Unit Roots

Maddala Ch6: very good exposition chapter 13 for more advanced treatment

Have mentioned the problem of spurious regression and the need to detrend. Now consider some of the issue involved in this more formally

In general we can define

$$E(u_t) = 0$$
$$var(u_t) = \sigma^2 \text{ for all } i$$
$$cov(u_t, u_{t-k}) = \sigma^2_{\rho_k}$$

as a covariance stationary, or just stationary series, where ρ_k is serial correlation of lag k but depends only on k and not on t.

Many economic time series are non stationary

- their mean and variance depend on time
- they depart from any given value as time goes on
- if the movement is predominantly in one direction then they exhibit a trend

Non stationary series are frequently detrended by:

- regressing on time
- successive differencing

More formally

$$y_t = f(t) + u_t$$
$$y_t = \alpha + \beta t + u_t$$

is trend stationary and the trend eliminated series is $\hat{u}_t$

$$y_t - y_{t-1} = \beta + \varepsilon_t$$
$$\Delta y_t = \beta + \varepsilon_t$$

is a random walk/difference stationary. The trend eliminated series in $\hat{\varepsilon}_t$

eg Gibrat's law

Now both trend and difference stationary will exhibit a trend, but the method of eliminating it is different.

Dickey Fuller test is used to find out to which a series belongs.

$$y_t = \alpha + \rho y_{t-1} + \beta t + \varepsilon_t$$

- DSP if $\rho = 1; \beta = 0$
- TSP if $|\rho| < 1$
- So test $\rho = 1; \beta = 0$ against $|\rho| < 1$
- Normally we simply test $\rho = 1$
- This is not a straightforward test as under the DSP the LS estimate is not distributed around

1, though the bias decreases as the number of observations increases.

- This means in practice that have to use higher values for the t and F tests than usual.
- Dickey and Fuller give tables.

Many economic series have been found to be difference stationary, suggesting that the trend stationary model is only relevant if we assume the errors u_t are highly autocorrelated.

Maddala emphasizes that using regression on time trend when the series is difference stationary is very dangerous and argues always better to work with difference than levels:

- if difference stationary and do levels problem of increasing variance over time which means LS and test of significance are invalid
- if levels are true and do differences all will do is to introduce a moving average error. Ignoring this will at worst lead to inefficient estimators. Can see this easily:

$$\begin{aligned} y_t &= \beta_0 + \beta_1 x_t + \beta_2 t + u_t \\ \Delta y_t &= \beta_1 \Delta x_t + \beta_2 + v_t \\ v_t &= \Delta u_t \end{aligned}$$

Testing for unit roots:

Testing for difference stationarity was a major concern of econometricians in the 1980s as if a series is difference stationary the effect of any shock is permanent. In the model

$$y_t = y_{t-1} + \varepsilon_t$$

where ε_t is zero mean stationary process. If at any time period there is a jump in ε_t then y_t will also increase and stay there. The effect is permanent. But with

$$y_t = \beta y_{t-1} + \varepsilon_t$$
$$|\beta| < 1$$

any shock will fade away over time.

In new classical macroeconomics fluctuations in real GDP should be explained by real shocks, monetary ones should be transitory, so testing whether in the above autoregression whether the root $\beta = 1$ or $\beta < 1$ i.e. whether there is a unit root became important.

To test for a unit root we test $\beta = 1$ in

$$y_t = \alpha + \beta y_{t-1} + u_t$$

using the Dickey Fuller test.
Occurs when DW statistic is close to zero

If $\beta = 1$ then

$$y_t = \alpha + y_{t-1} + u_t$$
$$y_t - y_{t-1} = \alpha + u_t$$
$$\Delta y_t = \alpha + u_t$$

Cant use the usual reported t statistic, or F to test the restriction, rule of thumb is around 3.2

Augmented Dickey Fuller Test

If we take

$$y_t = \alpha + \delta t + \beta y_{t-1} + u_t$$

and u_t is not white noise we need to modify the Dickey Fuller test:

$$y_t = \alpha + \delta t + \beta y_{t-1} + \sum_{j=1}^{k} \theta_j \Delta y_{t-j} + u_t$$

where lags of y are added to move any serial correlation. After estimating the revised equation the same tests can be used. This is the augmented Dickey-Fuller test.