Functional relationships

- We frequently need to represent economic quantities as a mathematical function of one or more variables, e.g.:
    - A firm's costs as a function of output
    - Market demand for a good in terms of price and other factors
    - A consumer's utility in terms of quantities of different goods purchased.

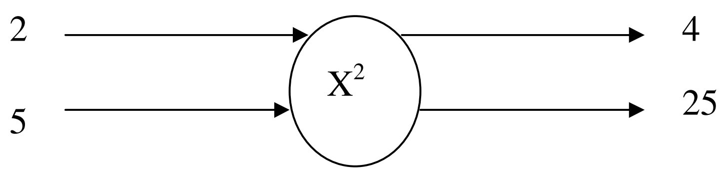
A *function* is a rule that takes one or more numbers as inputs (arguments) and gives another number as output.

e.g. $f(x) = x^2$

Here, "f" is the name given to the function, x represents the input, x^2 tells us how to calculate the answer in terms of the input.

e.g. $f(2) = 2^2 = 2 \times 2 = 4$, $f(5) = 5^2 = 5 \times 5 = 25$ etc.

A function can be seen as a 'black box' for converting numbers into other numbers:



The variable or number in brackets is referred to as the *argument* of the function.

If we want to write another variable as a function of x, we may write $y = f(x)$, or just (in this case) $y=x^2$.

E.g. we could express a demand function by

$Q = 20 - 5P$

Where Q is quantity demanded and P is price. Thus quantity is expressed as a function of price.

Functions of more than one variable

We may have functions of two or more variables. For example

$F(x,y) = xy + 2x$

This function requires two numbers input to produce an answer. For example,

$F(4,5) = 4*5 + 2*4 = 28$

Here x is 4 and y is 5.

Here, the function has two arguments.

For example in economics, a firm's output may depend on their input of Labour (L) and Capital (K), for example a Cobb-Douglas type production function:

$Q = 100L^{0.5}K^{0.5}$

Where Q is units of output, L is number of workers and K is, perhaps, number of machines.

For example, if L=9 and K=16, then

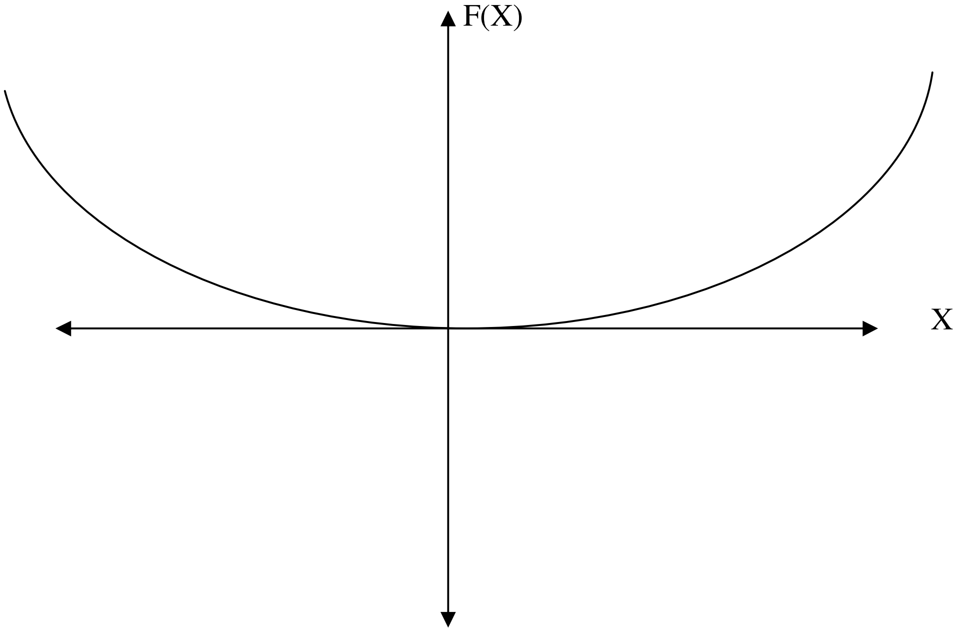
$Q = 100*(9^{0.5})*(16^{0.5}) = 100*3*4 = 1{,}200$ units.

Graphs of functions

Functions of 1 variable can easily be represented on a graph. E.g. if we have

$F(x) = x^2$

This has a graph something like:



The simplest sorts of functions are linear. These are of the form

$Y = a + bX$

Where **a** and **b** are constants, e.g. $Y = 5 – 2X$.. The graphs of these functions are straight lines. For example if we have the demand function

$Q = 20 –5P$

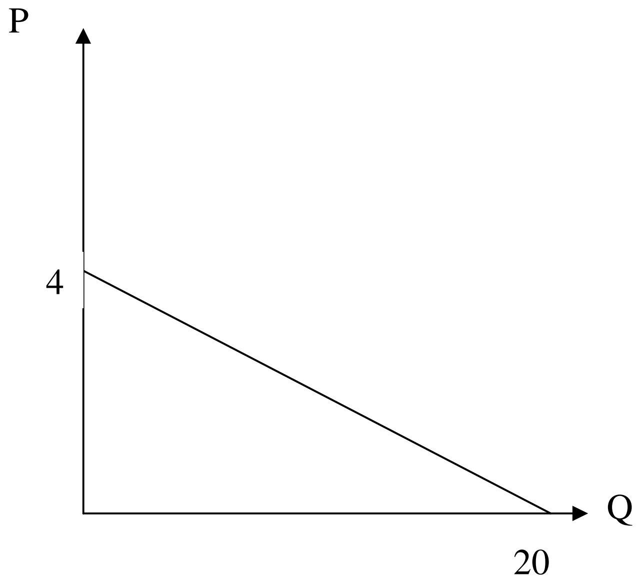
First, because we always put price on the vertical axis, we need to get P on to the left hand side of the equation. Thus

$Q + 5P = 20$

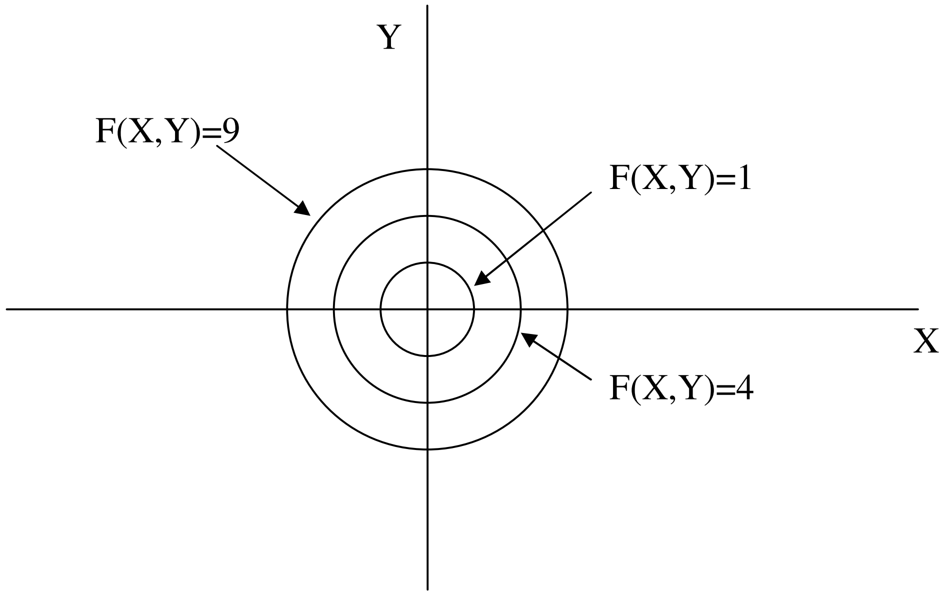
$5P = 20 – Q$
$P = (20/5) –(Q/5)$

$P = 4 –Q/5$

This has a graph:



With functions of two variables, the graph will in fact be a three-dimensional surface – for example the function $F(X,Y)=X^2+Y^2$ will be a dome-shape, which it may be possible to sketch. Another way of graphically representing functions of two variables is by a contour map that draws curves showing combinations of values of X and Y that give the same value of F(X,Y). For example, an indifference curve map is simply the contour map of a Utility function. To give another example, let $F(X,Y)=X^2+Y^2$. Then the contour map for F(X,Y) consists of a series of concentric circles around the origin, with further out circles representing higher values of F(X,Y), as follows:



Of course, it is not really possible to draw graphs of functions of more than two variables.