Convergence towards a Steady State Distribution

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Abstract

The convergence literature frequently presupposes some unidentified steady state distribution. This paper presents a new method to identify the presence and rate of convergence to a steady state distribution. The method is illustrated with application to UK regional male wages.

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1. Introduction

Although not exclusive to the economic growth literature, studies that investigate convergence frequently study the movement over time of a cross-section of economies towards a steady state distribution that is neither explicitly presented nor detailed. Studies often find the rate of convergence of the distribution to the hypothetical steady state is about 2% per annum (Barro, 1991; Barro and Sala-i-Martin, 1994), but there is a growing literature which suggests otherwise; examples include the identification of non-convergent economies (Togo, 2002) and swings in the trend from convergence to divergence and back again (Webber and White, 2003). Moreover, the gap might be getting wider in some sense but converging in another; this might occur when the differences between successive distributions is widening due to continual growth even though the ratios are diminishing over time (see Webber and White, 2005, for an analysis of ‘strong’ and ‘weak’ definitions of convergence).

A sample of economies could plausibly be moving towards a steady state distribution that has a greater standard deviation, more positive skew or lower kurtosis value than at present. The purpose of this paper is to propose a new method that will permit an investigation into whether or not a distribution is converging to some limiting constant value (indicating convergence in ratio). For illustrative purposes the method is applied to data for average male manufacturing wages across regions of the UK. The rate at which the sample converges towards its steady state is also investigated.

This definition of convergence is different from that which typically has connotations for the gap between two economies getting smaller and ultimately the gap being equal to zero. If the gap between two economies becomes successively smaller over time then it follows that the ratio of rewards tends to some finite value. However if the ratio of rewards between two economies tends to some limiting value then it does not necessarily follow that the gap is getting smaller. Convergence in ratios may still apply to systems which show continual growth or contraction (or other distributional features) and which may have sporadic “shocks” to the system.

2. Notation

Consider a set of K economies with rewards at time t denoted by $Y_{k,t}$ ($k=1,\cdots\cdots,K;\ t=1,\cdots\cdots,T$). Let $Y_{[k,t]}$ denote the *k*-th percentile of the distribution at time *t* and let $R_{[k,k-l,t]}=\dfrac{Y_{[k,t]}}{Y_{[k-l,t]}}>1$ where $k>l$. That is to say, for the distribution of rewards at time *t*, $R_{[k,k-l,t]}$ denotes the ratio of rewards of the percentiles indexed by *k* relative to the percentile indexed by *k*-*l*. Consider a model of the form

$$R_{[k,k-l,t]}=\Delta_{k,l}-R_{[k,k-l,t-1]}\exp(\beta_0+\beta_1 t) \qquad (t=1,.....,T) \qquad (1)$$

Note that in (1), if $\beta_1<0$ then in the limit as $t\rightarrow\infty$, $R_{[k,k-l,t]}\rightarrow\Delta_{k,l}$. That is to say under this condition the two percentile converge in ratio to $\Delta_{k,l}$ and that the rate of convergence is dictated by β_1. Note that in (1) if $\beta_1>0$ then in the limit as $t\rightarrow\infty$, $R_{k,k-l,t}$ is unbounded indicating divergence in ratios.

Fitting model (1) to sample data and comparing their relative merits indicates whether the ratio of percentiles tend to indicate convergence or divergence in the relative ratio of percentiles. Using percentiles means that general inferences about the distribution may be made. Rather than use percentiles of the distribution actual observations of two economies *k* and *l* may be used and this would mean that highly specific inferences about the economies that comprise the distribution could be made.

3. Results

An analysis of average male wages across regions of the United Kingdom is important because it provides information that can be used when considering inter-regional re-distributive policies. In an attempt to identify whether average male regional wages are tending to converge in ratio, the above model was fitted using data on average male wages across eleven regions of the UK between 1981 and 2002 (*t* = 1, ……, 22). Data were collected from the New Earnings Survey. In this sample the average male wage for the Greater London area is consistently higher than every other region and for this reason each of the other 10 regions were compared to the Greater London area using the models referred to in Section 2. Model fitting was performed using the non-linear regression routine in SPSS. The results of the empirical analysis are presented in Table 1.

Table 1: Fitted models relative to Greater London using non-linear regression

Region	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\Delta}_{K,l}$	Steady State	R^2 (%)
South East	-1.827	-0.059	1.345	74.3	91.0
West Midlands	-1.585	-0.118	1.488	67.2	87.6
North West	-1.350	-0.089	1.513	66.1	88.6
Scotland	-1.073	-0.112	1.521	65.7	93.3
East Midlands	-1.227	-0.081	1.607	62.2	91.8
Yorkshire and Humberside	-1.025	-0.082	1.644	60.8	94.0
Wales	-1.016	-0.101	1.645	60.8	90.3
East Anglia	-1.024	-0.050	1.673	59.8	94.8
North East	-0.883	-0.073	1.703	58.7	91.2
South West	-0.990	-0.041	1.717	58.2	89.4

Parameter estimates for β_0 and β_1 in Table 1 are all statistically significantly different from zero and in each case the negative sign for β_1 shows that the ratio of rewards for Greater London and each region converge to some limiting finite value shown by $\hat{\Delta}_{K,l}$. In each model the coefficient of determination (R^2) is high and range from 87.6 to 94.8. This suggests a high level of within sample predictive power.

Based on the estimated parameters, $\hat{\Delta}_{K,l}$, the other regions have been ranked according to their steady state ratio from Greater London. Table 1 shows that the region which will eventually receive the lowest average male wage belongs to the Southwest, where the average male wage would be 41.8% lower than Greater London in the steady state. The Southeast can be regarded as a region with a substantial amount of workers who migrate to Greater London to take advantage of relatively high average wages and therefore also has a relatively high average male wage. All other regions cluster in a position that is about 35% less than Greater London (give or take 5%), suggesting that differences in average male wages across the UK outside of Greater London (and the Southeast) are relatively small in the steady state.

Profiles to illustrate the path of convergence to the steady state for each region are shown in Figure 1. It becomes clear that the rate of convergence to the steady state differs for each region; some regions' average wages are falling faster relative to Greater London. One region which exemplifies this issue is the Southwest; her average male wage profile falls slower than the other regions although the fall lasts for a longer period of time until she eventually becomes the region with the lowest average regional male wage. This is in accordance with column 4 in Table 1, which provides a description of the steady state position.

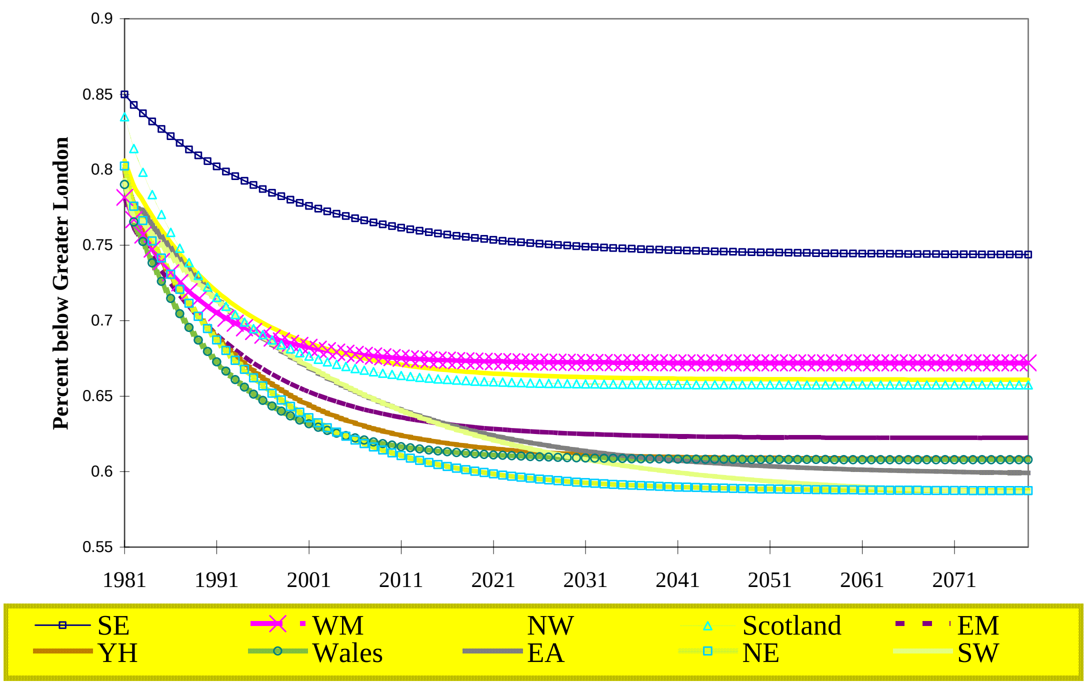

Figure 1. Summary of fitted models.

4. Conclusion

The purpose of this paper was to present a new method that is designed to identify the steady state position of a sample and the rate that each economy converges to their individual steady state position. The method is applied to average male regional wage data for the UK.

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