1. Introduction

In this lecture we examine the theory of decision-making under uncertainty and its application to the demand for insurance. The undergraduate treatment of these topics is typically rather brief and the diagrammatic treatment doesn't allow certain interesting questions to be asked. It allows you, for example, to show that a risk averse person will prefer to take out insurance, if offered it on an actuarially fair and full basis, but it doesn't allow you to analyse how much insurance cover the individual would choose and how this desired level of cover depends on things such as the premium charged, the person's income, and so on. The undergraduate treatment also makes it difficult to analyse topics such as moral hazard and adverse selection — topics we return to later in the course.

The first part of the lecture goes through the theory of decision-making under uncertainty. The second part applies it to the demand for insurance.

2. Decision-making under uncertainty

2.1. Prospects

In the analysis of decision-making under uncertainty, we talk about "prospects". A prospect consists of a vector of probabilities, indicating the chances of each of the possible states occurring, and a vector of incomes associated with each of these states. Denoting the vector of probabilities by \( \pi \) and the vector of incomes associated with these probabilities by \( y \), we have

\[
P = (\pi, y).
\]

For example, if there are just two possible states, "bad" and "good", with probabilities \( \pi \) and \( (1-\pi) \), and the incomes associated with these two states are \( y_b^0 \) and \( y_G^0 \), we have

\[
P^0 = (\pi, y_b^0; (1-\pi), y_G^0).
\]

In the two-state case, we can show the prospect diagrammatically as in Fig 1. The horizontal axis shows income in the good state and the vertical axis income in the bad state. The 45° line shows the certainty line, since along this line income is the same in both states. Prospect \( P^0 \) lies below the certainty line, since income is greater in the good state than in the bad.
2.2. Expected income

Another important line, apart from the certainty line, is the line labelled $E^0$ passing through prospects $P^0$ and $P^1$ in Fig 1. Prospects lying on this line are all associated with the same expected income as prospects $P^0$ and $P^1$. The line $E^0$ is known as the iso-expected-income line. The individual's expected income is defined as

$$E[y^0] = \bar{y} = \pi y^0_B + (1- \pi)y^0_G = y^0_G - \pi(y^0_G - y^0_B).$$

Points on $E^0$ thus satisfy the equation

$$\pi y^0_B + (1- \pi)y^0_G = \bar{y}.$$

Totally differentiating (4) gives

$$\pi dy^0_B + (1- \pi)dy^0_G = 0,$$

from which we get the slope of the iso-expected-income line

$$\frac{dy^0_B}{dy^0_G} = -\frac{1-\pi}{\pi}.$$

The slope is often known as the "odds ratio", since this is how gambling odds are expressed. So, for a given prospect, we can always find the expected income associated with it by

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1 For example, if a bookmaker offers you odds of 10 to 1 against a certain horse winning, he is saying he will give you £10 if it wins and you will pay him £1 if it loses. Thus the odds, $(1-\pi)/\pi$, are equal to 10, and hence $\pi$, the probability of the horse winning according to the bookmaker, is equal to 0.091.
drawing an iso-expected income line from the prospect to the certainty line with a slope equal to the odds ratio. Obviously, changing the odds, will cause the iso-expected-income line to pivot around \( P_0 \) — for example, a rise in the probability of the bad event happening pivots the iso-expected-income line anti-clockwise, making the line flatter and pushing prospect \( P^1 \) towards the origin.

### 2.3. Risk aversion and indifference curves

We now need to introduce preferences. In the undergraduate textbook analysis of decision-making under conditions of certainty, we use indifference curves and assume they are convex. In the case of decision-making under uncertainty, we can, under certain assumptions, also use indifference curves. Whether or not indifference curves under uncertainty are convex to the origin depends on the individual’s attitude towards risk. If she is risk averse, they will be convex. You can see this from Fig 2. Risk aversion means that the individual would prefer prospect \( P^1 \) to prospect \( P^0 \), even though they both lie on the same iso-expected income line. Thus \( P^1 \) must lie on a higher indifference curve to \( P^0 \). Of two prospects with the same expected income, individuals thus prefer the less risky prospect.

This is an appropriate point to introduce a couple of new concepts. The first is the concept of certainty equivalent. This is the level of income, \( y^c \), associated with prospect \( P^2 \) in Fig 2 — it is the amount of income, which, if received with complete certainty, would generate the same expected utility as the uncertain prospect \( P^0 \). The second concept is that of risk premium. As we have seen, the individual would prefer the certain prospect \( P^1 \) to the uncertain prospect \( P^0 \). Suppose she faces the certain prospect \( P^1 \). The risk premium, \( r \), is the amount of income she would be prepared to give up rather than face the risky prospect \( P^0 \). Thus in the case of Fig 2, we have

\[
(14) \quad r = y - y^c.
\]

This is obviously positive in the case where the individual is risk averse.
2.4. Expected utility and the slope of the indifference curve

How do we interpret the slope of the indifference curve in Fig 2? Here we need to introduce the expected utility function. Provided certain axioms are satisfied (see GR ch 19C for details), preferences over prospects can be represented by an expected utility function, which in the two-state case, can be written:

\[
V = E(u) = \pi u(y_B) + (1-\pi)u(y_G),
\]

where \(u(y_B)\) is the utility derived from \(y_B\). The indifference curves are contours of this function, showing combinations of \(y_G\) and \(y_B\) giving rise to a given level of expected utility.

What determines the shape of the typical indifference curve of an individual? We assume \(u(.)\) is at least twice differentiable. Differentiating (15) totally and setting the derivative equal to zero yields

\[
\frac{dy_B}{dy_G} = -\frac{(1-\pi)u'(y_G)}{\pi u'(y_B)},
\]

where \(u'\) means the derivative of \(u\) and is to be interpreted as the marginal utility of income. From (16) it is clear that the slope of a person's indifference curve at any arbitrary point on the diagram will depend on the individual's attitude towards risk (i.e. on the shape of the function \(u(.)\)) and on the risk of the bad outcome occurring (i.e. on the probability \(\pi\)).

Before exploring these factors, note that at the point where the indifference curve crosses the certainty line we have \(y_G=y_B\). Thus the slope of the indifference curve at this point is equal to

\[
\frac{dy_B}{dy_G} = -\frac{(1-\pi)}{\pi}.
\]

So, where it cuts the certainty line, the indifference curve has a slope equal to the odds ratio. Indifference curves are therefore always tangential to the iso-expected income line where they cut the certainty line, irrespective of the individual’s attitude to risk as captured by her utility function \(u(.)\).

2.4.1. Attitudes to risk and indifference curves

Let’s consider, then, how the individual’s attitude to risk affects the shape of the indifference map. Suppose we have two people, one of whom is more risk averse than the other. How do their indifference curves compare? In Fig 3, person i’s indifference curve associated with prospect \(P^0\) is labelled \(I^0_i\) and person j’s by \(I^0_j\). Note that both indifference curves must have the same slope, \(-(1-\pi)/\pi\) at \(P^0\). Nonetheless, person j is more risk averse than person i, since i is indifferent between the certain prospect \(P^0\) and the risky prospect \(P^1\), whilst person j is not. Person j would want more income in at least one of the two states in order to be left indifferent between the certain prospect \(P^0\) and the risky prospect. A prospect such as \(P^2\), which has more income in both states than \(P^1\), would be regarded by j as being as
good as $P^0$. Thus the straighter the person's indifference curve is, the less risk averse they are.

We can make this a bit more precise. From Fig 3, we can see that as we approach $P^0$ from above the certainty line, the slope of $j$'s indifference curve is getting less negative faster than the slope of $i$'s is. In other words, as we approach $P^0$ from above the 45° line, the rate of change of the slope of both $i$'s and $j$'s indifference curves is positive, but is larger in $j$'s case than $i$'s. GR pp.571-2 show that on the 45° line, the rate of change of the slope of an indifference curve is equal to

\[
\frac{d^2 y_B}{dy_G^2} = -\frac{u''(y)}{u'(y)} \frac{(1 - \pi)}{\pi^2}
\]

Eqn (18), coupled with what we said earlier, also implies that the larger $A(y)$ is, the more risk averse is the individual. Recall we said above that at $P^0$ the slope of $j$'s indifference curve is getting less negative faster than the slope of the slope of $i$'s — i.e. the LHS of eqn (18) is larger for $j$ than for $i$. Since the probabilities are the same for $i$ and $j$, $A(y)$ must therefore be larger at $P^0$ for $j$ than for $i$. Thus the larger $A(y)$ is, the more risk averse is the individual. It is worth noting that $A(y)$ is a more useful measure of risk aversion than $u''$ — the measure often used in undergraduate texts. This is because whilst expected utility is unique only up to a positive linear transformation, $A(y)$ is invariant with respect to such transformations (cf. GR p.566).

2.5. The indifference map

When we're performing comparative statics, it will be useful to know more about the factors that determine the shape of the individual's map. We know that convexity of
indifference curves means risk aversion and we know that the more bowed out the individual's indifference curve is, the more risk averse she is. But what can we say about the factors that determine how the shape of her indifference curves changes as we move away from the origin?

2.5.1. **Absolute risk aversion**

Consider first the case where we increase the incomes in the two states but keep the absolute difference between them, \(|y_G - y_B|\), unchanged. (We say in such a case that we keep the individual’s absolute risk unchanged.) In Fig 5 we start at prospect \(P^0\) and move to prospect \(P^1\). In prospect \(P^1\), incomes in both states are higher, but the difference between \(y_G\) and \(y_B\) is the same in \(P^1\) as it is in \(P^0\). Along the line \(P^1P^0\), which is parallel to the certainty line, the income difference is thus a constant and we can therefore write \(y_B = y_G - k\). The slope of the indifference curves along the line \(P^1P^0\) are equal to

\[
(19) \quad \frac{dy_B}{dy_G} = \frac{-\left(1 - \pi\right)u'(y_G)}{\pi u'(y_G - k)}.
\]

Differentiating this with respect to \(y_G\) shows how the slope of the indifference curve alters as we move out along the line \(P^1P^0\). GR pp.573-4 show that this derivative can be written

\[
(20) \quad \frac{d}{dy_G} \left[\frac{-\left(1 - \pi\right)u'(y_G)}{\pi u'(y_G - k)}\right] = \frac{\left(1 - \pi\right) u'(y_G)}{\pi u'(y_B)} [A(y_G) - A(y_B)],
\]

i.e. the negative of the slope of the indifference curve times the difference between \(A(y_G)\) and \(A(y_B)\). Eqn (20) shows that if, as one moves along the line \(P^1P^0\), \(A(y)\) does not vary as income varies, the indifference curves all have the same slope — i.e. the slope of the indifference curve \(I^0\) at \(P^0\) is the same as the slope of the indifference curve \(I^1\) at \(P^1\), etc.
What does a constant value of $A(.)$ mean? We saw earlier that $A(y)$ is a measure of risk aversion and reflects the curvature of the individual’s indifference curves — a very risk averse person will have indifference curves that are very bowed in. The slope of the indifference curve tells us the individual’s willingness to accept a reduction in income in one state in exchange for an increase in income in the other state. In effect, then, it tells us the individual’s marginal willingness to accept an increase in absolute risk. Saying that $A(.)$ doesn’t vary with $y$ thus means that we’re saying that the individual’s marginal willingness to accept an increase in absolute risk doesn’t change as income in both states increases.

It may well be more realistic to assume that $A(.)$ falls as income in both states increases — i.e. the indifference curves get more bowed in as one moves out along the $45^0$ line. In this case, as one moves along $P^0P^1$, a given reduction in $y_B$ can be compensated by an ever smaller increase in $y_G$. If $A(.)$ does fall as income in the two states rises, the term in square brackets in eqn (20) will be negative and indifference curves will get steeper as one moves out along $P^0P^1$.

2.5.2. Relative risk aversion

We consider now the case where we increase the incomes in the two states but keep the relative risk (i.e. the ratio of $y_G$ to $y_B$) unchanged. This is equivalent to moving out along a ray from the origin. In effect, then, we are asking what happens to the slope of the indifference curves as we move out along such a ray — see Fig 6. Along a ray such as that passing through $P^0$ and $P^1$ in Fig 6, we have $y_G$ being proportional to $y_B$. Hence we can write $y_B = \beta y_G$. The slope of the indifference curves along the ray through $P^1$ and $P^0$ is equal to

$$\frac{dy_B}{dy_G} = -\frac{(1-\pi)u'(y_G)}{\pi u'(\beta y_G)}.$$  

Differentiating this with respect to $y_G$ shows how the slope of the indifference curve alters as we move out along the ray through $P^1$ and $P^0$. GR p.575 show that this derivative can be written

$$\frac{d}{dy_G} \left[ -\frac{(1-\pi)u'(y_G)}{\pi u'(\beta y_G)} \right] = \frac{(1-\pi)}{\pi} \frac{u'(y_G)}{u'(y_B)} \frac{1}{y_G} [R(y_G) - R(y_B)],$$

where $R(y)$ is known as the coefficient of relative risk aversion. $R(y)$ is defined as $A(y)y$ and hence is equal to $-u''(y)/u'(y)y$. This is, of course, the negative of the elasticity of the marginal utility of income. $R(y)$ thus measures the responsiveness of marginal utility to changes in income. However, unlike $A(y)$, $R(y)$ measures it in a way that is independent of the units in which income is measured. Eqn (22) shows that if $R(y)$ does not vary as income varies, as one moves along the ray through $P^0$ and $P^1$, the indifference curves all have the same slope — i.e. the slope of the indifference curve $I^0$ at $P^0$ is the same as the slope of the indifference curve $I^1$ at $P^1$, etc. Again, this may not be a realistic assumption.
3. The demand for insurance

Armed with these tools, we can proceed to examine the demand for insurance. It’s convenient to alter the notation lightly. Let’s define the incomes in the two states associated with the initial prospect $P^0$ as

\begin{align}
  y^0_G &= y \\
  y^0_B &= y - L,
\end{align}

where $L$ is the loss that would occur if an accident occurred. We can now think about the insurance contracts the individual might be offered and what her optimal response will be.

3.1. The insurance decision

3.1.1. Actuarially fair full insurance

The simplest (and least realistic!) insurance contract is one in which the individual is offered insurance against the loss occurring on a full and actuarially fair basis. This means that the insurer will reimburse the full amount of the loss and charge a premium equal to

\begin{align}
  AFP &= \pi L.
\end{align}

The incomes in the good and bad states under this insurance contract are therefore equal to:

\begin{align}
  y^1_G &= y - \pi L = \bar{y} \\
  y^1_B &= y - \pi L = \bar{y}.
\end{align}
Thus the individual receives income $\bar{y}$ with certainty if she accepts the contract. This prospect, which we’ll call $P^1$, obviously lies on the certainty line. Note that in Fig 7 the horizontal distance, $y - \bar{y}$, is equal to the insurance premium, $AFP$.

As in standard consumer theory, we analyse choices by superimposing indifference curves on budget constraints. In this case, the budget constraint consists of two points: the uninsured option ($P^0$) and the take-it-or-leave-it insurance option ($P^1$). From Fig 7, it is clear that if the individual were given a choice between these prospects, she would choose $P^1$, since this allows her to get onto a higher indifference curve. This is a familiar conclusion you almost certainly reached in undergraduate microeconomics. You can arrive at it mathematically by noting that

$$V(P^1) = \Pi(G, u(y^0)) = u(\bar{y}) = u(\Pi G, + (1 - \pi)y^0)$$ if insured.

By the definition of concavity, $V(P^1) > V(P^0)$. Note that the strict equality implies that the individual would be prepared to pay more than $AFP$ to obtain full cover.

3.1.2. Actuarially fair insurance with variable cover

Suppose now that the individual were offered insurance on an actuarially fair basis but that she has cover only for a fraction $k$ of the loss $L$. Thus if $q$ is the amount of cover,

$$q = k(y^0 - y^0) = kL.$$

It may be that $k$ is decided by the insurer, in which case $(1-k)$ is known as the coinsurance rate. Or it may be that the insured is free to choose $k$ and hence $q$. Whatever the reason, the incomes in the good and bad states are now respectively
(30) \[ y_G = y - \pi q \]

(31) \[ y_B = y - \pi q - L + q = y - L + (1 - \pi)q \]

and the expected income is

(32) \[ E[y] = (1 - \pi)(y - \pi q) + \pi(y - L + (1 - \pi)q) = y - \pi L = \bar{y}, \]

so that the expected income associated with this contract is precisely the same as that associated with actuarially fair full insurance. The difference between these two cases is that with actuarially fair full insurance, \( \bar{y} \) is received with certainty, whilst when \( q < L \), the incomes in the two states are different. By varying \( k \), we move along the iso-expected-income line \( E_0 \) in Fig 7. If \( k = 1 \), the individual obtains full cover and we get the full cover prospect \( P_1 \). If \( k = 0 \), the individual has no cover and we get back to the uninsured prospect \( P_0 \). Points on the iso-expected-income line \( E_0 \) to the north-west of \( P_0 \) thus represent the range of actuarially fair insurance policies that are feasible, varying according to the extent of cover \( q \). Note that if the individual were to end up to the north-west of \( P_1 \), her level of cover would exceed the loss she would incur if the bad outcome occurred.

Suppose the individual can choose her level of cover. We can also conclude that if given the choice between policies with different levels of cover at actuarially fair premiums, the individual would choose to take out a policy offering full cover, since at \( P_1 \) she is on a higher indifference curve than she is anywhere else along the line \( E_0 \). (Recall the individual's indifference curve is tangential to the iso-expected-income line at the certainty line.) This conclusion can be reached mathematically too. The individual will choose the level of cover that maximises her expected utility:

(33) \[ V = (1 - \pi)u(y_G) + \pi u(y_B) = (1 - \pi)u(y - \pi q) + \pi u(y - L + (1 - \pi)q). \]

The optimal level of cover satisfies the condition:

(34) \[ \frac{\partial V}{\partial q} = V_q = -(1 - \pi)u'(y - \pi q^*)\pi + \pi u'(y - L + (1 - \pi)q^*)(1 - \pi) = 0 \]

which requires that

(35) \[ u'(y - \pi q^*) = u'(y - L + (1 - \pi)q^*), \]

which in turn requires that \( q^* = L \).

3.1.3. **Loaded premiums with variable cover**

In reality the insurer is unlikely to offer the individual an insurance contract that puts her on the iso-expected-income line \( E_0 \). In general this line needs to be distinguished from the "budget line". Suppose now that the individual is not offered insurance on an actuarially fair basis, so that the premium charged is not \( AFP \) but rather

(36) \[ NAFP = pkL = pq, \]
where \( p \) is the premium rate — i.e. the amount charged per £ of cover. The incomes in the good and bad states are now respectively

\[
(37) \quad y_G = y - pq
\]

\[
(38) \quad y_B = y - pq - L + q = y - L + (1 - p)q,
\]

and the expected income associated with this contract is

\[
(39) \quad E[y] = (1 - \pi)(y - pq) + \pi(y - L + (1 - p)q) = y - \pi L - (p - \pi)q = \bar{y} - (p - \pi)q,
\]

which is less than the expected income associated with the actuarially fair contract if \( p > \pi \) (i.e. if the premium is loaded). Solving (37) for \( q \), substituting the resultant expression in (38) and then rearranging gives:

\[
(40) \quad y_B = (y - L) - \frac{(1 - p)}{p} (y_G - y),
\]

which can be interpreted as the equation of the budget line under this insurance contract, assuming \( q \) can be chosen by the insured. This is shown as \( E^1 \) in Fig 8. When \( y_G = y \), no insurance cover is obtained and \( y_B = y - L \) — here we are at point \( P^0 \). Taking out cover means reducing \( y_G \) below \( y \) but increasing \( y_B \) above \( y - L \). The ratio \( (1 - p)/p \) shows the rate at which income in the good state can be transformed into income in the bad state. The higher the premium rate, the smaller the amount of income that can be obtained in the bad state from a £1 reduction in income in the good state.

What choice will the individual now make? Suppose the premium rate, \( p \), exceeds the probability, \( \pi \), and therefore the available options are \( P^0 \) and all points to the north-west of it along the line \( E^1 \). \( P^0 \) is clearly not the best she can do — she could do better by obtaining partial insurance cover, moving from \( P^0 \) to point \( A \). This puts her on a higher indifference curve than she is at \( P^0 \).
We can derive this result mathematically. The individual will choose the level of cover that maximises her expected utility, which in this case is equal to:

\[ V = (1-\pi)u(y_c) + \pi u(y_b) = (1-\pi)u(y-pq) + \pi u(y-L+(1-p)q) \]

The optimal level of cover satisfies the condition:

\[ \frac{\partial V}{\partial q} = -(1-\pi)u'(y-pq*)p + \pi u'(y-L+(1-\pi)q*)(1-p) = 0 \]

which requires that

\[ \frac{-(1-\pi)u'(y-\pi q*)}{\pi u'(y-L+(1-\pi)q*)} = \frac{-(1-p)}{p} \]

The LHS is the slope of the indifference curve and the RHS the slope of the budget line. Rearranging (43) gives:

\[ \frac{u'(y-\pi q*)}{u'(y-L+(1-\pi)q*)} = \frac{(1-p)/p}{(1-\pi)/\pi} \]

Since \( p>\pi \), the RHS must be less than one, and so, therefore, must the LHS. This requires that

\[ u'(y-pq*) < u'(y-L+(1-p)q*) \]

which, given that \( u(.) \) is concave, in turn requires that

\[ y-pq* > y-L+(1-p)q* \]

This can only be true if \( q*<L \).

A point such as \( d \) in Fig 9 on the budget line \( E^i \) corresponds to a particular level of cover. We can find the level of cover as follows. Having a level of cover corresponding to point \( d \) reduces income in the good state by an amount equal to the premium, \( pq \), but raises income in the bad state by an amount equal to \( q-pq=q(1-p) \). Thus the distances \( af \) and \( df \) are equal to \( pq \) and \( (1-p)q \) respectively. Now draw the line \( de \) parallel to the certainty line. Clearly the angles of the triangle \( def \) are \( 45^0, 45^0 \) and \( 90^0 \). Hence the distance \( df \) must be equal to the distance \( ef \). The distance \( ea \) is thus equal to

\[ ea = (1-p)q + pq = q \]

i.e. the level of cover.
3.2. Comparative statics of the demand for insurance

The budget line under insurance will change when the parameters underlying it change, namely the individual's initial income, \( y \), the premium rate, \( p \), the accident probability, \( \pi \), and the size of the loss, \( L \).

3.2.1. The effect of income changes

Let's start with the effect of a change in income on the demand for insurance. In Fig 10 the individual is initially at point \( d \), buying an amount of cover, \( q \), equal to the distance \( ae \). If \( y \) rises, with the loss, \( L = y_i - y_h \), unchanged, the budget line shifts up to \( a'g' \), which is parallel to the old budget line \( ab \). As it is drawn, Fig 10 shows the new equilibrium, \( d' \), on the continuation of the line \( de \). Given this, the distance \( a'e' \), which is the new level of cover, is the same as the distance \( ae \), the old level of cover. We know from our discussion of indifference maps that indifference curve \( I_1 \) will have the same slope at \( d' \) as \( I_0 \) has at \( d \) if the individual's index of absolute risk aversion, \( A(y) \), is constant. Thus we can conclude that if \( A(y) \) is constant, the demand for insurance cover will be unaffected by a rise in income. If, by contrast, \( A(y) \) is decreasing in income, the indifference curve \( I_1 \) will be steeper at \( d' \) than the budget line and the new equilibrium will lie to the right of \( d' \), meaning that the demand for cover will have fallen. If, on the other hand, \( A(y) \) is increasing in income, the demand for cover will rise following an increase in income.
This result too can be derived mathematically. Think of the first-order condition (42) as an implicit function relating $q^*$ to the various parameters of the insurance problem. Then from the implicit function theorem (see e.g. Chiang p.220), we can write

$$\frac{\partial q^*}{\partial y} = -\frac{\partial V_q / \partial y}{\partial V_q / \partial q} = \frac{V_{qq}}{V_{qy}}.$$  

For $q^*$ emerging from (42) to be a maximum, we require that $V_{qq} < 0$. Hence

$$\frac{\partial q^*}{\partial y} > (<) 0 \quad \text{as} \quad V_{qq} > (<) 0.$$  

Differentiating (42) with respect to $y$ yields:

$$\frac{\partial V}{\partial y} = V_{qy} = -(1-\pi)u''(y_G)p + \pi u''(y_B)(1-p).$$  

From the first-order condition (42), we get

$$(1-\pi)p = \pi(1-p)\frac{u'(y_B)}{u'(y_G)}.$$  

Substituting (51) in (50) and rearranging gives

$$\begin{align*}
\frac{\partial V}{\partial y} &= (1-p)\pi u'(y_B) \left[ -u''(y_G) + \frac{u''(y_B)}{u'(y_G)} \right] \\
&= (1-p)\pi u'(y_B) \left[ A(y_G) - A(y_B) \right].
\end{align*}$$
But we know the individual takes out less than full cover and hence $y_G > y_B$. So, the term in square brackets is positive — and hence so is the entire derivative — if $A'(y) > 0$ (i.e. if the individual displays increasing absolute risk aversion). This was the conclusion we reached earlier.

3.2.2. The effect of premium changes

Consider next the effect of a rise in the premium rate, $p$. The budget line rotates anti-clockwise around point $a$ in Fig 11. We can break the effect on the demand for insurance cover into substitution and income effects. The substitution effect is shown by the move from the initial equilibrium $d$ to point $b$ — this serves to reduce the demand for insurance cover and can be explained in terms of the relative price of income in the bad state having risen, causing the individual to substitute away from it towards income in the good state. The income effect is shown by the move from point $b$ to $d'$, which, in Fig 11, lies on a line that is parallel to the certainty line. Evidently the level of cover at $b$ is the same as that at $d'$, so the income effect is, in this case, zero. The overall effect in the case illustrated is a reduction in the demand for insurance cover.

Fig 11 is, of course, a special case and corresponds to the case where $A(y)$ is constant. If instead $A(y)$ had been decreasing in $y$, the indifference curve $I^1$ would have been flatter at $d'$ than at $b$, and hence the new equilibrium would have been to the left of $d'$ — the income effect would offset, at least partially, the substitution effect. Whether, in this case, the demand for cover falls or rises cannot be said a priori.

3.2.3. The effects of other changes

The other factors affecting the demand for insurance are the accident probability, $\pi$, and the loss, $L$. If $\pi$ rises, the iso-expected income line $E^0$ in Fig 8 rotates anti-clockwise around $P^0$. The effects on the demand for insurance are left as a class exercise. If $L$ falls, with $y$ unchanged, the uninsured prospect, $P^0$, moves vertically up towards the certainty line.
Assuming \( \pi \) and \( p \) are unaltered, the new lines corresponding to \( E^0 \) and \( E^1 \) run parallel to the old lines starting now from the new \( P^0 \). The effects on the demand for insurance are left as a class exercise, as are the effects in the case where \( p \) is altered accordingly when \( \pi \) changes.