Game Theory and Oligopoly

- Decision makers with strategic interdependence i.e. outcome of one depends on the decisions of the other
- Rather different to the simple market analysis we have used so far
- If we have 2 firms A and B with outputs perfect substitutes, identical constant MC of £1 and zero fixed costs and market demand

  $$\begin{aligned} p &= 41 - q \\ q &= q_A + q_B \end{aligned}$$

  we can show that the total output that maximises their joint profit

  $$40q - q^2$$

  is 20 units
- In many cases gives insights into market interactions
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- Abstract game:: described by key elements that determine its solution.
  - set of players: decision takers
  - set of actions
  - timing of choices
  - payoffs of players
  - set of strategies available determine how game will be played:
    * pure is complete specification of action at each point: simultaneous or sequential
    * mixed is probability distribution over pure strategies: defines probability which each pure strategy will be played
  - information set: compete or incomplete
  - feasibility of binding commitments:
    * cooperative games: all players can commit to choosing a particular strategy
    * Non-cooperative: cant commit
- Two ways of representing games
  - Extensive form: tree
  - Intensive form: matrix

- **Prisoners dilemma:** most common example two prisoners in separate cells , they have been arrested for robbery, but evidence not adequate to convince and need a confession. If they don't confess they get away with it and if one confesses they get off lightly, but if both confess they both suffer...

		B	
		Confess	Don't confess
A	Confess	10, 10	20, 0
	Don't confess	0, 20	0, 0

  Best solution is confess but...... So expected outcome is the both confess -dominant strategy

- Consider the firms A and B with each firm having a capacity of 12 units and only know what output the other has produced after they have made their own output decision. If they only consider 2 output levels 10 units and 12 units then they can collude to maximise joint profits with equal shares, or one can renege on agreement and maximise its own profits and produce 12. So payoff matrix

		Firm B	
		10	12
Firm A	10	200.200	180,216
	12	216,180	192,192

  now regardless of what B chooses As 'dominant strategy' is to choose 12. So in the absence of a binding agreement both will choose 12, but best outcome is to choose 10

- **Stackelberg game:** Leader-Follower game. If no capacity constraints and firm A produces today, so commits to some output level, and firm B tomorrow, so responding. Being the leader is an advantage. Suppose firm A considers producing 20 or 13.5,then B has to consider what A has done and respond giving 4 possible strategies for B

  - 1. if A chooses 20, B can choose 10; if A choose13.3 , B can choose 10: call $s_1^B = (10, 10)$
    2. if A chooses 20, B can choose 10; if A choose13.3 , B can choose 13.5: call $s_2^B = (10, 13.5)$
    3. if A chooses 20, B can choose 13.3; if A choose10 , B can choose 10: call $s_3^B = (13.5, 10)$
    4. if A chooses 20, B can choose 13.3; if A choose110 , B can choose 13.5: call $s_4^B = (13.5, 13.5)$

Giving with some rounding

			Firm B			
			A produces 20		A produces 10	
			s_1^B	s_2^B	s_3^B	s_4^B
			10	13.3	10	13.3
Firm A	20	s_1^A	200,100	200,100	133,89	133,89
	13.3	s_2^A	222,167	177,177	222,167	177,177

Extensive form suggests if A produces 20 the best B can do is produce 10, giving A a payoff of 200, but if A produces 13.3, B will also produce 13.3 giving A payoff of 177. If A predicts what B will do then A will produce 20.

Nash equilibrium: Clearly there is no dominant strategy as in the prisoners dilemma, so need concept of Nash equilibrium. Let

$$v^A(s_i^A, s_i^B) and\ v^B(s_i^A, s_i^B)$$

be the payoffs to A and B when A chooses strategy s_i^A and B s_i^B A strategy pair $v^A(s_n^A, s_n^B)$is a Nash equilibrium when:

$$v^A(s_n^A, s_n^B) \geqslant\ (s_i^A, s_n^B); i = 1, 2$$

and

$$v^A(s_n^A, s_n^B) \geqslant\ v^B(s_n^A, s_i^B); i = 1, 4$$

that is the Nash equilibrium is the mutual best replies to the others actions. In the payoff matrix highest payoff to A is highest value in column and highest payoff to B the highest value in the row

		Firm B			
		s_1^B	s_2^B	s_3^B	s_4^B
Firm A	s_1^A	200,**100**	**200,100**	133,89	133,89
	s_2^A	**222**,167	177,**177**	**222**,167	**177,177**

- so $s_1^A$is the best reply to s_2^B and $s_2^A$the best reply to the rest of Bs strategies, while s_1^B and s_2^B are the best replies to s_1^A and s_3^B and s_4^B are the best replies to s_2^A.These are given in bold and show two Nash equilibria $(s_1^A s_2^B)$ and $(s_2^A s_4^B)$
  - $(s_1^A s_2^B)$ is when A chooses 20 and B chooses 10, which can see is reasonable in extensive form
  - $(s_2^A s_4^B)$ is when A chooses 13.3 and the best B can do is choose 13.3,
  - but one of these is not a credible Nash equilibrium: s_4^B means B produces 13.3 if A produces 20 and this will mean that A will get 133 rather than the 200, so can think of this as a threat by B to get A to produce less. If A believes the threat it should produce less, but B will do better by producing 10 when A produces 20, so it is not a credible threat.

- So only one of the two Nash equilibria is reasonable: A would not produce 13.3

- **Entry game:** Consider firm A as an incumbent monopolist and firm B a potential entrant, who must decide whether to build up capacity and enter and if so decide how much to produce. Neither firm knows what the other has produced until total supply is on the market and price determined.

  - simplify by assuming B produces 13.3 if it enters or none at all
  - if both simultaneously choose when firm B enters then only Nash equil will be (13.3,13.3)
  - consider that this is what firm B offers firm A this outcome and that firm A then decides across three possible outcomes accommodating = 13.3; monopoly=20; warlike =27.7. The last will lead to losses for both.
  - game of perfect info. From extensive form if B enters with 13.5 As best response is to accommodate
  - considering strategic form (B has 2 possible outcomes, but A now has nine we search for nash equilibria

			Firm B	
			$s_1^B = 0$	$s_2^B = 13.3$
	Firm A	$s_1^A = (13.3, 13.3)$	356,0	**178.178**
		$s_1^A = (13.3, 20)$	356,0	133,**89**
		$s_1^A = (13.3, 27.6)$	356,0	-27.7,-13.3
a		$s_1^A = (20, 13.3)$	**400**,0	**178.178**
		$s_1^A = (20, 20)$	**400**,0	133,**89**
		$s_1^A = (13.3, 27.6)$	**400**,0	-13.3
		$s_1^A = (27.6, 13.3)$	341,0	**178.178**
		$s_1^A = (27.6, 20)$	341,0	133,**89**
		$s_1^A = (27.6, 27.6)$	341,**0**	-13.3

  there are 4, but only one seems plausible if reason as before, but can formalise how to do this

- **Sub game perfect equilibrium:** use equilibrium concept of strict dominance implies also nash equilibrium,

  - but some nash equilibria not plausible: threats not credible
  - define sub game as a game that begins at a certain node and contains all the info sets that can be reached from that node
    * prisoners dilemma has no sub games
    * Stackelberg has 2 sub games: B's choice between 10 and 13.3

    * Entry game has 2 sub games: A's choice between 13.3, 20, and 27.6

  - sub game perfect equilibrium is a list of strategies (one for each player) which must yield a Nash equilibrium in every sub game of the game (NB the game is actually a sub game of itself) Not all Nash equilibria are SPE
  - find SPE strategy by backward induction...

- **Repeated games**: Important to recognise that repeating game can make things rather different e.g. prisoners dilemma.

Oligopoly

- Market with few sellers -important point is that firms interdependent - affected or believe they are affected by other firms behaviour.
- Still try to maximise profits, but have to consider the reactions of others to changes they make in output or price
- Could model as before and gain some insights considering hypotheses on the actions and reactions of the firms, but could end up with multiple equilibria or indeterminate solutions
- But development of game theory revolutionize the study

  Consider duopoly -two firms with cost functions

$$\begin{aligned} C_i &= c_i q_i \\ c_i &> 0 \end{aligned}$$

inverse demand function is:

$$p_i = \alpha_i - \beta_i q_i - \gamma q_j$$

where $\gamma > 0$ meaning that the goods are substitutes. If perfect substitutes $\alpha_1 = \alpha_2 = \alpha$ and $\gamma = \beta_1 = \beta_2$ and

$$p = \alpha - \gamma(q_1 + q_2)$$

Consider the profit functions

$$\begin{aligned} \pi_i(q_1, q_2) &= p_i q_i - c_i q_i \\ &= (\alpha_i - \beta_i q_i - \gamma q_j) q_i - \beta_i q_i^2 \end{aligned}$$

if not perfect subs can get demand function from the inverse demand function:

$$q_i = q_i(p_1, p_2) = a_i - b_i q_i - \varphi q_j$$

note the profit function is strictly concave in own output with given other company output, and linear and decreasing in other company output for fixed own output and maximum profit is given by

$$q_i = \frac{\alpha_i - c_i - \gamma q_j}{2\beta_i}$$

Now consider some models using this framework

Cournot Model

Assume each firm makes a decision on the output it will produce without consultation the prices are then determined in the market by the interaction of this supply with demand and the firms make profits. The maximum profit output gives firm 1 best output given firm 2s decision

$$q_i = \frac{\alpha_i - c_i - \gamma q_j}{2\beta_i}$$

so this is really a response function and we can write it as

$$\begin{aligned} q_i &= A_i - B_i q_j \\ A_i &= \frac{\alpha_i - c_i}{2\beta_i}; B_i = \frac{\gamma}{2\beta_i} \end{aligned}$$

The slopes are negative and we can graph easily and the intersection of the curves is

$$q = \frac{A_i - A_j B_i}{1 - B_i B_j}$$

when the outputs are homogeneous

$$q = \frac{\alpha - c}{3\gamma}$$

which Cournot proposed as equilibrium arguing as in diagram that one firm picks an output the other firm chooses its profit maximisation based on that and then the first firm responds....

- Not a convincing argument as requires outputs being chosen sequentially over time and each firm to behave myopically , expecting the other to keep output constant next time despite the fact they never do
- Game theoretic interpretation is better -the outputs the firms produce are the Nash equilibrium output choices. A Nash equilibrium pair (q_1^*, q_1^*) means:

  $$\pi_1(q_1^*, q_2^*) \geqslant \pi_1(q_1, q_2^*) \text{ and } \pi_2(q_1^*, q_2^*) \geqslant \pi_2(q_1^*, q_2)$$

  and this is clearly satisfied by the pair of outputs at the intersection. Could also consider that the firms realise that the other firm is going to react to their chice changing their output decision and realise that there is only one combination at which this will not matter/happen the intersection

- Comparing the Cournot Nash with other possible output pairs perfect competition where price equals marginal cost

$$p_i = \alpha_i - \beta_i q_i - \gamma q_j = c_i$$

$$\max[\pi_1(q_1, q_2) + \pi_2(q_1, q_2)]$$

and joint profit maximisation. Can show that when there is product differentiation the competitive output is greater than the oligopoly output which is greater than the joint profit maximising output. So if firms want to maximise profit why don't they collude -would only work if they could make a binding commitment as its a one shot game and the outcome is not a Nash equilibrium -it will always be in the interest one firm to renege.

- **Stackelberg model**

  Rather than simultaneous output choices, allow firm 1 to announce output as a market leader and firm 2 to respond

$$\max[\pi_1(q_1, q_2)] \text{ st } q_2 = A_2 - B_2 q_1$$

first order conditions

$$\begin{aligned} \alpha_1 - c_1 - \gamma q_2 - 2B_2 q_1 - \lambda B_2 &= 0 \\ -\lambda q_1 - \lambda &= 0 \\ q_2 &= A_2 - B_2 q_1 \end{aligned}$$

give firm 1s Stackelberg output as

$$q_1 = \frac{[2\beta_2(\alpha_1 - c_1) - \gamma(\alpha_2 - c_2)]}{4\beta_1\beta_2 - 2\gamma^2}$$

- which is larger than the Cournot and in the homogeneous case is

$$q_1 = \frac{\alpha - c}{2\gamma}$$

and in this case total profits are not maximised and are lower than the Cournot, but firm 1's are greater reflecting first mover advantage.

- Can see this diagrammatically... (G&R)
- **Bertrand model**

  In many oligopolistic markets forms are concerned with setting prices rather than outputs and then selling what the market demands. In monopoly can do in terms of prices or outputs and get the same result but as Bertrand showed not in oligopoly

- **Developments**

  there has been a massive growth in the analysis of oligopolistic structure using game theory, with the analysis of the effect of capacity constraints, the use of repeated games and the analysis of firm entry -industrial organisation literature deals with these see G&R, Stephen Martin's book.