Structural Stability Testing

- We consider 3 tests for this.

1. Chow Test.
2. Predictive Failure test (Chow's second test).
3. Prediction test using dummies.

Chow Tests for Structural Stability

- The *Chow Test* is often used to see if there is evidence of a structural break in the data. Suppose we have data from 1950 to 1980 and we suspect that there is a structural break in the year 1966. We can split the sample into two sub-periods, 1950-1965 and 1966-1980 and perform a Chow Test. Call the number of observations in the first subsample n_1 and the number in the second subsample n_2.

- Suppose the equation we are estimating is:

$$y_t = \beta_0 + \beta_1 x_t + u_t \quad t = 1950 \ldots 1980$$

We can form an F-test by specifying an unrestricted and restricted regression and testing the restrictions.

- The restricted model is where we force the intercepts and slopes to be the same in the two subperiods. To run this, we estimate the above regression over the whole sample 1950-1980. The RSS from this regression is the RRSS.

- The unrestricted regression is when we allow the slopes in the two subperiods to be different (i.e. to vary freely without restriction). We can do this by running the two equations for each subperiod separately and adding the RSS for each subperiod. Call the residual sum of squares for 1950-1965 RSS_1 and that for 1966-1980 RSS_2. Then $\text{URSS} = \text{RSS}_1 + \text{RSS}_2$.

- We can now form an F-test in the usual way. But there is one thing we need to check first. We need to make sure that the variances of the u_t in the two sub-periods are equal because an F-test assumes that the variance of the restricted regression and the unrestricted regression are the same. We can test this using the following F-test for variance equality:

  $$\mathcal{V} = \frac{\hat{\sigma}_1^2}{\hat{\sigma}_2^2} \sim F_{(n_1-k-1, n_2-k-2)}$$

  where $\hat{\sigma}_1^2$ is the variance estimate from subsample 1 and $\hat{\sigma}_1^2$ is the variance estimate for subsample 2. If we reject this, we cannot use the Chow test.

- To perform the Chow test, just form the usual F-test (note there are n_2 restrictions:

$$\frac{(RRSS - URSS)/n_2}{URSS/(n_1 + n_2 - 2k - 2)} \sim F(n_2, n_1 + n_2 - 2k - 2)$$

Microfit Predictive Failure Test

- Sometimes there are too few observations in the second subsample to perform the two subsample estimations. In this case, one can form an adjusted Chow test known as Chow's second test which *Microfit* calls the Predictive Failure test.
- A dummy variable is added *for each period* in the second subsample and then these dummies are tested. Suppose there are 2 observations in the second subperiod. Make dummies for these periods called D_1 and D_2. Note that $n_2 = 2$ here.
- The unrestricted model is run over the whole sample ($n_1 + n_2$ observations):

$$y_t = \beta_0 + \beta_1 x_t + \delta_1 D_1 + \delta_2 D_2 + u_t$$

- The restricted model is run over just the first subsample (n_1 observations).

$$y_t = \beta_0 + \beta_1 x_t + v_t$$

- Then we just use an F-test to test $\delta_1 = \delta_2 = 0$.

- The test statistic is the usual one (note that the number of restrictions is equal to n_2.):

$$F = \frac{(RRSS - URSS)/n_2}{RSS_1/(n_1 - k - 1)} \sim F(n_2, n_1 - k - 1)$$

- The LM version of the test (which is χ^2 distributed) is the same but is just multiplied by the degrees of freedom in the numerator:

$$LM = n_2 \, F \sim \chi^2(n_2)$$

Prediction Test Using Dummies

- This test is similar to the Chow test. It is performed by using dummy variables (both intercept and slope dummies are possible) to allow the equation to shift for the second subsample. We need to form a dummy which is 0 for the first subsample and then 1 for the second. We then add this in to the equation (intercept dummy) and we add in our old variables multiplied by this dummy (slope dummies).

$$y_t = \beta_0 + \beta_1 x_t + \beta_3 D_t + \beta_4 D_t x_t + u_t$$

- We can then estimate over the whole sample period and test the significance of the dummies. The above is the unrestricted regression (we allow the slopes and intercepts to shift freely without restriction).
- The restricted regression is the original model.