# Dynamic Models

1. The problem with specifying the dynamic form of a regression model is that normally the theory provides little information on lag lengths, nature of adjustments etc. So seems better to use the theory to specify the variables to be included, but to allow the data to determine what the dynamic model should look like
    a. Consider the consumption function being used in the exercises:

$$C_t = \beta_0 + \beta_1 Y_t + \beta_2 Y_{t-1} + \beta_3 C_{t-1} + u_t$$

    This encompasses a number of different models all with different dynamic structures they are 'nested' in this model meaning the restrictions on the parameters can be written down and they can be tested.
2. Static model: absolute income hypothesis $\beta_2 = \beta_3 = 0$

$$C_t = \alpha_0 + \alpha_1 Y_t + v_{1t}$$

3. AR(1) model: first order autoregression $\beta_1 = \beta_2 = 0$

$$C_t = \alpha_0 + \alpha_1 C_{t-1} + v_{2t}$$

4. Partial adjustment/habit persistence model $\beta_2 = 0$

$$C_t = \alpha_0 + \alpha_1 Y_t + \alpha_2 C_{t-1} + v_{3t}$$

    This is a habit model if $\alpha_2 > 0$ and a partial adjustment model if $|\alpha_2| < 1$
5. Distributed lag model $\beta_3 = 0$

$$C_t = \alpha_0 + \alpha_1 Y_t + \alpha_2 Y_{t-1} + v_{4t}$$

6. First difference $\beta_3 = 1$ and $\beta_2 = -\beta_1$

$$\Delta C_t = \alpha_0 + \alpha_1 \Delta Y_t + v_{5t}$$

    where $\Delta C_t = C_t - C_{t-1}$and $\Delta Y_t = Y_t - Y_{t-1}$
7. Error correction model

$$\Delta C_t = \alpha_0 + \alpha_1 \Delta Y_t + \alpha_2 (C_{t-1} - Y_{t-1}) + v_{6t}$$

    where $\beta_3 = 1 + \alpha_2$ and $\beta_2 = -(\alpha_1 + \alpha_2)$

This is now a very commonly used model, because of its use in cointegration analysis

- All of these restrictions can be tested and you will do this in the exercise
- Look for most parsimonious
- Another issue is the long run solutions of these models
- Set $Y_t = Y_{t-1} = Y$ and do the same for $X$ gives the long run solution
- Note that the first difference equation does not have one so if you were to find this was the best model you would only have short run dynamics.
- It is possible that this problem can be dealt with but need to discuss the concept of cointegration

# Forms of Dynamic Models

To help in describing the form of more complex dynamic models some conventions and techniques have been developed and employed to make it easily to write down and analyse relatively complex dynamic models.

AR: Autoregressive

$$AR(1) : w_t = \alpha_1 w_{t-1} + \varepsilon_t$$
$$AR(k) : w_t = \alpha_1 w_{t-1} + \alpha_2 w_{t-2} + ... + \alpha_k w_{t-k} + \varepsilon_t$$

MA: Moving average

$$MA(1) : w_t = \varepsilon_t + \beta_1 \varepsilon_{t-1}$$
$$MA(l) : w_t = \varepsilon_t + \beta_1 \varepsilon_{t-1} + \beta_2 \varepsilon_{t-2} + ... + \beta_l \varepsilon_{t-l}$$

DL: Distributed lag

$$DL(1) : w_t = \gamma_1 x_t + \gamma_2 x_{t-1} + \varepsilon_t$$
$$DL(m) : w_t = \gamma_1 x_t + \gamma_2 x_{t-1} + ... + \gamma_m x_{t-m} + \varepsilon_t$$
$$= \sum_{j=0}^{m} \gamma_j x_{t-j} + \varepsilon_t$$

Differencing

$$First : \Delta w_t = w_t - w_{t-1}$$

Distinguish

$$\Delta^2 w_t = \Delta\Delta w_t$$
$$= \Delta(w_t - w_{t-1})$$
$$= (w_t - w_{t-1}) - (w_{t-1} - w_{t-2})$$
$$= w_t - 2w_{t-1} + w_{t-2}$$

and

$$\Delta_2 w_t = w_t - w_{t-2}$$

Combinations of these processes are possible and are used in empirical research

$$ARDL(k,m) : w_t = c_0 + c_1 x_t + c_2 x_{t-1} ... + c_m x_{t-m} + a_1 w_{t-1} + ... + a_k w_{t-k} + \varepsilon_t$$
$$ARMA(k,n) : w_t = a_1 w_{t-1} + ... + a_k w_{t-k} + \varepsilon_t + b_1 \varepsilon_{t-1} + ... + b_n \varepsilon_{t-n}$$
$$ARIMA(p,d,q) : \Delta^d w_t = a_1 \Delta^d w_{t-1} + ... + a_p \Delta^d w_{t-p} + \varepsilon_t + b_1 \varepsilon_{t-1} + ... + b_q \varepsilon_{t-q}$$

Other things you might see:

- lag operators are often used in exposition. They take the form

$$Lw_t = w_{t-1}$$
$$L^2 w_t = w_{t-2}$$
$$L^n w_t = w_{t-n}$$

- Forms of distributed lag: Polynomial; Rational; Koyk transformation
- ARCH and GARCH models

More detail in Pindyck and Rubinfeld