How Strong is the Relationship between Defence Expenditure and Private Consumption? Evidence from the United States

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Abstract

A long run conditional demand model is specified to provide empirical evidence on the relationship between government defence expenditure and private consumption in the United States. By assuming that government defence expenditure is exogenously determined with respect to private consumption decisions, the empirical results show a significant impact on the utility function of households and substitutable or complementary effects for specific categories of private expenditure. The findings are in line with the evidence that in aggregate it is possible to obtain a weak impact of defence expenditure on consumption.

KEY WORDS: Military Expenditure, Consumption, crowding out/in

J.E.L.: D12, H31, H4

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1. Introduction

The empirical analyses carried out by Ashauer (1985), Barro (1981) and Kormendi (1983) have shown that government consumption activities are transformed by a direct channel into consumption of goods and services for private use, with significant substitution effects between aggregate consumption and government expenditure.

Meanwhile, the public economic literature on the same topic has highlighted that the division of government consumption into functional categories provides mixed results concerning their impact on private expenditure (Graham, 1993; Ni, 1995; Karras, 1994; Kuehlwein, 1998; Fiorito and Kollintzas, 2004). It is evidently important for policy makers to know the way in which fiscal policy affects the level of private expenditure on goods and services as well as whether or not composition changes lead to re-allocative effects on consumption.

An important question raised by the aforementioned issue is the specific nature of the relationship between government defence expenditure and the private components of households’ consumption. While the impact of military expenditure on output, private investments and growth is well-known by means of crowding-out effects (Dunne, 1990; Gerace, 2002; Gadea et al., 2004; Atesoglu, 2002; Dunne et al., 2002; Dunne et al. 2005; Dunne et al., 2002), few investigations have been carried out to assess its direct influence on the economic agent welfare and on households’ decisions regarding consumption. In particular the investigation carried out by Smith (1977) testing if investments (private and non-military public physical capital formation) or consumption (non-military government expenditure and private consumption) fluctuate in response to military expenditure changes, concluded that the expansion of the military sector in the United States generated the reduction of private investments. Instead of this empirical result, Boulding (1973) and Edelstein (1990) found little evidence of a military expenditure-investment trade-off, while they highlighted that consumption fluctuated in response to military expenditure changes. Gold (1997) has stressed the previous hypothesis by a time series approach. Not-rejecting a cointegration relationship between military expenditure and private consumption in the United States, he confirmed the role of military expenditure in the allocation of non-military government expenditure and private consumption.
Following a utility-maximization process, the model proposed in this paper assumes a significant long run relationship between military expenditure and private consumption. Contrary to the orthodox analysis of military expenditure, in which private consumption is “jointly determined with decisions on the appropriate level of government activity” (Fleissig and Rossana, 2003), the theoretical background of this paper refers to the theory of consumer behaviour under quantity constraints that considers government expenditure exogenous to optimizing households’ consumption (Pollak, 1969, 1971; Neary and Roberts, 1980; Deaton, 1981; Deaton and Muellbauer, 1981). It is worth remarking that with respect to previous works (Smith, 1977, 1980), private decisions in households’ expenditure are fully separated from government’s non-military expenditure. In fact the allocation process of both non-military and military expenditures is a priori determined by government.

Two empirical remarks are needed. Firstly, with respect to the “trade-off defence-consumption approach” (Smith, 1977;1980), the private category of non-durables and services has been separated. The decomposition of private consumption is a useful way to measure the different impacts of defence expenditure on heterogeneous categories of consumption. Secondly, while empirical macroeconomic investigations have found significant relationships between similar categories of private and public goods and services (Karras, 1994; Ni, 1995; Kuehlwein, 1998), the net substitutability or complementarity effects between private categories and military expenditure cannot be directly tested since the defence expenditure category does not contain a counterpart in private components. On the other hand, it could be interesting to assess the effects of defence expenditure changes on their private counterpart of non-military government expenditure, i.e. expenditure on medical care and recreation, since a shock in defence expenditure could generate significant substitutability or complementarity effects on private components by the income effect.

The organization of the remainder of this paper is as follows. Section 2 presents the econometric specification and the identification of a long run demand system conditioned to government defence expenditure, in which budget shares of private categories, relative prices and income, as well as government defence expenditure are assumed to be non-stationary\(^2\). Thus, the defence expenditure enters the exact-identified cointegrating private demand system as an exogenous \(I(1)\) forcing variable (Johansen,
1992; Boswijk, 1994, 1995; Urbain, 1995; Harbo et al., 1998; Pesaran and Shin, 2002). Section 3 describes data and presents the unit-root and cointegration tests. Section 4 reports the estimation results and shed some light on the nature of the substitutability and complementarity effects between government defence expenditure and the different categories of the private demand system. Concluding remarks are offered in Section 5.

2. Theory and Econometric Specification

Recently, there has been a renewed academic interest in estimating dynamic demand models. Static specification of demand models have resulted in rejecting the theoretical restrictions of homogeneity and symmetry or have provided different elasticity estimations for the same sample, making the robustness of estimations questionable. One strand of the literature has shown that the previous problems were due to the inability to specify long run dynamic adjustments (Blundell, 1988, Ng, 1995), while other research programs concentrated their efforts on accounting for endogeneity of prices and total expenditure (Keen, 1986), or on specifying private demand models with non-separability preferences with respect to the labour supply (Browning and Meghir, 1991) or government expenditure (Deaton, 1981).

Since in this paper we are interested in analyzing substitutability or complementarity effects of a specific government expenditure, i.e. the effects of federal defence expenditure in the United States on private consumption categories, we extend the theory of consumer behaviour under quantity constraints (Pollak, 1969, 1971) in a long run perspective embodying together the previous suggestions. Private consumption expenditures are treated as choice variables by the representative household in an optimizing framework, while government defence expenditure is exogenously established by government decisions, although they could directly confer utility on the consumer.

Moreover, given this setting, the existence of statistical properties that assume private demand system variables as well as government defence expenditure to be non-stationary leads us to derive a long run conditional demand model by means of identifying restrictions of a vector error correction model (VECM). In the next sub-
paragraphs, the theory and the econometric specification of the long run conditional demand model are discussed in depth.

2.1. The demand system

As usual in the demand system estimation it is assumed that consumers follow a two-stage budgeting process. They decide how much to save and how much to spend on leisure and only in the second stage do they allocate the total expenditure among private goods and services. Thus, if a representative household obtains utility from goods and services provided by the government, the separability hypothesis cannot not be reasonable.

Following Pollak (1971), the model assumes that a representative household wishes to maximize the utility function in which the vector of government expenditures are rationed goods, i.e. preferences are not parameterized. Formally:

\[
\max \left[ u = u(q; g) \right] \quad \text{s.t. } q' p = e
\]

in which \( q \) is a vector of non durable goods and services, \( p \) represents the vector of prices of the freely chosen goods and \( e \) equals disposable income; \( g \) denotes the vector of quantities of publicly provided goods and services and is assumed to be entirely financed by tax revenue.

Solving the constrained maximization problem, we derive the following uncompensated conditional demand function:

\[
q = q(e, p; g)
\]

The government provision of goods and services exerts two effects on the demand for privately purchased goods: an income effect, whereby an increase in the provision financed by taxing reduces the amount of income available to purchase freely chosen items, and a net substitution/complementarity effect, whereby the consumer rearranges his expenditures on freely chosen goods following a change in the quantity constraint.

There is a testable implication that arises from a conditional demand function. The standard demand function \( q = q(e, p) \) is simply a special case of function [2], correctly specified when the vector of government expenditure \( g \) is separable from private expenditures \( q \). Thus, even though our specific investigation concerns how government defence expenditure affects private expenditures, we need to test preliminarily the separability hypothesis between private components of expenditure and government
defence expenditure. In order to do this, the flexible functional form of a demand system is specified as an Almost Ideal Demand System (Deaton and Muellbauer, 1980), in which government defence expenditure enters the utility function. This conditional demand specification, by a first order approximation of any demand system, provides a test to establish whether households obtain utility from the services provided by defence expenditures and calculations to determine substitution or complementarity effects.

The cost function of the conditional Almost Ideal Demand System can be written as:

$$\log C(u, p, g) = \alpha_0 + \sum_{i=1}^{n} (\alpha_i + \sum_{j=1}^{m} \theta_{ij} g) \log p_i + \frac{1}{2} \sum_{i=1}^{n} \sum_{k=1}^{m} \gamma_{ik} \log p_i \log p_k + u(g, g) \beta \prod_{i=1}^{n} p_i^{\beta_i} \quad [3]$$

Investigating the government defence expenditure in which \( g_j = 1 \), the cost function \( C(u, p; g) \) is minimized, given market prices of the privately purchased goods, yielding demand equations in terms of budget shares:

$$w_i = \alpha_i + \sum_{k=1}^{n} \gamma_{ik} \ln p_k + \beta_i \left[ \log e - \log P \right] + \theta_i g, \quad i, k = 1, 2, ..., n \quad [4]$$

where \( \gamma_{ik} = (1/2)(\gamma_{ik}^* + \gamma_{ik}^*) \), \( p_k \) is the relative price of the \( k \)-th good, \( e \) represents total per capita expenditure, \( \log P = \alpha_0 + \sum_{i=1}^{n} (\alpha_i + \theta g) \log p_i + (1/2) \sum_{i=1}^{n} \sum_{j=1}^{m} \gamma_{ij} \log p_i \log p_k \) is a functional form usually approximated by the Stone index \( \left( \sum_{i=1}^{n} w_i \log p_i \right) \). The theoretical constraints of adding-up, homogeneity and symmetry imply restrictions that are directly imposed on the parameters of private goods of the model and a demand system that is singular by construction. In order to avoid any related problems, the usual procedure consists in dropping one equation from the system.

Finally, \( g \) represents an index of the real defence expenditure and the associated parameter is used to test separability from private consumption. Given a particular level of the production capacity, the resources for military expenditure can be obtained at the expense of consumption rather than investments, government expenditure or the balance of payments (Dunne, 1990), so that substitutability effects between government defence expenditure and aggregate private consumption are generally expected. For the United States, Edelstein (1990) shows that military expenditure crowded out the private economy at the expense of consumption, while its effects on investments were statistically significant only in the short run. In order to evaluate the disaggregate effects of military expenditure on private consumption, it must be kept in mind that the relative
responses to military expenditure shocks can determine complementarity behaviours in both the substitutability effects if the defence consumption has a pattern that is positively correlated with the disaggregated private category.

2.2. Identifying a long run conditional demand model

This sub-section provides support for specifying an empirical model in which demand adjusts gradually over time in response to shifts in relative prices (Pollak and Wales, 1992) and to exogenous shocks (Ng, 1997). It is known that, in order to recover statistical properties of many macroeconomic variables, long run models have been improved by the cointegration analysis in a vector autoregressive (VAR) approach. However, sometimes the theory suggests that one or more variables could be exogenously given in the (long run) model, so that a symmetric endogenous treatment of variables does not seem to be necessary. The validity of excluding variables from the cointegrating relationships or the assumption of a priori hypotheses of exogeneity leads to specify a long run conditional model.

Formally, a conditional vector error correction model (VECMc) is written as a parameterisation of the \( p \)-th order of a Gaussian VAR (Johansen, 1995; Urbain, 1995; Ericsson, 1995):

\[
\Delta y_t = \gamma_0 + \gamma_1 t + \Psi \Delta z_t + \sum_{i=1}^{p-1} \Phi_i \Delta x_{t-i} + \Pi_y x_{t-1} + u_t
\] [5]

in which \( \Pi_y \) is a conditional long run structural matrix, \( \Phi \) is the short-run matrix, \( \gamma_0 \) is a constant term, \( \gamma_1 \) is the coefficient of the deterministic trend and \( \Psi \) is the contemporaneous coefficient matrix of the exogenous variable. In Appendix A.1, a conditional (and marginal) long run model is formally derived and discussed. By Granger’s theorem of representation of the conditional model [5], the \( \Pi_y \) matrix is decomposed as \( \Pi_y = \alpha_y \beta' \) by the \( n \times r \) matrix of loadings \( \alpha_y \) and the \( m \times r \) matrix of cointegrating vectors \( \beta' \).

In line with the aim of this study to investigate substitutability (or complementarity) relationships between private categories of consumption and government defence expenditure, the appropriate method of allowing for such effects is to exogenously include an integrated version of the \( I(0) \) variables for government defence expenditure...
in the long run demand system context specified as in Pesaran and Shin (2002). As mentioned before, including government defence expenditure as an exogenous \(I(1)\) forcing variable was motivated by theory and the statistical properties of defence expenditure. It is worth remarking that in this framework private demand system categories are modelled to have an insignificant impact on the long run evolution of military expenditure.

Disregarding deterministic terms, the \(r\) cointegrating vectors of the AI model, which is conditional to military expenditure can be written:

\[
\text{cv}(r) = \beta' x_{t-1} = \beta' (w_{t-1}, ..., w_{n-t-1}, \ln p_{t-1}, ..., \ln p_{n-t-1}, \ln(e_{t-1} / P_{t-1}); g_t)
\]

Exact and over-identifying restrictions must be imposed on \(\beta'\) vectors, in order to recover a conditional demand system from the VECM, to test predictions of consumer theory and to examine long run relationships of substitutability or complementarity with government defence expenditure.

The adding-up constraint becomes a crucial theoretical assumption to identify a long run demand system specified in terms of budget shares (\(w_i\)). A rank condition is that the number of cointegrating relationships in the VECM should be equal to the number of non-singular demand equations \((r = n - 1)\). On the other hand, the aforementioned identifying restrictions exclude all the cases where \(r < n - 1\). Moreover, the exact identification of the cointegrating relations [6] requires the imposition of at least \(r\) \textit{a priori} restrictions on the parameters of each cointegrating vector in which the \(\text{rank}\{R(I_r \otimes \beta)\} = r^2\) (order conditions). In particular, the cointegrating relationships of the long run AI demand system in \(n\) equations require \(r^2 = (n-1)^2\) restrictions, so that their choice follows the adjustment dynamic of the non-singular budget shares in the long run equilibrium. Formally:

\[
H_{EI} = \begin{cases} 
\beta_{11} = -1 & \beta_{12} = 0 & \ldots & \beta_{1n-1} = 0 \\
\beta_{21} = 0 & \beta_{22} = -1 & \ldots & \beta_{2n-1} = 0 \\
\vdots & \vdots & \ddots & \vdots \\
\beta_{n-11} = 0 & \beta_{n-12} = 0 & \ldots & \beta_{n-1n-1} = -1 
\end{cases}
\]

[7]
Secondly, the theoretical hypotheses of homogeneity and symmetry in a conditional long run can be tested by imposing over-identifying restrictions directly on $\beta'$, in which the parameters of interest for the conditional demand system are those included in the cointegrating vectors. The maximum likelihood estimator has been found to be consistent in order to estimate $\beta'$ conditioned for government defence expenditure, allowing us to make a correct inference on the cointegrating vector from the conditional model. The critical values for the rank test have been taken from Harbo et al. (1998)\(^5\).

Finally, from the empirical point of view, the inclusion of military expenditure as an additional exogenous $I(1)$ forcing variable allows us to measure the direct effect of government defence expenditure on private spending by the estimated parameters $\theta$\(^6\). The statistical assumption of a non-stationary behaviour for military expenditure seems to be appropriate since the allocation process of the policy maker follows the erratic dynamics of war-events or external threats.

Moreover, including the defence expenditure as a forcing variable in the long run demand system, a current (short run) expenditure shock in the critical periods (wars or external threats) generates an overshooting of the optimal long run level in the private allocation of household’s expenditures by the substitution (or complementarity) effects. The theoretical motivation lies in the fact that the commitment process, involved into war or national security programmes, generates an uncontrollable defence expenditure level in the future, so that policy makers under-discount this uncertainty because of precautionary motivations. Thus, this behaviour generates a misallocation of optimal defence expenditure, so that effective military expenditure input lowers with respect to current defence expenditure (Dunne et al., 1984).

In regards to the remark that within demand system framework the private consumption categories can simultaneously provide substitution or complementarity effects with respect to government defence expenditure, we stress that if this derivative of demand equations in terms of budget shares with respect to $g$ ($\partial w_i / \partial g$) is positive, goods $i$ and $k$ are defined as complements, while if this magnitude is negative they are substitute goods.
3. Data, Unit Root and Cointegration Tests

In this application a long run conditional demand model for three categories of private expenditure in the United States economy is identified by the cointegration test, extended to consider the effects of defence expenditure. Quarterly data (1957:1 to 2005:4) from the National Income and Product Accounts (NIPA) are used for private consumption and government defence expenditure. The functional classification used in this study for private consumption expenditures divides the aggregate into the categories of non-durables, services and medical care and recreation. This representation of private expenditures is a useful alternative in order to estimate preference parameters and to evaluate the substitution or complementarity effects of military expenditure on consumption. It is a stylized fact that, in developed economies, the functional category of private services has become ever more important with respect to non-durable goods with the consequence that their aggregation leads to a biased average effect on the relationship of substitutability or complementarity. Moreover, there is a great deal of evidence that supports the finding that there is a significant relationship between the functional categories of government expenditure and private expenditure with high substitutability in the expenditure categories that gives a similar utility (Karras, 1994; Ni, 1995; Kuehlwein, 1998). On the other hand, the defence expenditure category is a public good that does not contain a counterpart in private components. It is expected that the private medical care and recreation category, which contains a counterpart in non-military government expenditure, remains unaffected by a change in defence expenditure. Therefore, by including the latter private category in demand system, it is possible to test as a limited case how strong the income effect deriving from a change in defence expenditure is.

The variables of the demand system are completed by including per-capita real consumption expenditure \(e\). Prices are accounted for by their implicit deflators. Finally, defence expenditure is measured as the cost of inputs at the constant 2000 United States dollars and transformed into a quantity index. All variables used for estimations are adjusted for seasonality.

The descriptive analysis for the United States shows that the real private expenditure, used as a proxy for real income, has consistently grown in the last twenty years. This
increased purchasing power is expected to be devoted mostly to the consumption of merit goods and services rather than non-durable goods. These expectations are statistically confirmed in Figure 1, where the pattern of the budget share of non-durable consumption becomes relatively small, while we note an increase in the budget shares of the other private categories.

Figure 1 – *Budget share dynamics of the private categories of the demand system*

Figure 2 reports the time-series plots of the share of federal defence expenditure with respect to aggregate private consumption and real defence expenditure. What is immediately evident is that, while the share of defence has declined consistently in the last fifty years, the profile of the real United States federal defence expenditure appears to be event-driven (Gerace, 2002). Cyclical spikes corresponding to wars (or threat of Wars) are in opposition to a stationary use in the defence goods sector associated with a non-increasing defence expenditure in peace time. The levels of government defence expenditures show that a first peak in the data is accounted by the Vietnam War, the second peak by the re-emergence of Cold War Reagan while, after a ten-year downturn, the federal defence expenditure had an upswing in response to terrorist attacks.
Clearly, the United States financed its post-2\textsuperscript{nd} world war federal defence partly through deficits. This suggests the interesting hypothesis that long run defence expenditure might have crowded out (or crowded in) significant proportions of the private households’ expenditure by the consumption channel, larger in the period when the government defence expenditure is higher.

Figure 2 - \textit{Profiles of real Defence expenditure and the relative share of the United States Federal Defence with respect to private consumption}

Table 1 shows estimated parameters of time trends the private expenditure categories for the full sample and for a sub-sample that brings together the aforementioned critical wars or threat periods linked with higher levels of government defence expenditure. Considering that the estimated parameter of a time trend can be interpreted as a deterministic dynamic adjustment (Ng, 1995), the response parameters have absolute values higher in the sample connected with the critical (wars or threats) periods with respect to the estimated parameters in the full sample. The mechanism in action, generated by a (positive) shock in defence expenditure, leads to overshooting private categories of consumption in the short run\textsuperscript{6}, while their long run adjustments is achieved with a quicker dynamic.
Table 1 – The estimated parameters for time trends of the private categories

<table>
<thead>
<tr>
<th>Private expenditures</th>
<th>Full Sample</th>
<th>Sub-Sample War-Threats Periods</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Non-durables</td>
<td>-0.0128</td>
<td>-0.0273</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>(2) Medical Care &amp; Recreation</td>
<td>0.0009</td>
<td>0.0021</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>(3) Services</td>
<td>0.0003</td>
<td>0.0006</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
</tbody>
</table>

Note: standard errors in round brackets

In order to test the previous hypothesis, the statistical properties of the variables of [5] are investigated by the augmented Dickey-Fuller unit root test. The results reported in Appendix 2 concerning budget shares ($w_{1t}$, $w_{2t}$, $w_{3t}$), relative prices ($\log p_{1t}$, $\log p_{2t}$, $\log p_{3t}$), real total expenditure ($\log e_t - \log P_t$) and real defence expenditure ($g$) (run with a constant, and then a constant and trend, and with lags selected by using Schwarz information criterion in the equation) do not reject a unit root in all cases. Thus, we can test cointegration in the conditional demand system to identify an error correction demand system. This requires two cointegrating relationships among the six endogenous variables of the system, corresponding to the two non-singular budget share equations. A VECM that identifies a structural demand system is estimated with restricted intercepts and no trend is specified to ensure that steady state values for the budget shares exist both under the null and the alternative hypotheses (Pesaran and Shin, 2002). Indeed, the consumer theory predicts that budget shares converge (in the long run) towards a steady state value proxied by their parameters of constants.

The test for the presence of cointegrating relationships among the private demand variables is developed by using an LR test based on the maximal-eigenvalue of stochastic matrix (Johansen, 1995), in which the critical values are modified to take into account the exogenous defence expenditure $I(I)$ forcing variables (Harbo et al., 1998). Since the asymptotic distribution of the LR test statistic for cointegration does not have standard distribution and strictly depends on the assumptions made with respect to both the lag length and the deterministic components of the model, it is possible to determine
the rank of the long run multiplier matrix, $\Pi_y$, only when the lag in the VAR specification is defined.

The results of the cointegration tests presented in Table 2 do not reject the exact identification hypothesis of two cointegration relationships, supporting the necessary condition for the specification of a conditional cointegrated demand system$^9$.

Table 2 – Johansen’s Cointegration Rank Tests of Demand System conditional on Defence Expenditure

<table>
<thead>
<tr>
<th>No. of CE(s)</th>
<th>Max-Eigen. Statistic</th>
<th>95% critical value</th>
<th>90% critical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0$</td>
<td>$H_1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r = 0$</td>
<td>$r = 1$</td>
<td>81.1148</td>
<td>43.76</td>
</tr>
<tr>
<td>$r \leq 1$</td>
<td>$r = 2$</td>
<td>47.729</td>
<td>37.48</td>
</tr>
<tr>
<td>$r \leq 2$</td>
<td>$r = 3$</td>
<td>28.7732</td>
<td>31.48</td>
</tr>
<tr>
<td>$r \leq 3$</td>
<td>$r = 4$</td>
<td>20.1276</td>
<td>25.54</td>
</tr>
<tr>
<td>$r \leq 4$</td>
<td>$r = 5$</td>
<td>14.2694</td>
<td>18.88</td>
</tr>
<tr>
<td>$r \leq 5$</td>
<td>$r = 6$</td>
<td>6.8364</td>
<td>12.45</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>No. of CE(s)</th>
<th>Trace Statistic</th>
<th>95% critical value</th>
<th>90% critical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0$</td>
<td>$H_1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r = 0$</td>
<td>$r \geq 1$</td>
<td>198.8504</td>
<td>116.3</td>
</tr>
<tr>
<td>$r \leq 1$</td>
<td>$r \geq 2$</td>
<td>117.7356</td>
<td>86.58</td>
</tr>
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<td>$r \leq 2$</td>
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<td>60.0066</td>
<td>62.75</td>
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<td>$r \geq 4$</td>
<td>31.2334</td>
<td>42.4</td>
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<td>$r \geq 5$</td>
<td>15.1058</td>
<td>25.23</td>
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<td>$r \leq 5$</td>
<td>$r \geq 6$</td>
<td>6.8364</td>
<td>12.45</td>
</tr>
</tbody>
</table>

Note: the 95 and 90 percent critical values used for the rank cointegration test with exogenous I(1) are taken from Harbo et al. (1998).

4. Results

The long run estimated parameters of the conditional demand system provide a more complete representation of private consumption, that assume a significant influence of government defence expenditure. As is common in an extended version of demand analysis, before presenting the estimation, the separability hypothesis between government defence expenditure and private components of consumption is verified by an LR test for the estimated equations. The LR value equal to 62.8 (p-value=0.00092) is
higher than a one percent critical value, clearly rejecting the hypothesis of separability for the demand system and unambiguously indicating that consumer preferences are affected by federal defence expenditure.

In the first two columns of Table 3, the cointegrating vectors estimated for non-durables and medical care and recreation equations are reported with homogeneity and symmetry imposed in the parameters of the long run, while the services category is omitted and its parameters are then obtained by the adding-up constraints and shown in the third column.

<table>
<thead>
<tr>
<th></th>
<th>Vector 1</th>
<th>Vector 2</th>
<th>Vector 3*</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_1$</td>
<td>-1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$w_2$</td>
<td>0</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>$w_3$</td>
<td>0</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>$\log p_1$</td>
<td>.46341</td>
<td>-.41521</td>
<td>-.04819</td>
</tr>
<tr>
<td></td>
<td>(.0472)</td>
<td>(.0451)</td>
<td>(.0870)</td>
</tr>
<tr>
<td>$\log p_2$</td>
<td>-.41521</td>
<td>.12548</td>
<td>.28974</td>
</tr>
<tr>
<td></td>
<td>(.0451)</td>
<td>(.0486)</td>
<td>(.0848)</td>
</tr>
<tr>
<td>$\log p_3$</td>
<td>-.04820</td>
<td>.28973</td>
<td>-.24154</td>
</tr>
<tr>
<td></td>
<td>(.0870)</td>
<td>(.0848)</td>
<td>(.1664)</td>
</tr>
<tr>
<td>Income</td>
<td>.15976</td>
<td>.00998</td>
<td>-.16975</td>
</tr>
<tr>
<td></td>
<td>(.0258)</td>
<td>(.0331)</td>
<td>(.0398)</td>
</tr>
<tr>
<td>$g$</td>
<td>.48670</td>
<td>-.08431</td>
<td>-.40240</td>
</tr>
<tr>
<td></td>
<td>(.1297)</td>
<td>(.1818)</td>
<td>(.1922)</td>
</tr>
<tr>
<td>Intercept</td>
<td>.36890</td>
<td>.17333</td>
<td>.45778</td>
</tr>
<tr>
<td></td>
<td>(.0136)</td>
<td>(.0185)</td>
<td>(.0204)</td>
</tr>
</tbody>
</table>

Notes: standard errors in round brackets.

*Since the third cointegrating vector is a linear combination of the first two vectors the parameters of the third are obtained estimating the first and third equation of the long run demand. Standard errors in round brackets are asymptotically derived.

Analysing the estimated results, it is possible to notice that the prices and total expenditure coefficients are almost all significant in both equations. The positive estimated parameter of government defence consumption in the non-durables equation
provides evidence of a significant complementary relationship, while it appears statistically irrelevant for decisions of households in the medical care and recreation category. The estimated parameter for services is negative and highly significant, showing the existence of a substitutability effect between defence expenditure and services consumption\(^{10}\). In order to interpret the estimation results of defence expenditure in separated cointegrating vectors, it is necessary to keep in mind that the substitution or complementarity effects represent the utility demand for private goods \(i\) when the amount of government defence expenditure changes. Thus, since the intercept parameter in the cointegrated demand system represents a proxy for the level of steady state, we can check the non-separability effects of military expenditure by the shift of the estimated constant of the cointegrating vector with respect to the mean of the budget shares. It is possible to highlight that the effects of government defence expenditure on nondurable goods shift down to the steady state level. In fact, comparing the expected nondurable budget share for the separable specification (0.42) with the estimated steady state of nondurable category (0.369), we can remark that the added utility of military expenditure reduces the (relative) necessitate of allocation of the nondurable expenditure. On the contrary, substitution effects of defence expenditure on the services category provides support for a positive correction of long run equilibrium. The estimated intercept of the cointregtaing vectors (7) is 0.457, four points higher than the expected budget share of services (0.416) in a separable long run demand system of private goods.

The non-separability test and the almost significant estimated parameters of the cointegrating equations lead to a correct interpretation of the elasticity results of the conditional specification. Table 4 reports the estimated price and income elasticities of the private categories and the elasticity of substitution or complementarity relative to private consumption, in which asymptotic standard errors and the confidence intervals of elasticities are derived by bootstrap replications of the estimated parameters and standard errors.

Coherent with the theory the estimated price-elasticities are all negative in the diagonal matrix. This result shows that there are no violations of monotonicity and in general the elasticity sizes are well defined and plausible. Moreover, the compensated elasticity values are always lower than the uncompensated ones (available on request),
so that coherence of quasi-concavity on the parametric specification is implied. Finally, the response to income changes shows that the high aggregation of the private categories reduces the variability in the classification of goods as being necessary or luxury.

Table 4 – Long run estimated elasticities from the conditional demand system

<table>
<thead>
<tr>
<th>Private expenditures</th>
<th>Marshallian Price Elasticities</th>
<th>Income Elasticities</th>
<th>Defence Exp.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>(1) Non-durables</td>
<td>-0.068</td>
<td>-1.037</td>
<td>-0.268</td>
</tr>
<tr>
<td></td>
<td>(0.113)</td>
<td>(0.106)</td>
<td>(0.206)</td>
</tr>
<tr>
<td>(2) Medical care and Recreation</td>
<td>-2.602</td>
<td>-0.226</td>
<td>1.775</td>
</tr>
<tr>
<td></td>
<td>(0.292)</td>
<td>(0.004)</td>
<td>(0.533)</td>
</tr>
<tr>
<td>(3) Services</td>
<td>0.058</td>
<td>0.764</td>
<td>-1.413</td>
</tr>
<tr>
<td></td>
<td>(0.211)</td>
<td>(0.205)</td>
<td>(0.405)</td>
</tr>
</tbody>
</table>

Note: standard errors in round brackets

The estimated elasticities between defence expenditure and private consumption, computed at the sample mean, confirm the significant complementarity effects of defence consumption for non-durables (.12) while the substitutability relationships are obtained for the services (-.102) and medical care and recreation (-.055) categories. Thus, as expected, a shock to defence expenditure has opposite effects on the main categories of private consumption. The estimations are coherent with the results obtained by Kuehlwein (1998) using different aggregates of government expenditures. On the one hand, the results show that government defence expenditure directly enters the utility function of households by means of their significant relationship with non-durables and services. On the other hand, in order to compare the results obtained with the previous empirical defence literature, the aggregate effect of crowding out/in is obtained by using the sum of the average substitution elasticities found for each equation since, according to the definition of elasticity, each estimated parameter $\theta_y$ is weighted by the budget share. The computed aggregate substitution value, which is equal to -0.037 does not exclude a large amount of substitutability but, in aggregate, is
coherent with the perspective of a significant but weak relationship of government defence expenditure with respect to private decisions.

However, if it is relevant for policy makers to know the path of elasticity by means of demand side effects caused by the way preferences respond to a shift in defence expenditure, scholars also need to know why the consumption shares have an aggregate flat fluctuation in response to defence expenditure shocks.

The dynamics of the elasticity of private goods of the demand system are illustrated in Figure 3. Considering that if the absolute value of elasticity grows over time, so that the shift in the defence expenditure has a greater effects on private goods, the dynamic response of non-durable goods is very different from the way the consumers’ expenditure on services reacts to changes in defence expenditure.

Taking the longest view first, Figure 3a suggests a slightly growing pattern of complementary elasticity for non-durables, in which the peaks linked with war events are evident. The re-allocative effects of the budget shares shown in the descriptive analysis justify the complementarity relationship between defence expenditure and non-durables. With a decreasing trend of non-durables, a positive (negative) shock in defence expenditure in war time produces, in the short run, a (positive) overshooting of the private category with respect to the long run adjustment proxied by the trend coefficient as shown in Table 1. Moreover, with a negative cycle there is a reduction in the aggregate private expenditure, generating not only a stop in the decreasing of non-durables but, relative to the services category, an increase in the budget shares in the next one or two periods.

This empirical finding is particularly evident in the first two periods with high government defence expenditure, where the increase in defence expenditure has generated complementarity for non-durables. Coherent with the descriptive part in the war events, after this shock, the long run steady state leads to reallocating private goods more quickly in order to achieve a desired given level of private consumption. The range of long run complementarity elasticity, from 0.1 to 0.16, is coherent with the results found in the relationships between components of government expenditure and private consumption (Ni, 1995), in which the highest levels of elasticity are connected with the war events of the United States.
Instead of the relationship between defence and non-durable goods, the dynamic of elasticity between defence expenditure and service expenditure confirms the stability of the substitutability effect is around -0.12/-0.14 (Figure 3b). Coherent with the analysis
for non-durables, we find the services expenditure category to be substituted in the short run, overshooting the long run equilibrium. Thus, in order to achieve the long run steady state, the adjustment has to be stronger in the periods, when military expenditures are higher than in the overall sample analyzed. The crowding out effects find support from the significant statistical elasticities. An increase in government defence expenditure reduces (substitutes) private service consumption, so that consumers’ preferences have a relevant role in determining the nature of the relationship.

Finally, Figure 3c shows that the medical care and recreation category is unaffected by changes in defence spending. Since the elasticity is closed between slight substitutability (-0.05) and zero, with standard errors relatively large, we conclude that the impact of the government policy on allocating federal defence expenditure does not involve a significant relationship between defence expenditure and private medical care and recreation.

5. Conclusions

This paper provides some empirical evidence on the relationship between government defence expenditure and some categories of private consumption using United States data. In a first step, the exact identification of a long run conditional demand system is evaluated by using a cointegration approach. Secondly, by imposing the theoretical restrictions and specifying government defence expenditure as a non-stationary exogenous forcing variable, the elasticities of substitution or complementarity are obtained from the maximum likelihood estimated parameters of the long run conditional demand model.

The empirical results indicate that government defence expenditure enters the utility function of households, conditioning their decisions. This result implies that a significant relationship exists between defence expenditure and macro-categories of private consumption. The aggregate estimated elasticity between defence expenditure and private consumption is not completely in line with the previous empirical literature, since is lower in absolute terms. Moreover, by disaggregating private consumption, a complementary effect has been found for the non-durable category, while a substitution
effect characterizes the relationship between defence expenditure and private spending on services.

These findings are in favour of a more cautious attitude of government defence expenditure to produce in aggregate direct crowding out effects. Government defence expenditure changes have shown that the impact is quickly absorbed by long run dynamics of private consumption, so that military expenditure has only a slight influence on private decisions of consumption expenditures.
Footnotes

1 The terms government defence expenditure and military expenditure will be used as synonyms throughout the paper.
2 Many recent demand studies have confirmed non-stationary characteristics of the variables of demand system (Ng, 1995; Attfield, 1997; Lewbel and Ng, 2000; Pesaran and Shin, 2002).
3 A first order approximation of the private demand system, allows us to consistently estimate the effects for each period of the sample by means of the assessment of the curvature conditions required by the theory.
4 Formally:

Adding up: \( \sum_i \alpha_i = 1; \ \sum_i \gamma_{ik} = 0; \ \sum_i \beta_i = 0; \ \sum_i \theta_i = 0, \forall k, j \)
Homogeneity: \( \sum_i \gamma_{ik} = 0, \ \forall i \)
Symmetry: \( \gamma_{ik} = \gamma_{ki}, \ \forall i, k \)

Unlike homogeneity and symmetry, the adding-up constraint is endogenously satisfied.
5 The homogeneity and symmetry in the long run demand specification can be tested using the log-likelihood ratio statistic which is asymptotically distributed as a \( \chi^2 \) with degrees of freedom being equal to the number of over-identifying restrictions imposed.
6 Not-rejecting the separability hypothesis in the long run demand system, a nested private demand system is derived in which the relationships in the private utility function of households are negligible.
7 In this study we consider the federal defence expenditure for non-durable goods and services.
8 It is worth to compare the interruption of trends of the private expenditure categories at the start of high levels of defence expenditure linked with the critical (wars or threats) periods.
9 The selection of the order \( p \) is crucial in the VECM specification since it may affect the cointegration rank test. The VAR order is selected by using the Akaike Information Criterion (AIC), the Schwarz Bayesian Criterion (SBC) and a small-sample corrected likelihood ratio test. The results univocally indicate that the optimal lag order is equal to five ( \( p = 5 \)).
10 The standard errors are asymptotically calculated.
As mentioned before, in the United States only a part of the wars was financed by public debt.

The vectors and matrices of parameters in equation (A.12) are partitioned as:

\[ b_0 = (b'_{j0}, b'_{20})', \quad b_1 = (b'_{j1}, b'_{21})', \quad \Pi = (\Pi'_2, \Pi'_1)', \quad \Gamma_i = (\Gamma'_3, \Gamma'_2)' . \]
References


Appendix A.1

In this appendix a conditional long run model is derived from an unrestricted VAR when one or more variables are considered strictly exogenous. Applied to the long run demand system a VAR($p$) model is specified for the $m \times 1$ vector of variables $x_i$:

$$A_0 x_t = a_0 + a_1 t + A_1 x_{t-1} + ... + A_p x_{t-p} + \xi_t, \quad t = 1, 2, ..., T$$  \[A.1\]

where $a_0$ is a constant term, $a_i$ is the coefficient of the deterministic trend, $A_i$ ($i=1, 2, ..., p$) is a $m \times m$ matrix of unknown parameters and $A_0$ is a non-singular matrix. Since both private consumption variables and public consumption expenditures are found non-stationary, so that cointegration in private expenditure categories is a necessary condition for estimating a long run dynamic demand system. Rewriting the VAR($p$) (A.1) as a vector error correction model (VECM) it is obtained the following structure:

$$\Delta x_t = b_0 + b_1 t + \Pi x_{t-1} + \Gamma_1 \Delta x_{t-1} + ... + \Gamma_{p-1} \Delta x_{t-p-1} + \varepsilon_t$$  \[A.2\]

where $\Pi$ is the long run impact matrix which describes the long run relationships among the variables and $\Gamma_i$ are the short-run impact matrices, which account for the effects of short-run dynamics. The other elements of equation (A.2) represent the constant term ($b_0 = A_0^{-1} a_0$), the deterministic trend ($b_1 = A_0^{-1} a_1$) and the disturbances $\varepsilon_t = A_0^{-1} \xi_t$ (with $\varepsilon_t \sim iid(0, \Lambda)$).

In order to rewrite (A.2) as a conditional error correction model, we partition the $m \times 1$ vector $x_t$ into the $n \times 1$ vector $y_t$ and the $k \times 1$ vector $z_t$, that is $x_t = (y_t', z_t')'$, $t = 1, 2, ..., T$. By partitioning the error term $\varepsilon_t$ (and its covariance matrix) conformably to $x_t$ as $\varepsilon_t = (\varepsilon_t', \varepsilon_{zt}')'$, it is possible to express $\varepsilon_{zt}$ conditionally on $\varepsilon_{zt}$ as:

$$\varepsilon_{zt} = \Lambda_{zt} \varepsilon_{zt} + u_t$$  \[A.3\]

where the innovations $u_t$ are distributed as $N(0, \Lambda_{uu})$, with $\Lambda_{uu} = \Lambda_{yy} - \Lambda_{yz} \Lambda_{zz}^{-1} \Lambda_{zy}$, and are independent of $\varepsilon_{zt}$. Substituting (A.3) into (A.2), together with a similar partitioning of the other vectors and matrices of parameters\textsuperscript{12}, and assuming the process
\( \{z_t\}_{t=1}^\infty \) as weakly exogenous with respect to the long run impact matrix \( \Pi \), \textit{i.e.} \( \Pi_z = 0 \), we obtain a conditional long run structural matrix \( \Pi_{y,z} = \Pi_y \).

Rearranging the parameters, we obtain the conditional and marginal equations, respectively:

\[
\Delta y_t = \gamma_0 y_t + \gamma_1 z_t + \Psi \Delta z_t + \sum_{i=1}^{p-1} \Phi_i \Delta x_{t-i} + \Pi_y x_{t-i} + u_t \tag{A.4}
\]

\[
\Delta z_t = b_{z0} + \sum_{i=1}^{p-1} \Gamma_{zi} \Delta x_{t-i} + \epsilon_{zt} \tag{A.5}
\]

where \( \gamma_0 = -\Pi_y a_0 + (\Gamma_y - \Lambda_{zz}^{-1} \Gamma_z + \Pi_y) a_1 \), \( \gamma_1 = -\Pi_y a_1 \) and \( \Phi_i \equiv \Gamma_{yi} - \Lambda_{zz}^{-1} \Gamma_{zi} \).

The restriction \( \Pi_z = 0 \) clearly excludes cointegrating relationships in the marginal model (A.5). Moreover, this restriction makes the information available from model (A.5) redundant for efficient estimation and inference on \( \Pi_y \) as well as on \( \gamma_0 \), \( \gamma_1 \), \( \Psi \) and \( \Phi_i \). Thus, in line with Granger and Lin (1995), we define \( \{z_t\}_{t=1}^\infty \) as long run forcing for \( y_t \). By Granger’s theorem of representation constrained to the conditional model (A.4), the \( \Pi_y \) matrix is decomposed as \( \Pi_y = \alpha_y \beta' \) by the \( n \times r \) loadings matrix \( \alpha_y \) and the \( m \times r \) matrix of cointegrating vectors \( \beta' \). The estimated parameters of represent our interest to test the substitutability or complemenetarity effects in the relationship between government defence expenditure and the components of private consumption.