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# Conflict Success Functions and the Theory of Appropriation Possibilities



Standard economics treats individuals and groups (including nations) as enriching themselves through specialized production and trade. These are presented as peaceful activities because the resources used and the goods produced and traded are implicitly assumed to be secure from appropriation. We have seen in this book, however, that conflict over resources and goods abound. While previous chapters modeled conflict as a choice and considered the interdependence of economic and conflict variables, this chapter adds a new premise, namely, that appropriation stands coequal with production and trade as a fundamental category of economic activity. The chapter begins with an overview of the conflict success function, which is a key element of the theory of appropriation possibilities. We then present a model of conflict over a resource, which reveals, among other things, a paradox of power and incentives for peaceful settlement. The resource conflict model is then integrated with an Edgeworth box model of production and trade, showing how various economic variables are affected by appropriation possibilities.

## 12.1. Conflict Success Functions

A central building block for introducing appropriation possibilities into mainstream economic models is the conflict success function (CSF) (Hirshleifer 1995, Garfinkel and Skaperdas 2007). A CSF specifies how the weapons or fighting efforts of players combine to determine the distribution of a contested resource or good. Suppose, for example, that players $A$ and $B$ employ $M_A$ and $M_B$ units of military goods to determine the holdings of a resource such as land, oil, or water. Let $p_A$ be $A$'s conflict success in the resource dispute, with $p_B$ the same for $B$. Conflict success might be measured by the proportion of the disputed resource controlled





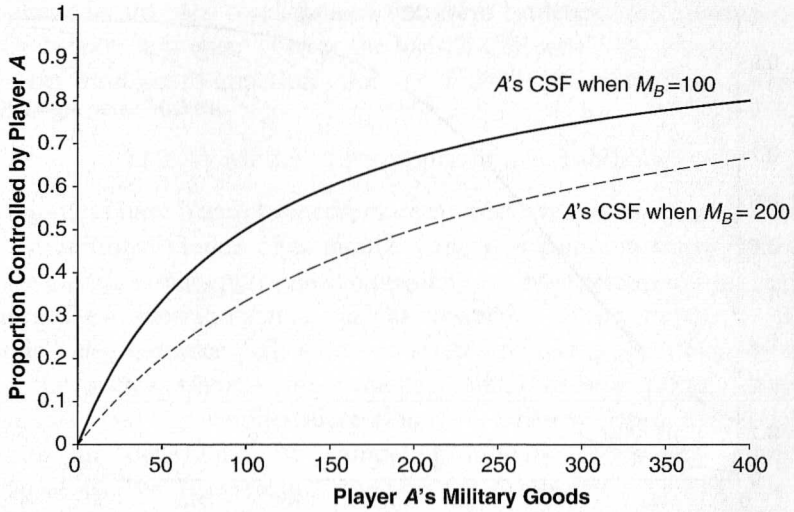


Figure 12.1. Ratio form conflict success functions for player $A$.

by a player or by the probability that a player controls the entire resource in a winner-take-all contest. The technology relating the military inputs $M_A$ and $M_B$ and the success outputs $p_A$ and $p_B$ is summarized by the CSF, which is assumed mathematically to take on either a ratio or logistic form.

According to the ratio form, the conflict successes of $A$ and $B$ are

$$p_A = \frac{(M_A)^m}{(M_A)^m + (ZM_B)^m}$$
$$\text{and} \quad p_B = \frac{(ZM_B)^m}{(M_A)^m + (ZM_B)^m}, \text{ with } m > 0, Z > 0. \tag{12.1}$$

Parameter $m$ is a decisiveness coefficient that captures the degree to which greater military input translates into conflict success, while parameter $Z$ represents the relative effectiveness of $B$'s military goods. Figure 12.1 illustrates the ratio form CSF for player $A$ when $m = Z = 1$, with $A$'s military goods $M_A$ measured horizontally and conflict success $p_A$ vertically. Assume first that $B$'s military goods are fixed at $M_B = 100$, which results in the solid curve in the figure. As seen, $A$'s conflict success rises at a diminishing rate as $M_A$ increases from zero along the horizontal axis. When $A$'s military goods reach $M_A = 100$, $A$'s conflict success $p_A$ equals 0.5; for values of $M_A$ above 100, $A$'s conflict success is greater than 0.5. Suppose now that player $B$'s military goods rise to $M_B = 200$. This causes $A$'s conflict success function to rotate downward, as shown by the dashed curve in Figure 12.1. In this case, $A$ does not reach a conflict success of 0.5 until $M_A$ is 200.

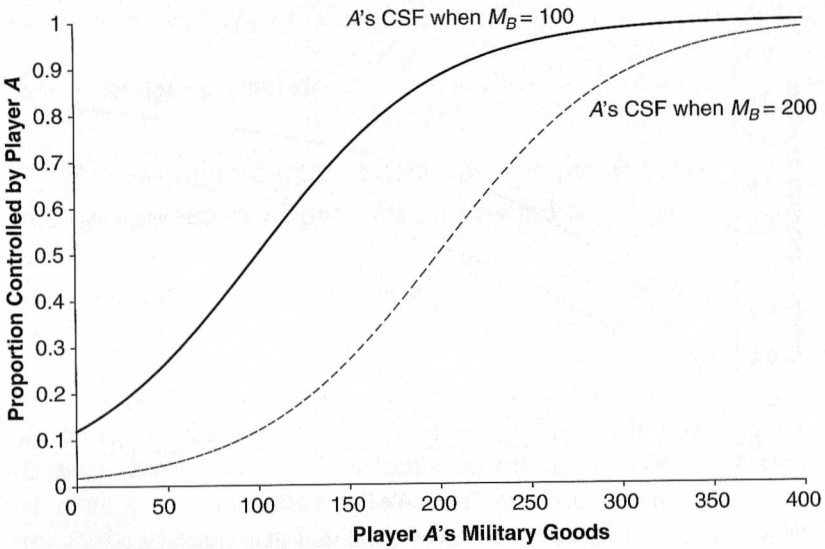


Figure 12.2. Logistic form conflict success functions for player $A$.

For the logistic form of the CSF, the conflict successes are

$$p_A = \frac{1}{1 + exp[m(ZM_B - M_A)]} \quad \text{and} \quad p_B = \frac{1}{1 + exp[m(M_A - ZM_B)]}, \tag{12.2}$$

where the second term in each denominator represents the natural constant $e$ raised to the power shown in brackets. Figure 12.2 illustrates the logistic form CSF for player $A$ when $m = 0.02$ and $Z = 1$. The solid curve occurs when $M_B = 100$, while the dashed curve holds when $M_B = 200$.

The ratio and logistic forms differ in two major ways. First, under ratio technology, conflict success depends on the *ratio* of military goods $M_A/M_B$, whereas under logistic technology, conflict success depends on the *difference* in military goods $M_A - M_B$ (Hirshleifer 1995, p. 176, Garfinkel and Skaperdas 2007, pp. 655–656). Second, the vertical intercept is zero for the ratio CSF but positive for the logistic CSF. This means that under ratio technology, a player with zero military goods will have zero success, even if the opposing player has only a negligible amount of military goods. In contrast, under logistic conflict technology, a player with zero military goods will still experience some degree of conflict success. Hirshleifer (1995, p. 178) notes that in the context of military combat the ratio CSF would



apply under the ideal conditions of a uniform battlefield, full information, and absence of fatigue, whereas the logistic CSF would be relevant when combat is subject to imperfect information or the presence of safe havens.

## 12.2. A Model of Appropriation Possibilities

Economists have begun to incorporate conflict over resources and goods into traditional models of economic activity. A common theme linking these models is that appropriation possibilities divert resources away from alternative economic activities such as production or consumption. Some models also consider destruction of assets and disruption of economic activities such as trade. Another theme linking economic models of conflict is the use of a conflict success function whereby appropriative outcomes are determined by competing military goods and conflict technology. We illustrate these themes by presenting a variation of a resource conflict model due originally to Skaperdas (2006, pp. 664–666).

### Basic Model of Resource Conflict

Suppose players $A$ and $B$ dispute control of a fixed resource $\tilde{R}$. The players also have respective holdings of secure and undisputed resources, $R_A$ and $R_B$. $A$ and $B$ are free to divert $M_A$ and $M_B$ units of their respective secure resources to produce military goods, which in turn can be used to fight over the disputed resource. For simplicity, we assume that each unit of resources diverted to conflict generates one unit of military goods. We assume for the same reason that fighting between $A$ and $B$ destroys a fixed proportion $\delta$ of the disputed resource, where $0 < \delta < 1$. Hence, $\tilde{R}(1-\delta)$ is the amount of the disputed resource that would remain following a fight. Assuming a ratio CSF with a decisiveness coefficient of $m = 1$, the net resources $NR_A$ and $NR_B$ controlled by the players if fighting occurs will be

$$\begin{aligned} NR_A &= (R_A - M_A) + \left(\frac{M_A}{M_A + ZM_B}\right)\tilde{R}(1-\delta) \\ &= \left[R_A + \left(\frac{M_A}{M_A + ZM_B}\right)\tilde{R}(1-\delta)\right] - M_A \end{aligned} \tag{12.3a}$$

$$\begin{aligned} NR_B &= (R_B - M_B) + \left(\frac{ZM_B}{M_A + ZM_B}\right)\tilde{R}(1-\delta) \\ &= \left[R_B + \left(\frac{ZM_B}{M_A + ZM_B}\right)\tilde{R}(1-\delta)\right] - M_B. \end{aligned} \tag{12.3b}$$

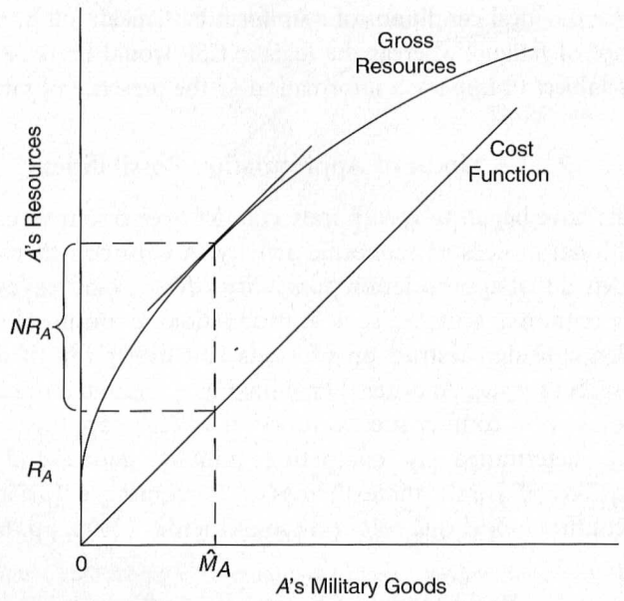


Figure 12.3. $A$'s optimal allocation of resources to military goods.

Equations (12.3a) and (12.3b) show that each player's amount of net resources is calculated as the secure resource holding minus the diversion of resources to military goods plus the portion of the remaining disputed resource claimed in the fight. We assume that the CSF determines the share of $\tilde{R}(1-\delta)$ seized, so that equations (12.3a) and (12.3b) show the net resources controlled by $A$ and $B$ with certainty. If we assume alternatively that the CSF determines the probability of capturing the resource in a winner-take-all contest, then $NR_A$ and $NR_B$ equal expected net resources of $A$ and $B$.

### *Optimization Problem*

We focus on player $A$'s optimization problem, with $B$'s being analogous. Player $A$'s objective is to choose $M_A$ to maximize its net resources. The trade-off that $A$ faces in (12.3a) is that, for any given $M_B$, more $M_A$ will increase $A$'s share of the remaining disputed resource$\tilde{R}(1-\delta)$, but it will also divert additional resources away from $A$'s secure resource holding. Figure 12.3 shows $A$'s optimization problem graphically. The diversion of $A$'s resources to conflict, $M_A$, is measured horizontally, while the amount of resources is measured vertically. The gross resources schedule in Figure 12.3 reflects the square-bracketed term in equation (12.3a); it has



an intercept equal to $R_A$ and a positive but diminishing slope due to the CSF. The cost function in Figure 12.3 reflects the $M_A$ term shown to the right of the brackets; it has an intercept of zero and a slope of one. Given $M_B$, $A$ maximizes net resources in Figure 12.3 by allocating to military goods a level of resources $\hat{M}_A$, where the marginal amount of resources obtained in conflict equals the marginal cost of resources diverted to military goods. Geometrically, this occurs where the slope of the gross resources schedule is equal to the slope of the cost function. The vertical distance between the two functions at $\hat{M}_A$ measures the net resources controlled by $A$ in the conflict.

Changes in the gross resources or cost schedules can be considered in Figure 12.3 in much the same way that changes in revenues and costs were discussed in the net revenue model of Chapter 7. For example, an increase in the amount of the disputed resource $\tilde{R}$ would lead to an upward rotation of the gross resources function in Figure 12.3, causing an increase in $A$'s optimal level of military goods. Alternatively, an increase in $A$'s secure resource holding $R_A$ would shift $A$'s gross resources schedule upward in a parallel fashion, leaving the optimal amount of military goods unchanged at $\hat{M}_A$ but increasing $A$'s net resource holdings. On the cost side, suppose one unit of military goods could be acquired for less than one unit of resources. This would cause the cost function in Figure 12.3 to have a lower slope, leading to an increase in $A$'s optimal resource diversion to military goods.

### *Reaction Functions and Equilibrium*

The optimization problem just sketched gives rise to $A$'s reaction function, which shows the level of military goods that $A$ will choose given alternative levels of military goods for $B$. Algebraically, $A$'s reaction function is derived by differentiating equation (12.3a) with respect to $M_A$, setting the derivative to zero, and then solving for $M_A$. $B$'s reaction function is derived similarly using equation (12.3b). Assuming that each player's optimal resource diversion to military goods is less than its secure resource holding, the respective reaction functions of $A$ and $B$ are

$$M_A = \sqrt{ZM_B\tilde{R}(1-\delta)} - ZM_B \tag{12.4a}$$

$$M_B = \frac{1}{Z}\left(\sqrt{ZM_A\tilde{R}(1-\delta)} - M_A\right). \tag{12.4b}$$

Solving simultaneously the two reaction functions in equations (12.4a) and (12.4b) yields the equilibrium military goods $M_A^*$ and $M_B^*$, where:

$$M_A^* = M_B^* = \frac{Z\tilde{R}(1-\delta)}{(1+Z)^2}. \tag{12.5}$$

Equation (12.5) shows that the players' equilibrium military goods depend positively on the amount of the disputed resource $\tilde{R}$, positively (negatively) on the relative effectiveness of $B$'s military goods for $Z < 1$ (for $Z > 1$), and negatively on the destructiveness of war $\delta$. Substituting $M_A^*$ and $M_B^*$ into the CSF and multiplying by $\tilde{R}(1-\delta)$ shows that the amounts of the remaining disputed resource seized in a fight, $D_A^*$ and $D_B^*$, will be

$$D_A^* = \frac{\tilde{R}(1-\delta)}{1+Z} \tag{12.6a}$$

$$D_B^* = \frac{Z\tilde{R}(1-\delta)}{1+Z}. \tag{12.6b}$$

Similarly, substituting $M_A^*$ and $M_B^*$ back into equations (12.3a) and (12.3b), the final net resources controlled by the players in equilibrium after a fight can be shown to be

$$NR_A^* = R_A + \frac{\tilde{R}(1-\delta)}{(1+Z)^2} \tag{12.7a}$$

$$NR_B^* = R_B + \frac{Z^2\tilde{R}(1-\delta)}{(1+Z)^2}. \tag{12.7b}$$

### *Numerical Example*

As a numerical example, suppose the amount of the disputed resource is $\tilde{R} = 200$, the secure resource holdings of $A$ and $B$ are $R_A = R_B = 100$, the relative military effectiveness parameter is $Z = 1$, and the destructiveness of conflict is $\delta = 0.2$. Based on equation (12.5), each player diverts 40 units of secure resources to military goods. If they fight over the disputed resource, 20 percent of the disputed resource is destroyed, leaving 160 resource units. Based on the CSF, the players' military capabilities imply that each claims 50 percent of the remaining disputed resource, or 80 resource units each. The net resources controlled by each player in equilibrium is then 140 units, made up of the 60 units of secure resources not diverted to military goods plus 80 units of the remaining disputed



Table 12.1. *Numerical example of resource conflict model.*

| | |
|---|---|
| Parameters | |
| Secure resource holding of $A$ | $R_A = 100$ |
| Secure resource holding of $B$ | $R_B = 100$ |
| Amount of disputed resource | $\tilde{R} = 200$ |
| Relative military effectiveness of $B$ | $Z = 1$ |
| Destructiveness of conflict | $\delta = 0.2$ |
| Equilibrium values of the variables | |
| Military goods of $A$ and $B$ | $M_A^* = M_B^* = 40$ |
| Remaining disputed resources controlled by $A$ and $B$ | $D_A^* = D_A^* = 80$ |
| Final net resources controlled by $A$ and $B$ | $NR_A^* = NR_A^* = 140$ |

resource seized. This result is consistent with the equilibrium net resource values implied by equations (12.7a) and (12.7b). Table 12.1 summarizes the numerical example of the resource conflict model.

### *Paradox of Power and the Irrelevance of Initial Resource Holdings*

One might think that in a conflict over resources or goods, the poorer side would be at a disadvantage relative to the wealthier side. In a number of economic models of conflict, however, scholars have found what Hirshleifer called the paradox of power (POP). In the strong form of the POP, the players end up with identical amounts of the disputed item ($D_A^*/D_B^* = 1$) despite disparity of initial resource holdings (e.g., $R_A/R_B > 1$). In the weak form of the POP, the distribution of the disputed item is less dispersed than the initial distribution of resources (e.g., $1 < D_A^*/D_B^* < R_A/R_B$) (Hirshleifer 1995, p. 182).

In the resource conflict model developed here, a strong form of the POP is evident when $Z = 1$. Equations (12.6a) and (12.6b) imply that when $Z = 1$, each player ends up controlling the same amount of the disputed resource, regardless of initial holdings of secure resources. For example, if $\tilde{R} = 200$ and $\delta = 0.2$, in equilibrium each player will control 80 units of the remaining disputed resource, even if the secure resource holdings of $A$ and $B$ are unequal, say at $R_A = 100$ and $R_B = 50$. In this example we find that the disparity of initial resources in favor of $A$ ($R_A/R_B = 2$) does not translate into a greater share of the disputed resource controlled by $A$ in the conflict ($D_A^*/D_B^* = 1$).

The paradox of power does not hold generally when $Z \neq 1$. For example, suppose the parameters are $\tilde{R} = 200$, $R_A = 100$, $R_B = 50$, and $\delta = 0.2$, but now $Z = 1/3$. Equations (12.6a) and (12.6b) imply that $A$ will control 120 units and $B$ 40 units of the remaining disputed resource. Hence, contrary to the POP, the disparity of initial resources in $A$'s favor ($R_A/R_B = 2$) translates into an even greater disparity of the seized amounts of disputed resource in $A$'s favor ($D_A^*/D_B^* = 3$). Inspection of equations (12.6a) and (12.6b) reveals that the paradox of power is a product of the irrelevance of initial resources in determining the final distribution of the disputed item. Note that $R_A$ and $R_B$ do not appear in the two equations. In the resource conflict model presented here, the distribution of the remaining disputed resource in equilibrium is governed exclusively by relative military effectiveness $Z$. Specifically, equations (12.6a) and (12.6b) imply that the ratio of the amounts of the remaining disputed item controlled by $A$ and $B$ is $1/Z$. Hence, when $Z = 1$, disparity of initial resources in favor of $A$ ($R_A/R_B = 2$) corresponds to an equal distribution of the disputed resource ($D_A^*/D_B^* = 1$) because of the more general point that the final distribution is determined exclusively by the technology of conflict parameter $Z$.

## Settlement Opportunities in the Resource Conflict Model

To this point we have assumed that the players fight to determine control of the disputed resource. Given the destructiveness of conflict, however, each player can potentially gain from nonviolent settlement of the dispute. This is shown in Figure 12.4 using a linear version of Hirshleifer's bargaining model from Chapter 5, together with the parameters and equilibrium values of the resource conflict example in Table 12.1.

The horizontal axis in Figure 12.4 measures $A$'s net resources expected from fighting or settlement and the vertical axis does the same for $B$. If $A$ and $B$ fight, the net resources controlled by each player equal 140 units, as shown by point $E$ in the figure and the last row of Table 12.1. If fighting is avoided, however, $\delta\tilde{R} = 40$ units of the disputed resource will not be destroyed, which is a surplus available to the players from peaceful settlement. Assume for simplicity that under peaceful settlement the players distribute the disputed resource according to what Garfinkel and Skaperdas (2007, p. 674) call the "split-the-surplus rule of division." Under this division rule, the surplus from peaceful settlement $\delta\tilde{R}$ is split evenly, while the remaining disputed resource $(1-\delta)\tilde{R}$ is divided according to the players' military stocks and the conflict success function in equation



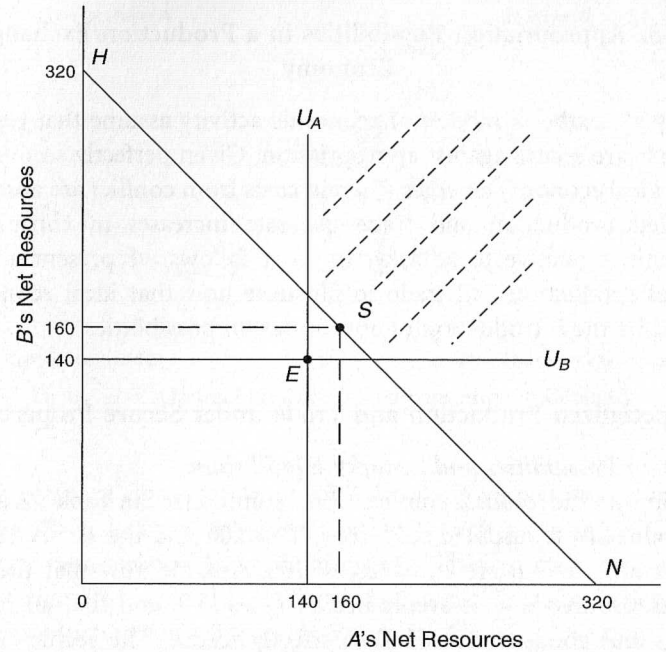


Figure 12.4. Resource conflict in Hirshleifer's bargaining model.

(12.1). Given the split-the-surplus division rule, each player's diversion of resources to military goods is the same whether they fight or settle (Skaperdas 2006, pp. 665–666). Based on Table 12.1 and the split-the-surplus division rule, each player will divert 40 units of secure resources to military goods under war or peace, but if war is avoided $\delta\tilde{R} = 40$ units of the disputed resource will not be destroyed. Hence, under peaceful settlement, 320 resource units will be available to the players, made up of 100 units of secure resources for each player, 200 units of disputed resources, less each player's diversion of 40 units of secure resources to military goods. Since 320 resource units are potentially available to the players' under peace, the settlement opportunity line $HN$ in Figure 12.4 has intercept values of 320.

Players are assumed to be strict egoists as indicated by their respective indifference curves $U_A$ and $U_B$ passing through point $E$. Since the settlement opportunity line intersects the region of mutual gain, the model predicts peaceful settlement over violence. Given that the players are fully informed, equally capable ($M_A^* = M_B^* = 40$ and $Z = 1$), and adopt the split-the-surplus division rule, the players are predicted to reach a peaceful settlement whereby each obtains 160 units of net resources, as shown by point $S$ in Figure 12.4.

## 12.3. Appropriation Possibilities in a Production/Exchange Economy

Virtually all textbook models of economic activity assume that resources and goods are secure against appropriation. Given perfectly secure property, an ideal economy emerges wherein costs from conflict are absent and specialized production and trade generate increases in consumption opportunities relative to autarky. In what follows we present a simple model of production and trade to illustrate how that ideal economy is reshaped by the introduction of appropriation possibilities.

### Specialized Production and Trade under Secure Property

*Production Possibilities and Autarky Equilibrium*

We begin with the resource conflict model summarized in Table 12.1, where the amount of the disputed resource is $\tilde{R} = 200$ and the secure resource holdings of $A$ and $B$ are $R_A = R_B = 100$. Assume now that the entire disputed resource is split evenly between $A$ and $B$ and that all resource holdings and goods produced are perfectly secure. The secure resource holdings of $A$ and $B$ are now $R_A = R_B = 200$. Given the assumption of perfect security, there is no incentive to produce military goods because there is no ability to take property from others and thus no need to defend. Hence, the full amount of the players' resources ($R_A = R_B = 200$) is available for producing goods.

We assume that the players can use their resources to produce two goods $X$ and $Y$. The production of each good is based on a production technology that specifies the number of units of resources required to produce one unit of a good. For example, let $a_X = 1$ and $a_Y = 2$ be player $A$'s unit resource requirements. These coefficients imply that $A$ needs one unit of resources to produce one unit of good $X$ and two units of resources to produce one unit of good $Y$. Hence, if $A$ allocated all 200 units of her resources to produce good $X$, she could produce 200 units of $X$. Alternatively, if $A$ allocated all 200 units of resources to produce good $Y$, she could produce 100 units of $Y$. Of course, $A$ might choose to produce some combination of both $X$ and $Y$. For example, if $A$ allocated half of her resources to the production of each good, she could produce $X_A = 100$ and $Y_A = 50$. For simplicity, assume player $B$'s unit resource requirements are the reverse of $A$'s, namely, $b_X = 2$ and $b_Y = 1$. Player $B$ could allocate all 200 units of his resources to produce good $X$ (giving $X_B = 100$) or good $Y$ (giving $Y_B = 200$), or he might divide his resources between the two goods to produce, say, $X_B = 50$ and $Y_B = 100$. In



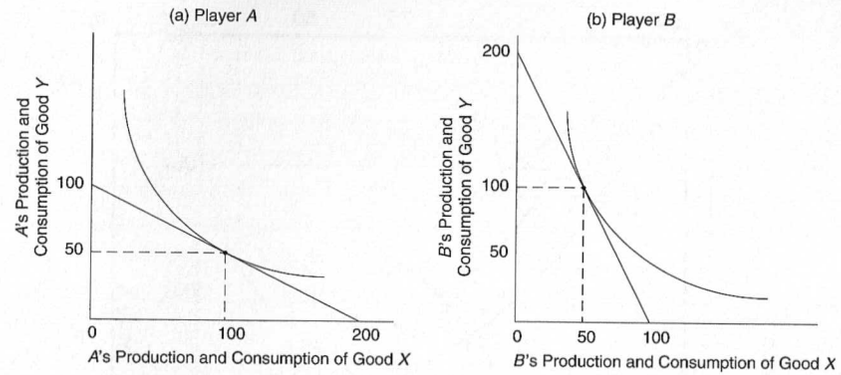


Figure 12.5. Optimal production and consumption in autarky.

general, player $A$'s production possibilities are governed by the constraint equation $R_A = a_X X_A + a_Y Y_A$ and $B$'s by $R_B = b_X X_B + b_Y Y_B$. Given the parameter values, player $A$'s production possibilities frontier (PPF) is shown by the straight line in panel (a) of Figure 12.5, while $B$'s is shown in panel (b).

The production possibilities frontiers in Figure 12.5 show possible production points for players $A$ and $B$, but not the specific production points that they would choose. To know where $A$ would like to operate on her PPF, we need to know $A$'s preferences over $X$ and $Y$, and likewise for $B$. For simplicity, assume that $A$ and $B$ have identical preferences represented by an equal weight Cobb-Douglas (CD) utility function $U = XY$. A convenient property of a CD utility function wherein each good is equally important is that a utility maximizer operating in autarky will allocate an equal amount of resources to each good. Hence, given $R_A = 200$, player $A$ will maximize utility in autarky by allocating 100 units of resources to produce good $X$ and 100 units of resources to produce good $Y$. With unit resource requirements $a_X = 1$, and $a_Y = 2$, this results in 100 units of good $X$, denoted $X_A^\wedge = 100$, and 50 units of good $Y$, denoted $Y_A^\wedge = 50$. Similarly for player $B$ with $R_B = 200$, $b_X = 2$, and $b_Y = 1$, $B$ will allocate 100 units of resources to the production of each good, leading to $X_B^\wedge = 50$ and $Y_B^\wedge = 100$. The determination of $A$'s and $B$'s optimal production and consumption in autarky are shown geometrically in Figure 12.5, where each player's indifference curve is tangent to her or his PPF.

*Gains from Trade*

Beginning from the autarky equilibrium in Figure 12.5, mutual gains are available to $A$ and $B$ from specialized production and trade. To

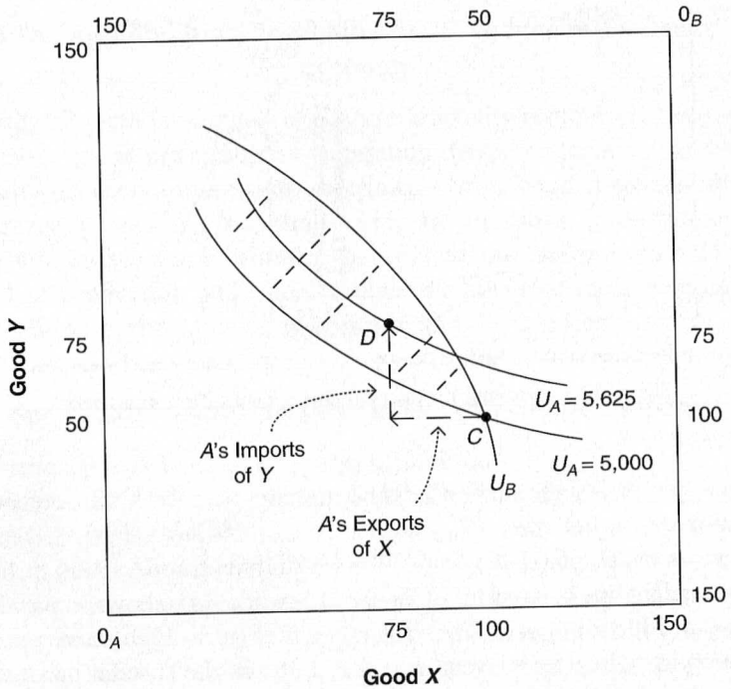


Figure 12.6. Gains from exchange in an Edgeworth box.

demonstrate this we rely on a graphical device known as the Edgeworth box. Figure 12.6 shows the box for the autarky equilibrium of Figure 12.5. The dimensions of the box reflect the total quantities of $X$ and $Y$ produced by $A$ and $B$ in autarky. Since $X_A^{\wedge} = 100$ and $X_B^{\wedge} = 50$ in Figure 12.5, the width of the box in Figure 12.6 is 150 units of $X$. Similarly, since $Y_A^{\wedge} = 50$ and $Y_B^{\wedge} = 100$ in Figure 12.5, the height of the box in Figure 12.6 is 150 units of $Y$. We measure $A$'s autarky production and consumption of $X_A^{\wedge} = 100$ and $Y_A^{\wedge} = 50$ in the usual manner from the lower-left origin $0_A$, leading to point $C$ in the Edgeworth box. For $B$'s autarky production and consumption of $X_B^{\wedge} = 50$ and $Y_B^{\wedge} = 100$, however, we measure left and down from the upper-right origin $0_B$. Because of the way the box is constructed, this places $B$'s autarky point also at $C$. In summary, the dimensions of the Edgeworth box in Figure 12.6 reflect the aggregate production of the two goods under autarky, while point $C$ reflects the distribution of this total production between $A$ and $B$.

Now consider $A$'s and $B$'s indifference curves passing through the autarky point $C$. From Figure 12.5, $A$'s indifference curve at the optimum



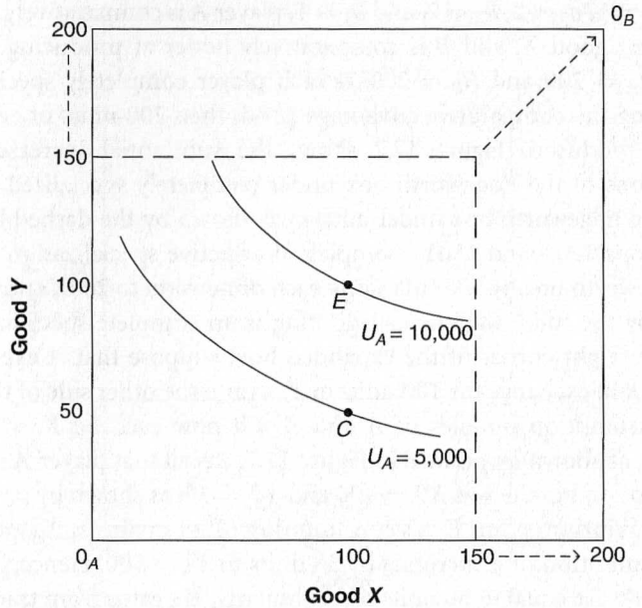


Figure 12.7. Gains from specialized production and exchange.

has a slope of $-1/2$ (equal to the slope of $A$'s PPF) while $B$'s has a slope of $-2$ (equal to the slope of $B$'s PPF). In Figure 12.6, these divergent slopes are shown by the intersection of $A$'s and $B$'s indifference curves at point $C$. Since the indifference curves cross at point $C$, a region of mutual gain arises to the northwest of $C$. Hence, both players have an incentive to work out a trade that moves them into this region. For example, beginning from point $C$, if $A$ were to export 25 units of $X$ to $B$ in exchange for 25 units of $Y$, the players' consumption bundles would be at point $D$. Both players would gain from this exchange. Player $A$'s utility would rise from $U_A = X_A Y_A = 100 \cdot 50 = 5{,}000$ in autarky at point $C$ to $U_A = X_A Y_A = 75 \cdot 75 = 5{,}625$ under trade at point $D$. Player $B$ would experience the same increase in utility when moving his consumption bundle from $C$ to $D$.

As noted in Chapter 2, there are generally two sources of increased wealth from trade: (1) gains from exchange and (2) gains from specialization. When moving from point $C$ to $D$ in Figure 12.6, we considered only gains from exchange. Specifically, we allowed the players to use exchange to redistribute their existing stocks of goods at point $C$ so that each was better off, but we did not allow them to alter production to take advantage of gains from specialization. Figure 12.7 shows what happens when we do.

Since $a_X = 1$, $a_Y = 2$, $b_X = 2$, and $b_Y = 1$, player $A$ is comparatively better at producing good $X$, and $B$ is comparatively better at producing good $Y$. Given $R_A = 200$ and $R_B = 200$, if each player completely specializes in producing the comparative advantage good, then 200 units of each good will be produced. Figure 12.7 shows the substantial increase in the dimensions of the Edgeworth box under completely specialized production. The Edgeworth box under autarky is shown by the dashed box with dimensions $150X$ and $150Y$. Complete productive specialization expands the Edgeworth box by 50 units along each dimension to $200X$ and $200Y$, as shown by the solid-lined box. Beginning from complete specialization at the lower-right corner of the expanded box, suppose that $A$ exports 100 units of $X$ in exchange for 100 units of $Y$, with $B$ the other side of the trade. The consumption bundles of $A$ and $B$ will now each be $X = 100$ and $Y = 100$, as shown by point $E$ in Figure 12.7. Recall that player $A$'s autarky consumption bundle was $X_A^{\wedge} = 100$ and $Y_A^{\wedge} = 50$, as shown by point $C$. At trade equilibrium point $E$, $A$'s consumption of $X$ remains at $X_A = 100$, but $A$'s consumption of $Y$ increases by 50 units to $Y_A = 100$. Hence, $A$'s gains from trade are equal to 50 units of $Y$. Similarly, $B$'s gains from trade are 50 units of $X$. The total gains from trade of $50Y$ and $50X$ are also reflected in the expansion of the Edgeworth box by these same amounts when moving from autarky to specialized production and trade. Note also that $A$'s utility rises from $U_A = 100 \cdot 50 = 5{,}000$ in autarky to $U_A = 100 \cdot 100 = 10{,}000$ under specialized production and trade. Player $B$ experiences the same increase in utility.

## Insecure Resources and Dissipation of the Production/Exchange Economy

We now consider how conflict radically changes the idealized production/exchange economy of Figure 12.7 by reintroducing the numerical example of resource conflict from Table 12.1. In the example, the amount of disputed resource is $\tilde{R} = 200$, $A$ and $B$ have respective secure resource holdings $R_A = R_B = 100$, the destructiveness of conflict is $\delta = 0.2$, and the relative military effectiveness is $Z = 1$. Whether there is fighting or a split-the-surplus settlement, each player diverts 40 units of secure resources to military goods. Under fighting, each player controls 140 units of resources, made up of the 60 units of secure resources not diverted to military goods plus 80 units of the remaining disputed resource claimed in the fight. Under settlement, each player controls 160 units of resources, consisting of 60 units of secure resources not diverted to military goods plus 100 units of



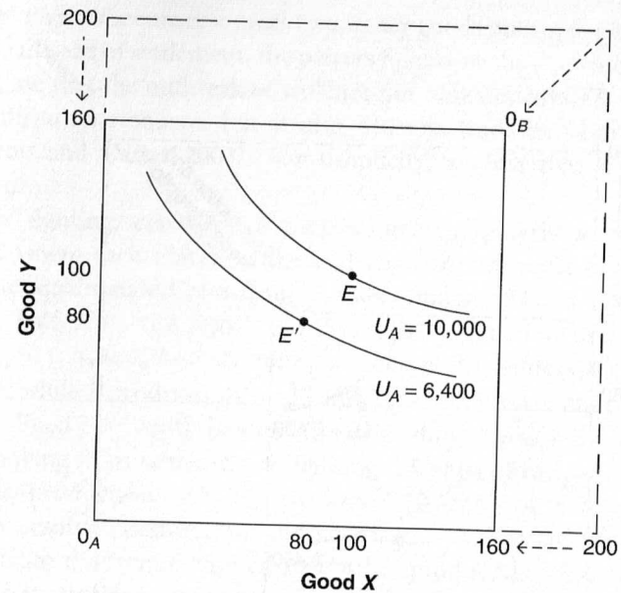


Figure 12.8. Effects of diversion of resources to military goods.

the disputed resource acquired in the settlement. For simplicity, we assume that potential gains from trade between $A$ and $B$ and war's diminution of such gains do not alter the parameters or equilibrium values of the resource conflict model in Table 12.1. This assumption allows us to illustrate, in the simplest way possible, how a resource conflict undermines the idealized production/exchange economy.

### *Diversion*

We repeat the idealized production/exchange economy as the large dashed Edgeworth box in Figure 12.8 with dimensions $200X$ and $200Y$. For reference purposes, we show again $A$'s indifference curve through consumption point $E$ with utility level $U_A = 100 \cdot 100 = 10{,}000$. From the resource conflict model, assume that the players reach a settlement so that the destructiveness of conflict is avoided. Under settlement, each player allocates 40 units of secure resources to military goods and controls 160 units of resources on net. Since violence is avoided, we assume that the players are able to maintain a trading relationship. The diversion of resources to military goods, however, shrinks the dimensions of the Edgeworth box as shown in Figure 12.8. Because specialized production and trade continue under settlement, $A$ produces $160X$ and $B$ produces

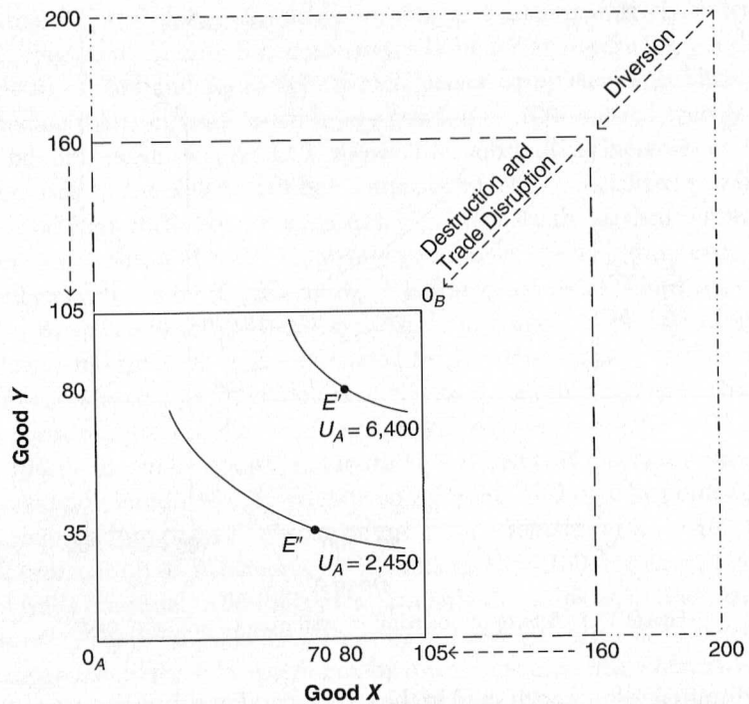


Figure 12.9. Effects of destruction and trade disruption.

160$Y$, causing the solid-lined box to emerge with $A$'s origin remaining fixed but $B$'s origin shifting inward. Suppose that $A$ exports 80$X$ in exchange for 80$Y$ with $B$ the other side of the trade. Thus, specialization and trade result in consumption bundles for both $A$ and $B$ at $X = 80$ and $Y = 80$, as shown by point $E'$ in the reduced Edgeworth box. Note that diversion of resources to military goods causes the Edgeworth box to shrink by 40 units along each dimension, with the result that both players consume 20 fewer units of each good relative to what they would in the idealized economy with perfectly secure property. As shown in Figure 12.8, $A$'s utility falls from $U_A = 10{,}000$ to $U_A = 80 \cdot 80 = 6{,}400$, and similarly for $B$.

### *Destruction and Disruption*

Figure 12.9 shows the effects of resource destruction and trade disruption when violence erupts. We repeat the idealized production/exchange economy as the large dashed-and-dotted Edgeworth box with dimensions 200$X$ and 200$Y$. Within the large box is the dashed Edgeworth box with dimensions 160$X$ and 160$Y$, which recall results when the players each



divert 40 units of secure resources to military goods but reach a settlement. Suppose instead of settlement, the players fight over the disputed resource. We assume that the outbreak of violence not only destroys $\delta\tilde{R} = 40$ units of the disputed resource, but it also disrupts trade between $A$ and $B$ (Anderton and Carter 2003). For simplicity, assume that trade ceases altogether.

Under fighting, cessation of trade causes the players to operate in autarky, where each player in the end controls 140 units of resources. Given the equal weight Cobb-Douglas utility function $U = XY$, each player allocates half of its 140 resource units to the production of each good. Given $a_X = 1$, $a_Y = 2$, $b_X = 2$, and $b_Y = 1$, player $A$ produces $X_A^{\wedge} = 70$ and $Y_A^{\wedge} = 35$, while $B$ produces $X_B^{\wedge} = 35$ and $Y_B^{\wedge} = 70$. Hence, under fighting, the solid-lined Edgeworth box emerges with dimensions 105$X$ and 105$Y$. When moving from settlement to fighting, 55 units of each good are lost from the production/exchange economy: 20 units of each good are lost from resource destruction, and an additional 35 units of each good are lost from the termination of specialized production. Since trade has ceased, consumption occurs at production point $E''$. Comparing points $E'$ (with settlement) and $E''$ (with fighting) reveals that player $A$'s utility falls from $U_A = 80{\cdot}80 = 6{,}400$ to $U_A = 70{\cdot}35 = 2{,}450$. Note that the amount of goods available to the players in *total* under fighting (105$X$ and 105$Y$) in Figure 12.9 is only slightly larger than the amount of goods consumed by *one* player (100$X$ and 100$Y$) in the idealized Edgeworth box economy in Figure 12.7. Polachek (1994, p. 12) characterizes violent conflict as "trade gone awry," and so it is in Figure 12.9.

## Appropriation Possibilities and Equilibrium in a Production/Exchange Economy

Figure 12.9 suggests that the textbook model of peaceful economic activity is but a special case of a more general model wherein appropriation possibilities both shape and are shaped by the traditional economic activities of production and trade. At one extreme of this general model, appropriation possibilities are ignored and the full potential of specialized production and trade is realized. This is the approach taken in standard economics texts. At the other extreme, gains from trade are ignored under actual or threatened violent conflict. Many theoretical models of conflict ignore potential gains from trade. Between these extremes lies a wide range of human behavior where specialized production and trade occur, but they are radically modified by appropriation possibilities. To show further the

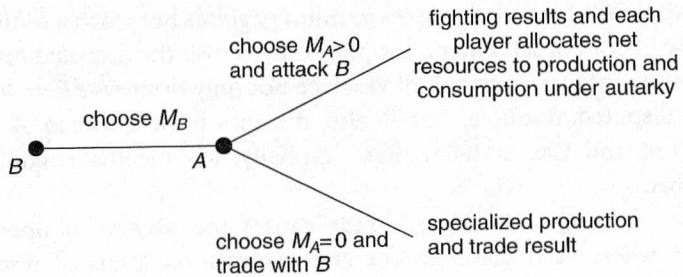


Figure 12.10. Predator/prey game.

interdependence of appropriation, production, and trade, and also to illustrate the emergence of equilibrium out of this interdependence, we conclude with a few numerical examples based on a model due originally to Anderton et al. (1999).

### *A Predator/Prey Model of Appropriation, Production, and Exchange*

We begin again with the resource conflict example, where the amount of the disputed resource is $\tilde{R} = 200$, the secure resource holdings of $A$ and $B$ are $R_A = R_B = 100$, and the destructiveness of fighting is $\delta = 0.2$. Assume now that the disputed resource is split evenly between $A$ and $B$ such that $R_A = R_B = 200$. Unlike earlier, assume that player $A$'s resource holding is secure but $B$'s resource holding is vulnerable to attack by $A$. This assumption casts player $A$ in the role of attacker or predator and player $B$ as defender or prey. One possible outcome of the predator/prey relationship between $A$ and $B$ is that a fight will ensue over $B$'s vulnerable resource holding. Another possibility is that $A$ and $B$ will avoid fighting and instead engage in specialized production and trade. For simplicity, assume that an attack by $A$ against $B$'s resources precludes the possibility of specialized production and trade. Moreover, suppose that a peaceful settlement of the predation is not possible, perhaps owing to a commitment problem.

Figure 12.10 provides a schematic of the predator/prey game. Player $B$ moves first, diverting some of its resources to produce military goods $M_B$ with which it defends its remaining vulnerable resources. Player $A$ moves second, taking as given $B$'s stock of military goods $M_B$. In particular, player $A$ either diverts some of its resources into military goods ($M_A > 0$) and attacks $B$'s remaining resources, or it produces no military goods ($M_A = 0$) and engages in specialized production and trade with $B$. The combined decisions of $A$ and $B$ result in either fighting or specialization and trade, as shown by the top and bottom branches of the game tree, respectively.



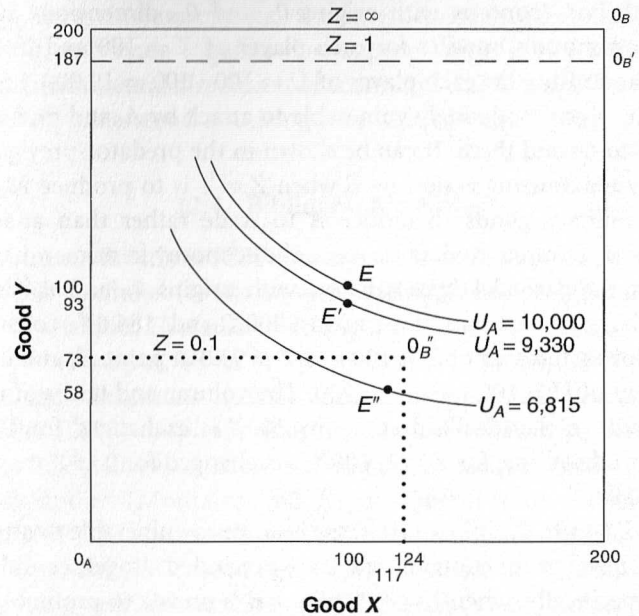


Figure 12.11. Equilibrium economies in the predator/prey game.

As the first mover, player $B$ anticipates $A$'s reaction and chooses a level of military goods that brings about the state of the world (fighting or specialization and trade) that yields $B$ the higher utility. When profitable to do so, player $B$ chooses a level of military goods that defends its resources to the point that $A$ prefers to trade with rather than to attack $B$. When it is not profitable to induce trade, $B$ chooses a level of military goods that minimizes its loss from $A$'s attack. If fighting occurs, the ratio conflict success functions, $p_A = M_A/(M_A + ZM_B)$ and $p_B = ZM_B/(M_A + ZM_B)$, determine the proportion of $B$'s surviving net resources $(R_B - M_B)(1 - \delta)$ claimed by $A$ and $B$, respectively. Following the fight, the players use their respective net resource holdings in autarky to produce goods $X$ and $Y$ according to the unit resource requirements $a_X = 1$, $a_Y = 2$, $b_X = 2$, $b_Y = 1$. If trade occurs, $A$ specializes in good $X$, $B$ in good $Y$, and trade ensues.

Under these assumptions, what type of economy will emerge in equilibrium, and what will be its production, consumption, and utility characteristics? In the predator/prey game, the $Z$ parameter in the conflict success functions reflects the security of $B$'s resource holdings. Figure 12.11 illustrates the equilibrium economies that emerge for $Z = \infty$, $Z = 1$, and $Z = 0.1$. For $Z = \infty$, $B$'s resources are perfectly secure, so neither player has an incentive to produce military goods. This generates the idealized

Edgeworth box economy with origins $0_A$ and $0_B$, dimensions $200X$ and $200Y$, consumption bundles for each player of $X = 100$ and $Y = 100$ at point $E$, and utility for each player of $U = 100 \cdot 100 = 10{,}000$. For $Z = 1$, $B$'s resources are moderately vulnerable to attack by $A$, and thus $B$ has an incentive to defend them. It can be shown in the predator/prey game that the utility maximizing action by $B$ when $Z = 1$ is to produce $M_B = 13.4$ units of military goods to induce $A$ to trade rather than attack. This diversion of resources to defense alters the economy in numerous ways, as shown by the dashed Edgeworth box with origins $0_A$ and $0_B'$ in Figure 12.11. The new box has dimensions $200X$ and $186.6Y$, consumption bundles for each player of $X = 100$ and $Y = 93.3$ at point $E'$, and utility for each player of $U = 100 \cdot 93.3 = 9{,}330$. The volume and terms of trade are also altered. In the idealized economy $100X$ is exchanged for $100Y$ at a terms of trade of one; for $Z = 1$, $100X$ is exchanged for $93.3Y$ at a terms of trade of 0.93.

When $Z$ falls to 0.1, player $B$'s resources are so vulnerable to attack by $A$ that specialized production and trade are precluded altogether and fighting ensues. Specifically, when $Z = 0.1$, player $B$ is unable to profitably induce player $A$ to prefer to trade. The best that $B$ can do is to defend itself with $M_B = 100$ units of military goods and fight it out with $A$. Player $A$'s optimal allocation for military goods is $M_A = 18.3$. Following the fight, the players' net resource holdings are $NR_A = 233.4$ and $NR_B = 28.3$, which lead to autarky production and consumption bundles of $X_A^{\wedge} = 116.7$, $Y_A^{\wedge} = 58.4$, $X_B^{\wedge} = 7.1$, and $Y_B^{\wedge} = 14.2$. Hence, when $Z = 0.1$, the dotted Edgeworth box emerges in Figure 12.11, with origins $0_A$ and $0_B''$, dimensions $123.8X$ and $72.6Y$, autarky consumption bundles shown at $E''$, and much reduced utilities of $U_A = 116.7 \cdot 58.4 = 6{,}815$ for the predator and $U_B = 7.1 \cdot 14.2 = 101$ for the prey.

*Discussion*

The broad lesson of Figure 12.11 is that appropriation, production, and trade are indeed deeply intertwined: appropriation possibilities determine the security of property on which specialized production and trade depend, while at the same time the production and trade possibilities shape the incentives for appropriation. Modern physics provides an analogy to the interconnectedness of appropriation, production, and trade in the economic realm. In the physical universe, space, time, and matter are profoundly intertwined because matter alters space-time and space-time in turn alters the paths of matter. Hence, the fundamental categories of physical reality are understood integrally in the general theory of relativity. In a similar manner, the interdependence of appropriation, production, and trade requires that appropriation join production and trade as a fundamental dimension of economic activity, and that economic outcomes be understood as arising from this economic triad.



## 12.4. Bibliographic Notes

Conflict success functions (CSFs) have been used in many areas of conflict analysis. See Garfinkel (1990) on arms racing, Nalebuff and Riley (1985) on wars of attrition, Garfinkel and Skaperdas (2000b) on conflict over the distribution of output, Usher (1989) and Collier (2000b) on civil conflict, Konrad and Skaperdas (1998) on extortion, and Anderson and Marcouiller (2005) on piracy. Ratio and logistic CSFs have been used to construct alternatives to the ethnolinguistic fractionalization index in empirical studies (Montalvo and Reynal-Querol 2005, Cederman and Girardin 2007). For general theoretical formulations of the CSF, see Dixit (1987), Skaperdas (1992), and Neary (1997). For an intuitive and graphical review of ratio and logistic CSFs, see Hirshleifer (1995). Skaperdas (1996) derives general and specific CSFs from easily interpretable axioms. A valuable review of theoretical properties of CSFs is provided by Garfinkel and Skaperdas (2007).

While previous chapters of this book emphasized the use of economic principles and variables in understanding conflict, this chapter added the theme that appropriation stands coequal with production and trade as a fundamental category of economic activity. This relatively new branch of conflict economics grows out of the seminal work of Bush (1972), Brito and Intriligator (1985), Hirshleifer (1988, 1991), Usher (1989), Garfinkel (1990), Skaperdas (1992), and Grossman and Kim (1995), who developed models of appropriation and production. Experimental tests of models with production and appropriation include Durham, Hirshleifer, and Smith (1998), Carter and Anderton (2001), and Duffy and Kim (2005). More recent work has introduced appropriation into models of exchange. See, for example, Anderton et al. (1999), Rider (1999, 2002), Skaperdas and Syropoulos (2002), Hausken (2004), Anderson and Marcouiller (2005), Garfinkel and Skaperdas (2007, pp. 682–690), and Anderton and Carter (2008b).

The results of conflict models are often surprising based on intuitions that tend to ignore appropriation possibilities. For example, in Hirshleifer's (1991) paradox of power model, agents with equal output productivities but unequal resource holdings can end up with an equal share of disputed

output. Skaperdas (1992) and Skaperdas and Syropoulos (1997) find that the more productive player in a conflict model can receive a smaller share of disputed output, even when the players have equal resource endowments. In a model of trade and appropriation, Anderton (2003) finds that increases in the productivity of each player in its area of comparative advantage can cause aggregate output to fall.

# APPENDIX A

## Statistical Methods

Statistical analysis involves populations, samples, luck of the draw, and systematic inferences. This appendix provides an informal and somewhat intuitive introduction to the basic statistical methods used in conflict economics.

### A.1. Populations and Samples

A population is just a collection of relevant objects. A simple example might be the citizens of a country. Associated with the citizens are variables that measure attributes like income, age, gender, political opinion, and so on. Thus, populations can be thought of as collections of values of various variables of interest. Summary measures of these population values are called parameters. Suppose the variable of interest is income. Then the parameters might include the mean $\mu$ and the standard deviation $\sigma$ of income. The mean is just the arithmetic average of the citizens' incomes. The standard deviation measures dispersion and indicates how far the income values typically lie above or below the mean. Alternatively, suppose the variable of interest is citizens' political opinion. Then a key parameter might be the proportion $\pi$ of citizens who oppose the current regime.

While the full population is of ultimate interest, researchers usually work with a subset of the population called a sample. To the extent that the sample is representative, it provides useful information about the larger population. Corresponding to population parameters are sample statistics. Just as parameters summarize the values in the population, statistics summarize the values in a sample. Thus, corresponding to the population mean $\mu$, population standard deviation $\sigma$, and population proportion $\pi$ are the sample mean $\bar{X}$, sample standard deviation $S$, and sample