A^A \geq 0$ and $x_B^B \geq 0$ since $S_A^0 \geq 0$ and $S_B^0 \geq 0$.

Figure 6 presents the effect of a change in the number of defense firms in ***A*** and ***B*** on the equilibrium world price, ***A***'s production and exports, and ***A***'s net defense cost (for two sets of parameters)[14], when the governments of ***A*** and ***B*** pay the world price for their own procurement from their defense industries. Generally, as expected,

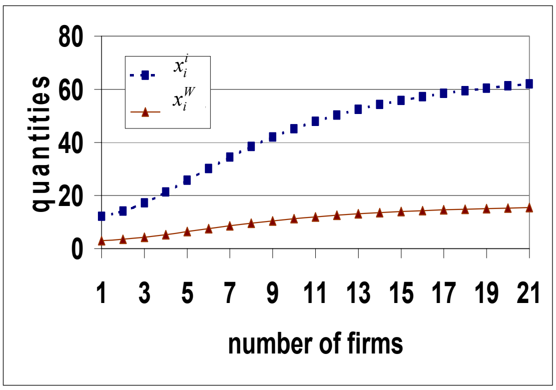
Figure 6: The effects of a change in the number of defense firms on equilibrium price, production, exports and net defense cost



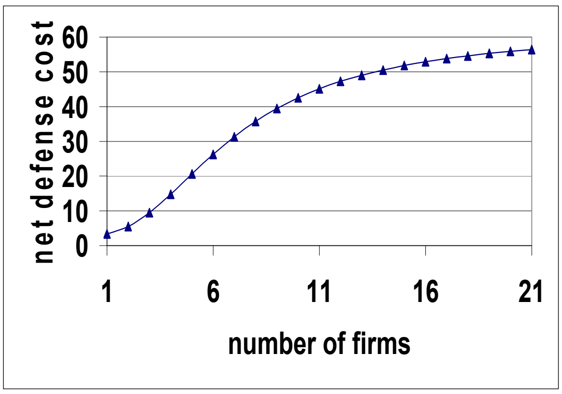
Data: a_0^N=20, α_1^N=0.1, a_1^N=-2.5, $S^0 = 2$, $c = 1$



Data: a_0^N=25, α_1^N=0.1, a_1^N=-2, $S^0 = 2$, $c = 1$



Data: a_0^N=20, α_1^N=0.1, a_1^N=-2.5, $S^0 = 2$, $c = 1$



Data: a_0^N=20, α_1^N=0.5, a_1^N=-5, $S^0 = 2$, $c = 1$

[14] Figure 6 is based on a particular set of parameter values. Clearly, other sets of parameter values would yield different numerical results. However, very extensive experiments, using different sets of parameter values, yielded the same pattern of results as in Figure 6.

an increase in the number of defense firms decreases the world price since more competition yields less monopoly power. The world price approaches marginal production cost when the number of defense firms is large. Consequently, an increase in the number of defense firms increases production and exports.

The effect of an increase in the number of defense firms on ***A***'s net defense cost is more interesting and somewhat less intuitive. Generally, an increase in the number of defense firms increases the country's net defense costs, thus providing support for the consolidation process that took place in the 1990s[15]. Clearly, an increase in the number of defense firms lowers equilibrium price and, hence, increases exports to the rest of the world which, in turn, increases ***A***'s expenditure on procurement of the defense good (since the target security level is fixed). Depending on the model parameters, country ***A***'s industry profits may initially increase at a faster rate than ***A***'s expenditure on defense (yielding a decline in ***A***'s net defense cost). However, when the number of firms is sufficiently large, industry profits always decline, causing an increase in the country's net defense cost. These two outcomes are depicted in Figure 6.

When the governments of ***A*** and ***B*** pay for their own procurement a price that equals the marginal production cost plus a markup, net defense costs are always minimal when each arms-producing country has one defense firm.

In summary, if the governments of ***A*** and ***B*** can choose the number of defense firms (and if net defense cost is the relevant welfare index) they should opt for a small

[15] In theory, full cooperation between ***A*** and ***B*** can prevent exports of the defense goods to the rest of the world, yielding zero defense expenditure in ***A*** and ***B*** (zero procurement). This solution is not possible here since we assume, as is the case in reality, that ***A*** and ***B*** do not cooperate.

number of firms. This choice results in a high world price and a small amount of exports to the rest of the world.

7. Applying the Model to Real Data

The effects of some of the exogenous variables on the model's solution are dependent on the model parameters. Therefore, in order to better understand and predict trends in global arms markets, one needs to apply the model to actual data. In this section we describe the use of actual data to calibrate the model parameters for the year 2000, and then use these parameter values to assess the effects of changes in several exogenous variables on the model solution.

The calibration of the model's parameters was carried out, under the assumption of symmetry (all firms in each arms-producing country are identical), using the following steps (see Appendix):

1. Using annual data for 1994-1996, expression (8) and the optimal solution of stage 2 (for N and K firms), and assuming an annual real price increase of about 6% to 10% in the USA and Western Europe[16], we estimated the parameters of the aggregate (linear) demand functions of the rest of the world for the defense goods of the USA and Western Europe. We then used these estimates to determine the ratio of the own- to the cross-price parameters in the demand functions of the two arms-producing countries (two sets of parameter estimates, yielding ratios of -1.1 and -2.1, are used in this section).

[16] On these estimates see Kirkpatrick (1995) and Golde and Tishler (2002). The calibration results described here are not too sensitive to the use of other reasonable values of the rates of change of the price indices of the defense goods, or to the number of defense firms in each arms-producing country.

2. In 2000, the actual security levels of the USA (about 1) and Western Europe (about 0.6) were estimated using the data in SIPRI 2001. Marginal production cost was estimated by Golde and Tishler (2002) to equal about 80% of the world price in 2000.

3. We set the number of firms in each arms-producing country at four.

4. We set the price of the defense good in the USA (Western Europe) in 2000 at 100 (90). Then, using the estimates of the marginal cost and the ratios of the own- and cross-price effects in each country, together with production and export data for 2000 (SIPRI, 2001), we estimated, using expression (11), the parameters of the demand functions (8), separately for the USA and Western Europe[17].

The calibrated parameters are used to analyze the effects on the model solution of changes in the number of defense firms, the level of the demand from the rest of the world, and a change in the target security levels of the arms-producing countries.

Figure 7 presents the effect of changing the number of defense firms on the model solution.

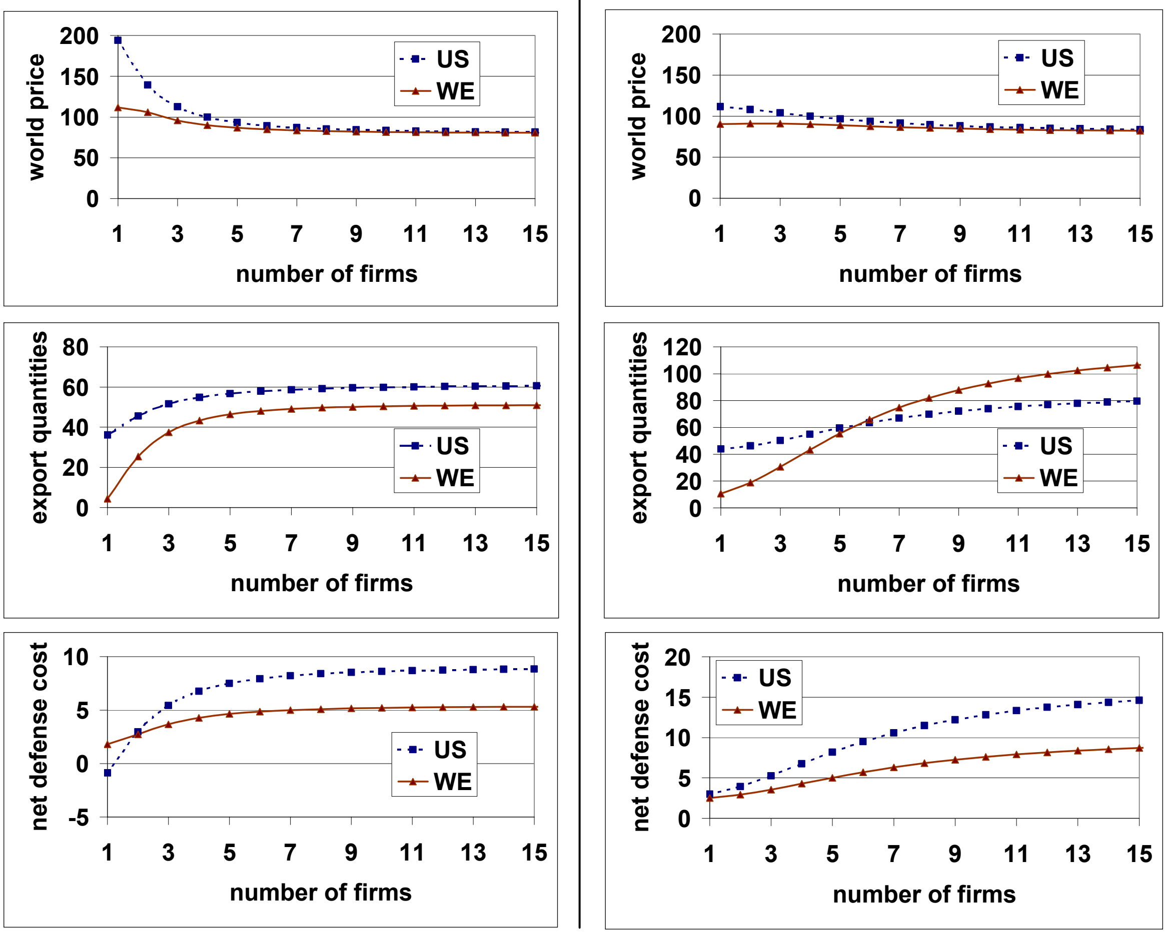
Clearly, an increase in the number of firms results in lower world prices (since monopoly power declines) and, as a consequence, an increase in the arms-producing countries' exports to the rest of the world. These effects, in turn, explain the resulting increase in the net defense costs of the arms-producing countries. Note that the government procurement in the USA and Western Europe increases due to the combined effects of the decrease in the world prices of the defense goods and the increases in their own procurement (due to the increase in the rest of the world's

[17] See Mantin (2001) for details.

purchases of the defense good). Thus, the consolidation process that took place in the defense industry during the 1990s may be partly due to the recognition that a "large" number of defense firms is detrimental to the arms-producing countries.

Figure 7: The effect of a change in the number of firms on the model solution

The ratio of own- to cross-price parameters equals -1.1 in Western Europe and the USA | **The ratio of own- to cross-price parameters equals -2.1 in Western Europe and the USA**



The direction of a change in the solution of the model, caused by an increase in the number of firms, is not dependent on the degree of substitution between the two defense goods (the ratio of the own- to cross-price parameters). The pattern and magnitude of the change in the solution of the model is dependent on this degree of substitution (and on the parameters' size); that is, the higher the degree of substitution between the defense goods (the closer to one, in absolute value, is the ratio of the own- to cross-price effects) the faster the world price converges to the value of the marginal cost. This result, in turn, implies smaller amounts of exports to the rest of the world and lower net defense costs of the arms-producing countries (when the number of defense firms is large).

Next we present the effect of a change in the target security levels of ***A*** and ***B*** on the optimal solution. The 2000 value of the security level of the USA is 1 and that of Western Europe is 0.6. Figure 8 shows the effect of a change (in percentage points) in the current security levels on the model solution. That is, 50% on the horizontal axis means that the target security levels in the USA and Western Europe are 0.5 and 0.3, respectively.

Generally, an increase in the target security levels in the USA and Western Europe results in higher world prices of the defense goods and an increase in the arms-producing countries' net defense cost (due, mainly to the increase in procurement levels that are required to match the higher target security levels). Total exports are lower since the increase in the world prices of the defense goods lowers the quantities demanded by the rest of the world. However, this decline need not be symmetric. Note that the exports of Western Europe increase (but those of the USA decrease) when the defense goods exhibit a high degree of substitution (the ratio of the own- to cross-price parameters is -1.1). This result is due to the asymmetry in the rest of the world's demand functions for the arms-producing countries' defense goods (the reservation price in the demand function for the defense good produced by Western Europe is much smaller than the reservation price of the demand function for the defense good produced by the USA). The most important implication of these results is simple – an increase in the target security levels of the arms-producing countries is costly to these countries (it increases their net defense costs).

Figure 8: The effects of a simultaneous change in the target security levels of the USA and Western Europe on the optimal solution

The ratio of own- to cross-price parameters equals -1.1 in Western Europe and the USA | **The ratio of own- to cross-price parameters equals -2.1 in Western Europe and the USA**

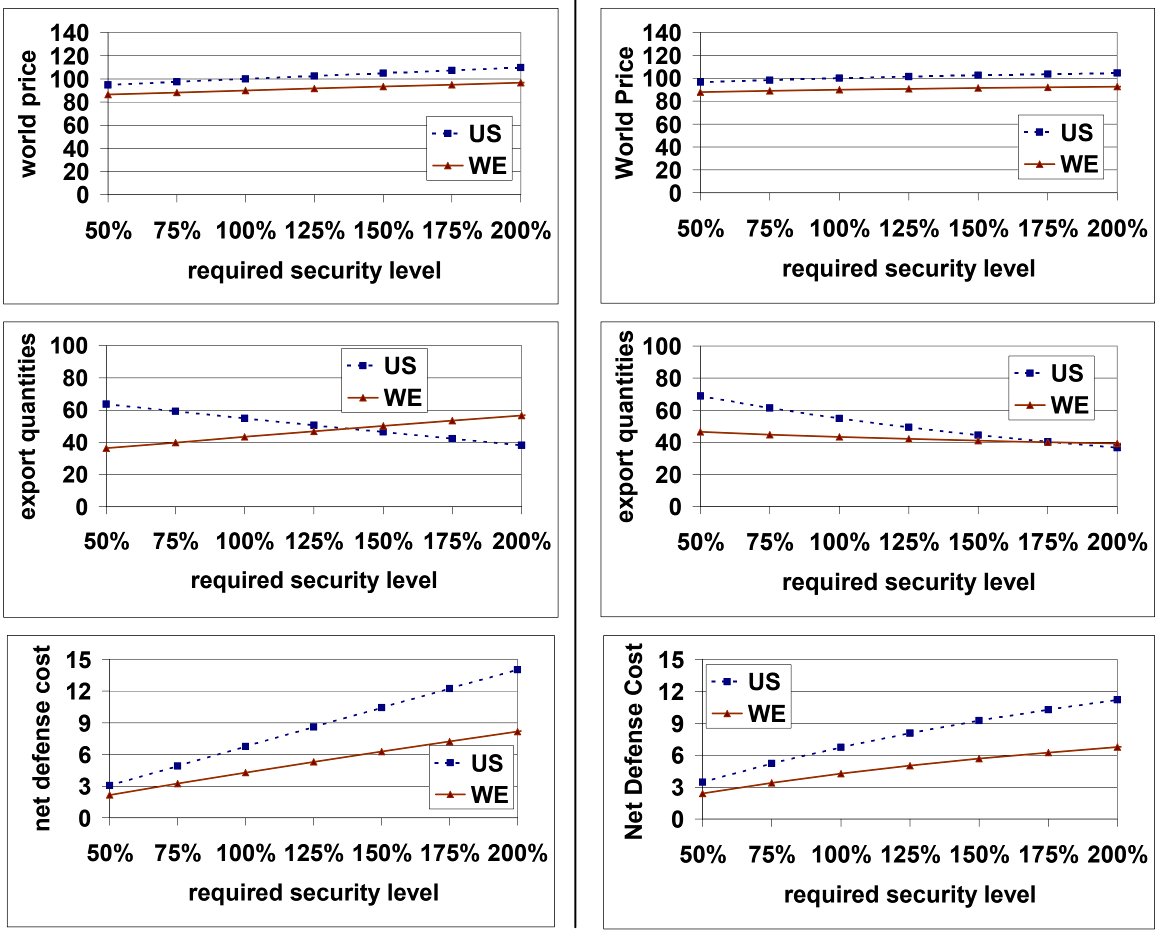


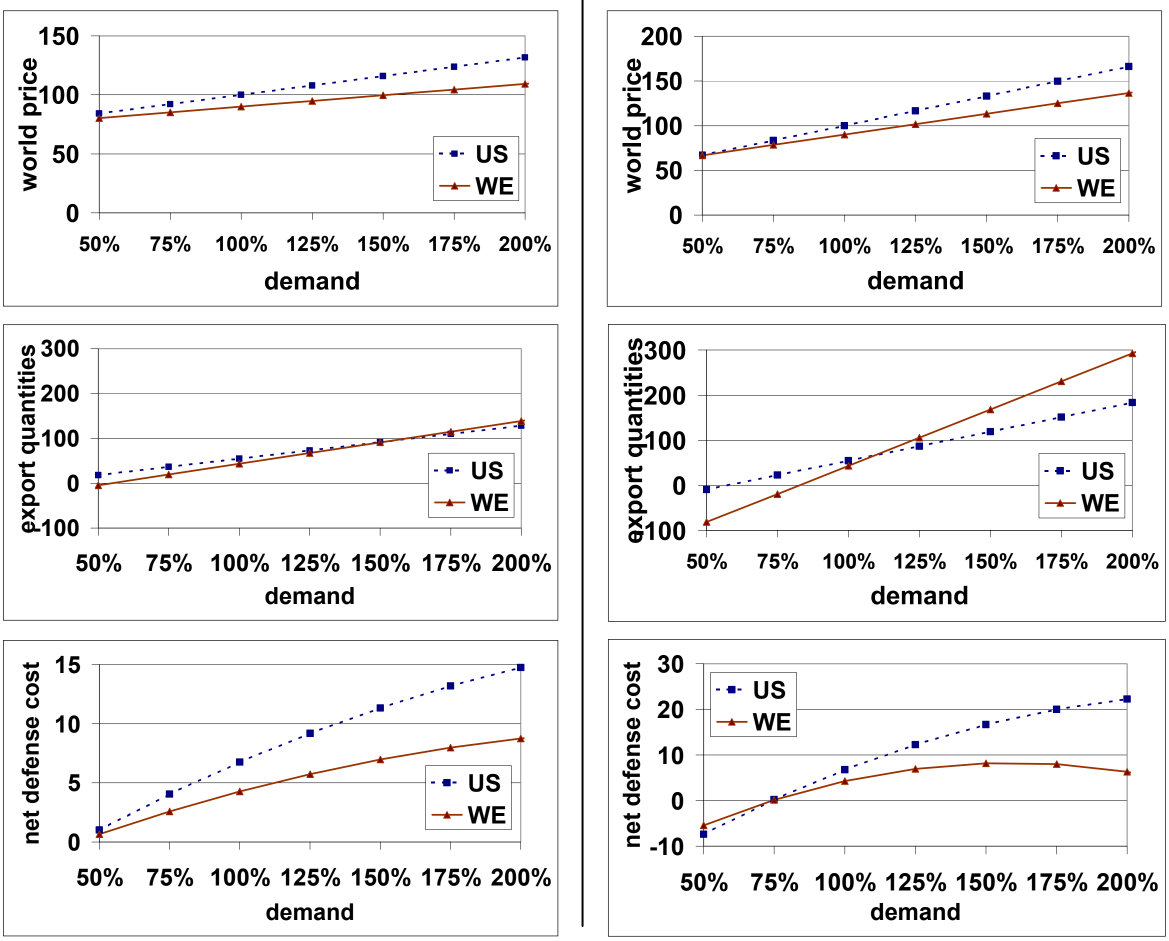
Figure 9 presents the effect of a change in the rest of the world's demands for the defense goods on the optimal solution of the model. The units on the horizontal axis of the graphs in Figure 9 depict the percentage increase in the value of the constants of the demand functions. That is, 150% on the horizontal axis means that the constant of the demand functions for the defense goods were increased by 50% on their estimated values.

Clearly, an increase in the demand of the rest of the world for the defense goods (due to an increase in local conflicts, or an increase in the rest of the world's

GDP) increases the exports to the rest of the world and, at the same time, raises the world prices of the defense goods. The increase in exports forces an increase in the arms-producing countries' procurement levels (since target security levels are unchanged). At the same time, the profits of the defense industries in the USA and

Figure 9: The effect of a simultaneous change in the constants of the demand functions for the defense goods of the USA and Western Europe

The ratio of own- to cross-price parameters equals -1.1 in Western Europe and the USA | **The ratio of own- to cross-price parameters equals -2.1 in Western Europe and the USA**



Western Europe increase. The net effect of these changes (higher procurement levels and higher profits in the defense industry) depends on the size of the increase in the levels of the demand functions. That is, at modest increases in the level of the demand functions (up to 150% of the 2000 levels if the degree of substitution between the defense goods is low, and up to about 250% of the 2000 levels if it is high) procurement levels increase at a higher rate than do the defense industries' profits, and the arms-producing countries' net defense costs increase. However, net defense costs in the arms-producing countries may decline if the increase in the rest of the world's demand is much higher.

Finally, similar patterns of results hold when the arms-producing countries pay their defense industries a price that is equal to the marginal cost plus a markup (Mantin, 2001).

8. Summary and Conclusions

This paper presents a model of the interactions between the choice of security levels and market structure. The model assumes that the target security levels in the arms-producing countries are exogenously determined by military and political decision makers who evaluate the potential external threat to their countries. The decisions in this model are taken in two stages. In stage 1, each of the arms-producing governments commits itself to the amount of the defense good that it will purchase from its defense industry. In stage 2, given the commitments of the governments to purchase the defense good from their own defense industries, and the rest of the world's demands for the defense goods, the defense firms play a Bertrand differentiated products game to decide on the prices of the defense goods. There exists a unique solution to the game.

The main results, and the implied policy recommendations, of this paper are as follows.

1. The net defense costs of the producer-countries (their expenditure on defense minus the profits of their defense industry) are minimal when the number of defense firms in the world is small. Thus, the consolidation process of the defense industry in the USA and Western Europe may be, at least in part, the outcome of an economic, rather than a political or security, process. Moreover, the consolidation is likely to continue in the future.
2. A rise in the arms-producing countries' target security levels, such as occurred at least in the USA after September 11, 2001, increases these countries' purchase of the defense goods and their world prices. As a result, we observe an increase in the producer-countries' defense expenditures, an increase in the defense industry's profit, and, generally, a rise in the producer-countries' net defense cost.
3. A rise in the rest of the world's demands for the defense goods (due, say, to increased regional conflicts or an increase in the rest of the world's GDP) increases exports of the defense goods to the rest of the world and equilibrium world prices. These changes, in turn, increase the defense industry's profits, but, at the same time, they also increase the arms-producing countries' net defense cost.
4. Increasing production costs and continuing consolidation of the defense industry act in favor of the USA and Western Europe insofar as these effects reduce their net defense costs. Thus, the trend would appear to be toward trans-Atlantic mergers and acquisitions, and continuing massive investments by the USA and Western Europe in military R&D, aiming in part at raising world prices to a level that is likely to crowd out the rest of the world from the market for modern

weapon systems. This outcome, in turn, may force countries in the rest of the world to develop and use "cheap and dirty" weapon systems.

Finally, the simulation results of this paper are somewhat similar, though not identical, to those in Blume and Tishler (2001), who employ a similar model but use the Cournot conjecture (in a market for a homogeneous defense good) in the second stage of the game (see Mantin (2001) for details). This outcome suggests that the basic set up of the two-stage model, and the definition of security that we use here, are robust. Furthermore, the model and simulation results are also in general agreement with those of Levine et al. (1994), Levine and Smith (1995, 1997b, 2000), Garcia-Alonso (1999, 2000) and Golde and Tishler (2002), which lends further support to the conclusions that we draw in this paper.

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Appendix

a. Solution of the Model – two firms

Stage 2

First, assume that the governments pay the world price for the defense goods (***WP***).

$$\partial \pi_A / \partial p = 0 \Rightarrow \qquad p = \frac{c_A a_1 - a_0}{2a_1} - \frac{x_A^A}{2a_1} - \frac{\alpha_1 q}{2a_1}. \tag{A1}$$

Similarly,

$$\partial \pi_B / \partial q = 0 \Rightarrow \qquad q = \frac{c_B b_1 - b_0}{2b_1} - \frac{x_B^B}{2b_1} - \frac{\beta_1 p}{2b_1}. \tag{A2}$$

Solving (A1) and (A2) for p and q yields:

$$p = A_1 + A_2 x_A^A + A_3 x_B^B, \tag{A3}$$

$$q = B_1 + B_2 x_A^A + B_3 x_B^B. \tag{A4}$$

Denote:

$$A_1 \equiv \frac{2a_1 b_1 c_A - 2b_1 a_0 - b_1 \alpha_1 c_B + b_0 \alpha_1}{4a_1 b_1 - \alpha_1 \beta_1},\ A_2 \equiv -\frac{2b_1}{4a_1 b_1 - \alpha_1 \beta_1},\ A_3 \equiv \frac{\alpha_1}{4a_1 b_1 - \alpha_1 \beta_1},$$

$$B_1 \equiv \frac{2a_1 b_1 c_B - 2a_1 b_0 - a_1 \beta_1 c_A + a_0 \beta_1}{4a_1 b_1 - \alpha_1 \beta_1},\ B_2 \equiv \frac{\beta_1}{4a_1 b_1 - \alpha_1 \beta_1},\ B_3 \equiv -\frac{2a_1}{4a_1 b_1 - \alpha_1 \beta_1}.$$

Since $|a_1| > \alpha_1 > 0$, $|b_1| > \beta_1 > 0$ we have $4a_1 b_1 - \alpha_1 \beta_1 > 0$. Clearly, $a_1 < 0$ and $b_1 < 0$ imply $A_1 \geq 0$, $A_2 \geq 0$, $A_3 \geq 0$, $B_1 \geq 0$, $B_2 \geq 0$, and $B_3 \geq 0$.

Substituting the values of p and q in (A3) and (A4) into the demand functions (2) we obtain:

$$x_A^W = \widetilde{A}_1 + \widetilde{A}_2 x_A^A + \widetilde{A}_3 x_B^B, \tag{A5}$$

$$x_B^W = \widetilde{B}_1 + \widetilde{B}_2 x_A^A + \widetilde{B}_3 x_B^B, \tag{A6}$$

where:

$\widetilde{A}_1 \equiv a_0 + a_1 A_1 + \alpha_1 B_1$, $\widetilde{A}_2 \equiv a_1 A_2 + \alpha_1 B_2$, $\widetilde{A}_3 \equiv a_1 A_3 + \alpha_1 B_3$, and

$\widetilde{B}_1 \equiv b_0 + \beta_1 A_1 + b_1 B_1$, $\widetilde{B}_2 \equiv \beta_1 A_2 + b_1 B_2$, $\widetilde{B}_3 \equiv \beta_1 A_3 + b_1 B_3$.

Thus, $\widetilde{A}_2 = a_1 A_2 + \alpha_1 B_2 = \dfrac{\alpha_1 \beta_1 - 2a_1 b_1}{4a_1 b_1 - \alpha_1 \beta_1} \leq 0$, $\widetilde{A}_3 = a_1 A_3 + \alpha_1 B_3 = \dfrac{-a_1 \alpha_1}{4a_1 b_1 - \alpha_1 \beta_1} \geq 0$.

Similarly, $\widetilde{B}_2 \geq 0$, and $\widetilde{B}_3 \leq 0$. Requiring $x_A^W > 0$ and $x_B^W > 0$ implies $\widetilde{A}_1 > 0$ and $\widetilde{B}_1 > 0$.

Second, assume that the governments of *A* and *B* pay a price equal to marginal cost plus a markup (***MU***).

Differentiating the profit function (4b) w.r.t. *p* and solving yields:

$$p = \frac{c_A a_1 - a_0}{2a_1} - \frac{\alpha_1 q}{2a_1}. \tag{A7}$$

Similarly, $\partial \pi_B / \partial q = 0$ yields:

$$q = \frac{c_B b_1 - b_0}{2b_1} - \frac{\beta_1 p}{2b_1}. \tag{A8}$$

Solving (A7) and (A8) for *p* and *q* yields expressions similar to those in (A3) and (A4) with

$$A_2 = A_3 = B_2 = B_3 = 0. \tag{A9a}$$

Substituting the values of *p* and *q* into the demand functions (2) yields (A5) and (A6) with

$$\widetilde{A}_2 = \widetilde{A}_3 = \widetilde{B}_2 = \widetilde{B}_3 = 0. \tag{A9b}$$

<u>**Stage 1**</u>

Substituting (A5) and (A6) into the definitions of security (1) yields:

$$S_A^0 = \frac{x_A^A}{x_A^W + x_B^W} = \frac{x_A^A}{D_1 + D_2 x_A^A + D_3 x_B^B}, \tag{A10a}$$

$$S_B^0 = \frac{x_B^B}{x_A^W + x_B^W} = \frac{x_B^B}{D_1 + D_2 x_A^A + D_3 x_B^B}, \tag{A10b}$$

where: $D_1 \equiv \widetilde{A}_1 + \widetilde{B}_1 \geq 0$, $D_2 \equiv \widetilde{A}_2 + \widetilde{B}_2 = \dfrac{\alpha_1\beta_1 - 2a_1b_1 - b_1\beta_1}{4a_1b_1 - \alpha_1\beta_1} \leq 0$, and

$$D_3 \equiv \widetilde{A}_3 + \widetilde{B}_3 = \frac{-a_1\alpha_1 + \alpha_1\beta_1 - 2a_1b_1}{4a_1b_1 - \alpha_1\beta_1} \leq 0.$$

Solving (A10) for the optimal x_A^A and x_B^B yields:

$$x_A^A = \frac{D_1 S_A^0}{1 - D_2 S_A^0 - D_3 S_B^0}, \tag{A11a}$$

$$x_B^B = \frac{D_1 S_B^0}{1 - D_2 S_A^0 - D_3 S_B^0}. \tag{A11b}$$

Clearly, (A11) is the optimal solution of **stage 1** under both ***WP*** (using (A1)-(A6)) and ***MU*** (using (A7)-(A9)).

Analysis of the solution under symmetry

By symmetry we mean that all the demand functions and all the cost functions in ***A*** and ***B*** are identical. Symmetry implies that $D_2 = D_3$ in (A10). That is, (A11) become:

$$x_A^A = \frac{S_A^0 D_1}{1 - (S_A^0 + S_B^0) D_2}, \; x_B^B = \frac{S_B^0 D_1}{1 - (S_A^0 + S_B^0) D_2}, \tag{A12}$$

where $D_1 \equiv 2\widetilde{A}_1 \geq 0$ and $D_2 \equiv \widetilde{A}_2 + \widetilde{A}_3 \leq 0$. Using (A12) we have:

$$\frac{\partial x_A^A}{\partial S_A^0} = \frac{D_1\left(1 - D_2(S_A^0 + S_B^0)\right) + S_A^0 D_1 D_2}{\left(1 - D_2(S_A^0 + S_B^0)\right)^2} = \frac{D_1(1 - D_2 S_B^0)}{\left(1 - D_2(S_A^0 + S_B^0)\right)^2} > 0, \tag{A13}$$

$$\frac{\partial x_B^B}{\partial S_A^0} = \frac{D_2}{\left(1 - D_2(S_A^0 + S_B^0)\right)^2} < 0, \tag{A14}$$

$$\frac{\partial x_A^W}{\partial S_A^0} = \widetilde{A}_2 \frac{\partial x_A^A}{\partial S_A^0} + \widetilde{A}_3 \frac{\partial x_B^B}{\partial S_A^0} < 0, \tag{A15}$$

$$\frac{\partial x_B^W}{\partial S_A^0} = \widetilde{A}_3 \frac{\partial x_A^A}{\partial S_A^0} + \widetilde{A}_2 \frac{\partial x_B^B}{\partial S_A^0} > 0. \tag{A16}$$

Analysis of the solution under symmetry <u>and</u> $S_A^0 = S_B^0$

Using symmetry and $S_A^0 = S_B^0 \equiv S^0$ in (A3)-(A6) and in (A11) we have:

$$p = q = A_1 + (A_2 + A_3) x_A^A \tag{A17}$$

$$x_A^W = x_B^W = \widetilde{A}_1 + (\widetilde{A}_2 + \widetilde{A}_3) x_A^A \tag{A18}$$

$$x_A^A = x_B^B = \frac{S^0 D_1}{1 - 2 S^0 D_2} \tag{A19}$$

The effect of an increase in S^0 on profits, government expenditure and NDC is as follows.

$$\frac{\partial \pi_A}{\partial S^0} = \frac{D_1}{(1-2S^0 D_2)^2}(1+D_2)\left(2x_A^A(A_2+A_3)+A_1-c\right)>0, \qquad \text{(A20)}$$

where $c_A = c_B \equiv c$. Since $1+D_2 = a_1/(2a_1+\alpha_1) > 0$,

$$\frac{\partial GE_A}{\partial S^0} = \frac{D_1}{(1-2S^0 D_2)^2}\left(2(A_2+A_3)x_A^A + A_1\right) > 0. \qquad \text{(A21)}$$

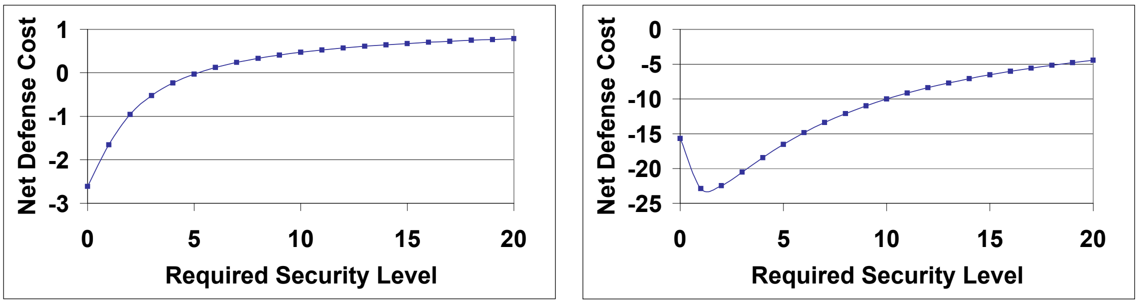
Finally, $NDC_A \equiv GE_A - \pi_A$, where $GE_A = p \cdot x_A^A$ and π_A is given by (4a), implies:

$$NDC_A = \frac{-2cD_1D_2(S^0)^2 + S^0\left(cD_1 + D_1D_2(A_1-c) - \widetilde{A}_1 D_1(A_2+A_3)\right) - (A_1-c)\widetilde{A}_1}{(1-2S^0D_2)^2}. \qquad \text{(A22)}$$

Hence, for $S^0 \to \infty$ NDC_A is positive. Differentiation of NDC in (A22) w.r.t. S^0 yields $\partial NDC_A/\partial S^0 > 0$ for $S^0 > \dfrac{D_2(A_1-c)+\widetilde{A}_1(A_2+A_3)-c}{D_2\left(2D_2(A_1-c)-D_1(A_2+A_3)-2c\right)}$. That is, $\partial NDC_A/\partial S^0 > 0$ when $a_0 \le ca_1(4a_1+3\alpha_1)/\alpha_1$. Otherwise, $\partial NDC_A/\partial S^0 < 0$ for small values of S^0, and $\partial NDC_A/\partial S^0 > 0$ for all values of S^0 which are greater than some value. This result is summarized in Figure A1.

Figure A1: Net defense cost vs. required security level (S^0)

$a_0 \le \dfrac{ca_1(4a_1+3\alpha_1)}{\alpha_1}$ $\qquad$ $a_0 > \dfrac{ca_1(4a_1+3\alpha_1)}{\alpha_1}$



Graph data: a_0 =10, a_1 =-10, α_1 =2, c = 0.1 $\qquad$ Graph data: a_0 =10, a_1 =-4, α_1 =3, c = 0.1

The effect of a change in a_0 on profits, government expenditure and NDC is as follows.

$$\frac{\partial \pi_A}{\partial a_0} = \frac{(1+2S)\left[(A_2+A_3)\left(x_A^A(2+D_2)+x_A^W\right)+(A_1-c)\right]}{1-2SD_2} > 0 \qquad \text{(A23)}$$

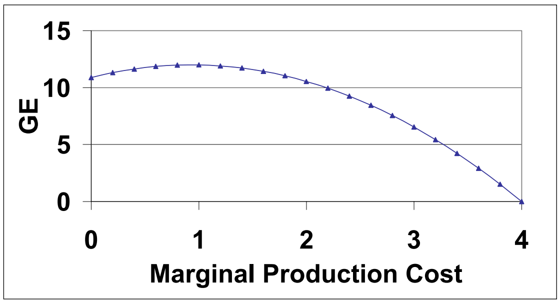
$$\frac{\partial GE_A}{\partial a_0} = (A_2+A_3)x_A^A + \frac{\partial x_A^A}{\partial a_0}\left(2x_A^A(A_2+A_3)+A_1\right) > 0 \qquad \text{(A24)}$$

Mantin (2001) shows that NDC_A is a quadratic function of a_0, that $NDC_A = 0$ at $a_0 = -c(a_1 + \alpha_1) > 0$ and $a_0 = -c(2S^0 + 1)(a_1 + \alpha_1) - (2cS^0 a_1)/(2S^0 + 1) > 0$, and that NDC_A reaches its maximum inside this interval.

The effects of a change in *c* on profits, government expenditure and NDC are as follows. It is straightforward to show that optimal profit, government expenditure and NDC are all quadratic functions of *c* (see Mantin (2001)). The optimal profit is positive at $c = 0$ and is a decreasing function of *c* as long as the optimal price is larger than *c*. When $S^0 < -a_1 / 2(a_1 + \alpha_1)$, GE_A increases for small values of *c* and then declines. When $S^0 \geq -a_1 / 2(a_1 + \alpha_1)$, GE_A decreases for all (relevant) values of *c* ($GE_A = 0$ when $c = -(a_0 /(a_1 + \alpha_1))$). Figure A2 summarizes these results.

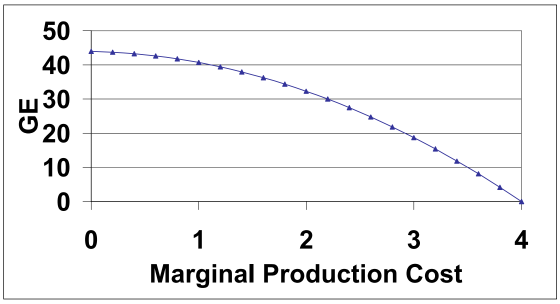
Figure A2: Government expenditure (GE) vs. marginal production cost (*c*)

$$S^0 < -\frac{\alpha_1}{2(a_1 + \alpha_1)}$$



Graph data: a_0=20, a_1=-1.5, α_1=10, S^0=0.3

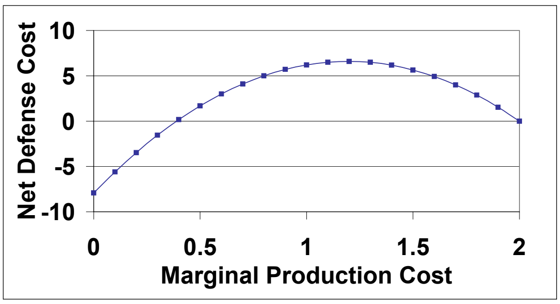
$$S^0 \geq -\frac{\alpha_1}{2(a_1 + \alpha_1)}$$



Graph data: a_0=10, a_1=-15, α_1=10, S^0=1.1

In addition, for small values of *c* NDC_A is an increasing function of *c* and for some $c \geq c_0$ NDC_A is a decreasing, function of *c*. The maximal value of NDC_A is obtained for a value of *c* for which $p > c$. This result is summarized in Figure 3A.

Figure A3: Net defense cost (NDC) vs. marginal production cost (c)



data: a_0=20, a_1=-15, α_1=5, S^0=1.5

Finally, using (A11) in (5), (6), (7) and (A13) it is straightforward to show that, under symmetry, the equilibrium price is higher, exports are lower, the government commitment is lower and production is lower under ***WP*** than under ***MU***.

b. Solution of the model: K and N firms in countries *A* and *B*

The solution of the model for K+N different firms (and K+N different demand functions) follows the same steps as in Section 4. The expressions for the solution are, however, cumbersome and difficult to use in both empirical and theoretical analysis since they include a multitude of (unknown) parameters. Thus, in order to provide insight into the effect of the number of firms in the defense market, we invoke the symmetry assumption. That is, we assume that all firms, separately in each country, are identical. In addition, we assume that the commitment of each government is divided equally among all the defense firms of the country ($x_{A1}^A = x_{A2}^A = ... = x_{AK}^A$, $x_{B1}^B = x_{B2}^B = ... = x_{BN}^B$). Specifically, symmetry implies:

$a_{k0} = a_0 > 0,\ k = 1,...,K\ ,\ b_{n0} = b_0 > 0\quad n = 1,...,N$,

$a_{kj} = a > 0,\ \forall k \neq j,\ k = 1,...,K\ ,\ b_{nj} = b > 0,\ \forall n \neq j,\ n = 1,...,N$,

$a_{kk} = \overline{a} < 0,\ k = 1,...,K\ ,\ b_{nn} = \overline{b} < 0,\ n = 1,...,N$,

$\alpha_{kj} = \alpha > 0,\ k, j = 1,...,K\ ,\beta_{nj} = \beta > 0,\ n, j = 1,...,N$,

$c_{Ak} = c_A > 0,\ k = 1,...,K\ ,c_{Bn} = c_B > 0,\ n = 1,...,N$.

The solution of the model under *WP*

<u>Stage 2</u>

Setting $\partial\pi_{Ak}/\partial p_k = 0,\ k = 1,...,K$ and $\partial\pi_{Bn}/\partial q_n = 0, n = 1,...,N$, imposing symmetry (after the differentiation), and solving (the K+N first order conditions) yields identical prices (denoted p) for the defense goods of the K firms in country ***A***, and identical prices (denoted q) for the N defense firms in country ***B***. Specifically, we obtain the following reaction functions:

$$p = \frac{\overline{a}c_A - a_0}{2\overline{a} + (K-1)a} - \frac{1}{2\overline{a} + (K-1)a}(x_A^A / K) - \frac{N\alpha}{2\overline{a} + (K-1)a}q\,, \tag{A25a}$$

$$q = \frac{c_B\overline{b} - b_0}{2\overline{b} + (N-1)b} - \frac{1}{2\overline{b} + (N-1)b}(x_B^B / N) - \frac{K\beta}{2\overline{b} + (N-1)b}p\,. \tag{A25b}$$

Solving (A25) for *p* and *q* yields the solution of the second stage:

$$p = A_1 + A_2(x_A^A / K) + A_3(x_B^B / N) \quad \text{(A26a)}$$

$$q = B_1 + B_2(x_A^A / K) + B_3(x_B^B / N) \quad \text{(A26b)}$$

where:

$$A_1 \equiv \frac{\sigma}{\varphi} > 0,\ A_2 \equiv -\frac{\gamma}{\varphi} > 0,\ A_3 \equiv \frac{N\alpha}{\varphi} > 0,\ B_1 \equiv \frac{\omega}{\varphi} > 0,\ B_2 \equiv \frac{K\beta}{\varphi} > 0,\ B_3 \equiv -\frac{\lambda}{\varphi} > 0,$$

and, $\sigma = (\overline{a}c_A - a_0)\gamma - N\alpha(\overline{b}c_B - b_0) > 0$, $\omega = (\overline{b}c_B - b_0)\lambda - K\beta(\overline{a}c_A - a_0) > 0$,

$\varphi = \lambda\gamma - N\alpha K\beta > 0$, $\lambda = 2\overline{a} + (K-1)a < 0$, $\gamma = 2\overline{b} + (N-1)b < 0$.

$A_1 > 0$ and $B_1 > 0$ imply $p > 0$ and $q > 0$. Substituting p and q in (A26) into the demand functions (8) we obtain,

$$x_{Ai}^W = \widetilde{A}_1 + \widetilde{A}_2 x_{Ai}^A + \widetilde{A}_3 x_{Bi}^B \quad \text{(A27a)}$$

$$x_{Bi}^W = \widetilde{B}_1 + \widetilde{B}_2 x_{Ai}^A + \widetilde{B}_3 x_{Bi}^B \quad \text{(A27b)}$$

where:

$\widetilde{A}_1 \equiv a_0 + (\overline{a} + (K-1)a)A_1 + N\alpha B_1$ $\qquad$ $\widetilde{B}_1 \equiv b_0 + K\beta A_1 + (\overline{b} + (N-1)b)B_1$

$\widetilde{A}_2 \equiv (\overline{a} + (K-1)a)A_2 + N\alpha B_2$ $\qquad$ $\widetilde{B}_2 \equiv K\beta A_2 + (\overline{b} + (N-1)b)B_2$

$\widetilde{A}_3 \equiv (\overline{a} + (K-1)a)A_3 + N\alpha B_3$ $\qquad$ $\widetilde{B}_3 \equiv K\beta A_3 + (\overline{b} + (N-1)b)B_3$

The analytical properties of the parameters of the demand functions (A27) are not easy to derive. However, if N=K, all cross-price effects are identical ($\alpha = \beta$) and the demand functions in ***A*** and ***B*** are identical ($\overline{a} = \overline{b}$ and $a = b$), then it is straightforward to verify that $\widetilde{A}_3 > 0$, $\widetilde{B}_2 > 0$, $\widetilde{A}_2 < 0$ and $\widetilde{B}_3 < 0$. Finally, positive exports to the rest of the world ($x_{Ai}^W > 0, x_{Bi}^W > 0$) imply $\widetilde{A}_1 > 0$ and $\widetilde{B}_1 > 0$.

Stage 1:

Substituting expressions (A27) into the definitions of the security (1) we obtain the solution of stage 1 (the commitments of the governments) as follows:

$$x_A^A = \frac{S_A^0 D_1}{1 - \frac{S_B^0 D_3}{N} - \frac{S_A^0 D_2}{K}}, \qquad x_B^B = \frac{S_B^0 D_1}{1 - \frac{S_B^0 D_3}{N} - \frac{S_A^0 D_2}{K}} \quad \text{(A28)}$$

where $D_1 \equiv K\widetilde{A}_1 + N\widetilde{B}_1 > 0$, $D_2 \equiv K\widetilde{A}_2 + N\widetilde{B}_2 < 0$ and $D_3 \equiv K\widetilde{A}_3 + N\widetilde{B}_3 < 0$.

c. Construction of the parameters of the demand and cost function for N+K firms

In order to meaningfully compare the results of the model under different number of firms, but a given size of the market, we need to construct the parameters of the demand and cost functions in a manner appropriate for a fixed size market. In the following we shall assume symmetry as in part b of the Appendix. That is, suppose that we have N firms in ***A*** and N firms in ***B***. Then, we assume:

$S_A^0 = S_B^0 \equiv S^0$, $\quad x_{A1}^A = x_{A2}^A = ... = x_{AK}^A$, $\quad x_{B1}^B = x_{B2}^B = ... = x_{BN}^B$, $a_{i0} = b_{i0} \equiv a_0^N > 0 \ \forall i$,

$a_{ij} = b_{ij} \equiv \alpha_1^N > 0 \ \forall i \neq j$, $a_{ii} = b_{ii} \equiv a_1^N < 0 \ \forall i$, $c_{Ai} = c_{Bi} \equiv c > 0 \ \forall i$.

Using these assumptions in (A26) implies $p = q$.

The demand functions for the defense good in a market which consists of only two firms (a monopoly in ***A*** and a monopoly in ***B***) are given by:

$$x_{A1}^W = x_{B1}^W = a_0^1 + (a_1^1 + \alpha_1^1)p \,. \qquad \text{(A29)}$$

The demand functions for the defense good in a market which consists of 2N firms (N firms in ***A*** and N firms in ***B***) are given by:

$$x_{Ai}^W = x_{Bi}^W = a_0^N + \left(a_1^N + (2N-1)\alpha_1^N\right)p \,. \qquad \text{(A30)}$$

To preserve consistency in the demand functions of the rest of the world when the number of firms changes, we determine that the sum of the parameters in each demand function will be the same for any number of firms in the market (the overall "size" of the market demand is not affected by the number of firms in the market). That is:

$$2Na_0^N = 2a_0^1 \quad \text{and} \quad 2N\left(a_1^N + (2N-1)\alpha_1^N\right) = 2(a_1^1 + \alpha_1^1) \,. \qquad \text{(A31)}$$

If the values of the parameters of the demand functions in a market which consists of two firms only (one firm in each country) are known (say, a_0^1, a_1^1, α_1^1), the parameters of the demand functions in a market with 2N firms are computed as follows:

$$a_0^N = a_0^1 / N \,, \qquad \text{(A32a)}$$

$$a_1^N = (a_1^1 + \alpha_1^1)/N - (2N-1)\alpha_1^N \,, \qquad \text{(A32b)}$$

and the cross-price effect, α_1^N, is any given constant ($\alpha_1^1 = \alpha_1^N$ for all N).

d. Calibration of the parameters for the real-world simulations

The model's parameters can be calibrated in several ways. Here we chose to present simulations of the model with N firms in ***A*** and K firms in ***B*** under symmetry. That is, we use the assumptions presented in part b of the Appendix. In other words, we assume that all firms, separately in each country, are identical. In addition, we assume that the commitment of each government is divided equally among all the defense firms of the country ($x_{A1}^{A} = ... = x_{AK}^{A}$, $x_{B1}^{B} = ... = x_{BN}^{B}$). Symmetry implies:

$a_{k0} = a_0 > 0$, $k = 1,...,K$, $b_{n0} = b_0 > 0$ $n = 1,...,N$, $a_{kj} = a > 0$, $\forall k \neq j$, $k = 1,...,K$,

$b_{nj} = b > 0$, $\forall n \neq j$, $n = 1,...,N$, $a_{kk} = \overline{a} < 0$, $k = 1,...,K$, $b_{nn} = \overline{b} < 0$, $n = 1,...,n$,

$\alpha_{kj} = \alpha > 0$, $k, j = 1,...,K$, $\beta_{nj} = \beta > 0$, $n, j = 1,...,N$, $c_{Ak} = c_A > 0$, $k = 1,...,K$,

$c_{Bn} = c_B > 0$, $n = 1,...,N$.

Generally, one can identify the relevant parameters by using the demand functions (8) and the expressions for the solutions of stages 1 and 2 (expressions (A26), (A27) and (A28)) using several years of data. Specifically, consistent data on local purchases and exports of the USA and Western Europe (x_A^A, x_B^B, x_A^W, x_B^W) are available for 1990-2000 (see Mantin (2001)).

According to Kirkpatrick (1995), the unit cost of defense systems (mainly planes) increases, on average, by 9% to 11.5% annually. Assuming that the American defense good is somewhat higher in quality than that of Western Europe, we set the price index for the US defense good at 100 in 1994, and that of the Western European defense good at 90.

Golde (2001) estimated the ratio of marginal production cost to the price of the defense good for 1994-1999. This estimate for 1999 is about 0.8. We set the US price of the defense good at 100. Thus, marginal production cost is calibrated to equal 80 in both markets.

The number of defense firms in 2000 in each bloc is assumed to be four (*Lockheed Martin, Boeing, Raytheon* and *Northrop Grumman* in the USA, and *BAE Systems, EADS, Thales* and *DaimlerChrysler* in Western Europe).

The security levels (S_A^0 and S_B^0) are derived from SIPRI 2001 data (for 2000):

	Defense Expenditure in 2000 (billions of US$, 1998 prices)	**Security Level**
USA	280.6	0.95
Western Europe	180	0.61
"The rest of the world"	295.4	-

Thus, the target security level of the USA is set equal to 1 and that of Western Europe to 0.6.

For the simulations in this paper we estimated the parameters of the demand functions (using N=4, K=4, $p = 100$, $q = 90$, $c_{US} = 80$, $c_{WU} = 80$, $S_{US}^0 = 1.0$, $S_{WE}^0 = 0.6$ for several groups of years during 1992-1999) and several values of the annual rate of change in real prices of weapon systems (from an increase of 3% per year to an increase of 13% per year).

The simulation results were quite robust for the various sets of parameter estimates. The preferred set of parameter values (using data for 1994-1996) of the demand functions under ***WP*** is[18]:

	Demand Const. (a_0^4, b_0^4)	Own Effect $(\bar{a}^4, \bar{b}^4)$	Cross Effect (α^4, β^4)
USA	94.308	-1.914	0.168
Western Europe	90.674	-2.557	0.224

The preferred set of parameter values (using data for 1994-1996) of the demand functions under ***MU*** is:

	Demand Const. (a_0^4, b_0^4)	Own Effect $(\bar{a}^4, \bar{b}^4)$	Cross Effect (α^4, β^4)
USA	42.612	-0.686	0.060
Western Europe	44.664	-1.083	0.095

[18] The simulations in the text employ this data set and another one which assumes a much smaller increase in the price of weapon systems over the relevant period.