**Functional relationships**

- We frequently need to represent economic quantities as a mathematical function of one or more variables, e.g.:
  - A firm’s costs as a function of output
  - Market demand for a good in terms of price and other factors
  - A consumer’s utility in terms of quantities of different goods purchased.

A *function* is a rule that takes one or more numbers as inputs (arguments) and gives another number as output.

e.g. \( f(x) = x^2 \)

Here, “f” is the name given to the function, \( x \) represents the input, \( x^2 \) tells us how to calculate the answer in terms of the input.

e.g. \( f(2) = 2^2 = 2 \times 2 = 4, f(5) = 5^2 = 5 \times 5 = 25 \) etc.

A function can be seen as a ‘black box’ for converting numbers into other numbers:

![Diagram](attachment:image.png)

The variable or number in brackets is referred to as the *argument* of the function.

If we want to write another variable as a function of \( x \), we may write \( y = f(x) \), or just (in this case) \( y = x^2 \).

E.g. we could express a demand function by
Q = 20 – 5P

Where Q is quantity demanded and P is price. Thus quantity is expressed as a function of price.

**Functions of more than one variable**

We may have functions of two or more variables. For example

\[ F(x,y) = xy + 2x \]

This function requires two numbers input to produce an answer. For example,

\[ F(4,5) = 4\times5 +2\times4 = 28 \]

Here x is 4 and y is 5.

Here, the function has two arguments.

For example in economics, a firm’s output may depend on their input of Labour (L) and Capital (K), for example a Cobb-Douglas type production function:

\[ Q = 100L^{0.5}K^{0.5} \]

Where Q is units of output, L is number of workers and K is, perhaps, number of machines.

For example, if L=9 and K=16, then

\[ Q = 100\times(9^{0.5})\times(16^{0.5}) = 100\times3\times4 = 1,200 \text{ units}. \]
**Graphs of functions**

Functions of 1 variable can easily be represented on a graph. E.g. if we have

\[ F(x) = x^2 \]

This has a graph something like:

![Graph of a quadratic function](image)

The simplest sorts of functions are **linear**. These are of the form

\[ Y = a + bX \]

Where \( a \) and \( b \) are constants, e.g. \( Y = 5 - 2X \). The graphs of these functions are straight lines. For example if we have the demand function

\[ Q = 20 - 5P \]

First, because we always put price on the vertical axis, we need to get \( P \) on the left hand side of the equation. Thus

\[ Q + 5P = 20 \]
\[5P = 20 - Q\]
\[P = \frac{20}{5} - \frac{Q}{5}\]
\[P = 4 - \frac{Q}{5}\]

This has a graph:

With functions of two variables, the graph will in fact be a three-dimensional surface – for example the function \(F(X,Y) = X^2 + Y^2\) will be a dome-shape, which it may be possible to sketch. Another way of graphically representing functions of two variables is by a contour map that draws curves showing combinations of values of \(X\) and \(Y\) that give the same value of \(F(X,Y)\). For example, an indifference curve map is simply the contour map of a Utility function. To give another example, let \(F(X,Y) = X^2 + Y^2\). Then the contour map for \(F(X,Y)\) consists of a series of concentric circles around the origin, with further out circles representing higher values of \(F(X,Y)\), as follows:
Of course, it is not really possible to draw graphs of functions of more than two variables.