Functional form

When estimate models often don't have information form theory on functional form.

- Use of linear is for simplicity of estimation
- Note that it is linear in the parameters that allows estimation by OLS not linearity in the variables
- There are methods of estimating non linear relations, but they are not straightforward so we try to find a way of transforming relation to make it linear in the parameters
- There are a wide variety of functional forms that can be approximated using the powers of the independent variables and applying OLS -though problems of multicollinearity possible

$$Y_i = \alpha_0 + \alpha_1 X_i + \alpha_2 X_i^2$$

- Can also use logs or reciprocal transformations

$$Y_i = \alpha_0 + \alpha_1 X_i$$

$$\ln Y_i = \alpha_0 + \alpha_1 \ln X_i$$

$$\ln Y_i = \alpha_0 + \alpha_1 X_i$$

$$Y_i = \alpha_0 + \alpha_1 \ln X_i$$

$$Y_i = \alpha_0 + \alpha_1 (1/X_i)$$

$$\ln Y_i = \alpha_0 + \alpha_1 (1/X_i)$$

- A common example is the Cobb Douglas production function which is multiplicative:

$$Q = AK^{\beta_1} L^{\beta_2}$$

  - to estimate we take logs and add an error

$$\log Q = \log A + \beta_1 \log K + \beta_2 \log L + u$$

  - and then rewrite

$$\log Q = \beta_0 + \beta_1 \log K + \beta_2 \log L + u$$

  - Note the error could have been assumed as $\exp(u)$ in the initial form .
- Using logs has another advantage in that it also give us the elasticities. Consider a demand function:

$$Q_t = \beta_0 + \beta_1 P_t + u_t$$

  - Now β_1 is the slope coefficient

$$\frac{\delta Q}{\delta P} = \beta_1$$

  but it is not the elasticity
  - Instead if we had taken logs

$$\log Q_t = \alpha_o + \alpha_1 \log P_t + e_t$$

  - then α_1 is the elasticity as

$$\frac{\delta \log Q_t}{\delta \log P_t} = \frac{\delta Q_t}{\delta P_t} \bullet \frac{P_t}{Q_t}$$

- Also the difference in logs approximates the growth rate

$$\Delta \log Y_t = \log Y_t - \log Y_{t-1} \approx \frac{Y_t - Y_{t-1}}{Y_{t-1}}$$

- Sometimes theory gives information on possible functional form but it is unusual for it o be specific enough
- May be able to compare using R^2as a guide or SER

  remember cant use R^2 to compares different dependent variables, such as linear vs log linear
- Microfit gives the Reset test which is designed to check the choice of an inappropriate functional form.
- if functional form not appropriate them might expect squares or higher powers of one or more regressors to improve the explanation of the dependent variable.
  - A simple test that wont lose as many degrees of freedom is to use the powers of $\hat{Y}$ in the regression as additional explanatory variables. For example

$$Y_i = \alpha_0 + \alpha_1 X_i + u_i$$

$$Y_i = \alpha_0 + \alpha_1 X_i + \alpha_2 \hat{Y}_i^2 + u_i$$

  - Have to rely o the large sample properties of the test procedures as the predicted Y involves estimates of parameters and is random (non-stochastic)
  - Usefulness of this test lies in acting as a general indicator that something is wrong. In practice it is not very good as detecting any specific alternative to a proposed model.